Modeling of Slag Entrainment and Interfacial Mass Transfer in Gas Stirred Ladles

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(Received on October 13, 2016; accepted on July 7, 2017)

In gas stirred steelmaking ladles, entrainment of liquid slag into liquid steel and vice versa takes place. Qualitatively, slag entrainment is known to increase slag-metal interfacial reaction rates. However, it is unknown how the entrainment is quantitatively affected by gas stirring rate and the properties of the slag and metal. In this paper the modelling of slag entrainment is resolved into several sub-models to determine the interfacial reaction rates between slag and metal for the droplets and other interfaces. The model predictions are in line with literature values for refining rates and also rationalize the effects of ladle size and number of gas injection plugs.

KEY WORDS: secondary steelmaking; slag entrainment; large eddy simulation; interfacial mass transfer; Gerris flow solver.

1. Introduction

Ladle Metallurgy is the step after primary steelmaking to fine tune the steel to the desired composition before casting. Typically argon gas is injected from the base of the ladle to stir the liquid steel that is also covered by a synthetic slag, as shown in Fig. 1; this figure also has various nomenclature used in this paper. The injected gas breaks into bubbles and rise along with the liquid metal which is called a plume. The uprising plume pushes the top slag radially creating a opening called the ‘slag eye’. At the eye, the gas in the plume escapes creating a ‘spout’ while the liquid metal flows radially and then downwards near the slag metal interface. The relative motion between the slag and metal in the eye region causes entrainment of slag into metal. As in the classic plot of gas flow rate vs interfacial mass transfer rate of Ishida et al.1 in Fig. 2 and through many other water modeling studies, the sudden increase in mass transfer rate has been shown to be due to the increase in slag metal interfacial area caused by slag entrainment. Despite this beneficial effect, entrainment can potentially lead to metal cleanliness and yield loss issues.

An extensive review of the past work to characterize the entrainment phenomena can be found elsewhere.2 However, they can be summarized as follows: significant effort has been dedicated, but limited mostly, to finding the critical conditions for the onset of entrainment of the upper phase, namely the critical velocity near the interface and the critical gas injection rate. Much work needs to be done to characterize the birth rate, size and residence time of entrained droplets, beyond the critical conditions. From numerous mass

Fig. 1. Schematic of gas stirred ladle showing various fluid zones. Arrows in the slag eye region depict the liquid flow direction. LP—lower phase; UP—upper phase; ULflat and ULeye are the interfaces between LP and UP at more or less horizontal and slag eye regions, respectively.

Fig. 2. Plot of mass transfer rate constant (k) versus specific gas flow rate (Q) obtained from a 2.5 ton converter.3

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1962
transfers the role of entrainment on mass transfer is well understood in water models; however, the rate parameters have not been related to entrainment parameters, the fluid properties and the scale of the system. Developments in numerical simulations of the two phase flow phenomena is still in its infancy due to the lack of modeling techniques, robust representation of interface topology, stable and efficient solvers and finally necessary computational resource to study entrainment in full scale industrial systems.

In the present work, to characterize slag entrainment in metal, the ‘degree of entrainment’ is defined through the following parameters:

1. Rate of entrainment, which is percentage volume of upper phase entrained per unit time, $V_{PU}$,
2. Size distribution of entrained drops and
3. Residence time distribution of entrained drops.

To date, there are no reliable methods to measure these parameters even in water models, let alone actual ladles. So Computational Fluid Dynamics (CFD) based modeling appears to be the only viable route. However the wide range of scales (from ladle size to droplet size) demands prohibitive computational resources. So it was decided to limit the CFD simulation only to estimate the entrainment rate and droplet size distribution in the vicinity of entrainment, which allows a reduced simulation domain; use a Lagrangian particle tracking method to estimate residence times for various droplet sizes using the mean flow field obtained from the inexpensive Reynolds Averaged Navier-Stokes (RANS) simulation for the entire ladle; finally, use a kinetic model to compute interfacial mass transfer rates using the entrainment parameters estimated through above steps.

This rigorous approach is first of its kind wherein all the entrainment parameters are estimated through dedicated models. The model results show the dramatic effect that stirring gas flow rate and slag-metal interfacial tension have on droplet formation. Further they show good agreement with the interfacial mass transfer rates observed in full-scale ladle and also present logical explanations to long-standing questions regarding scale-up from small scale ladles, dependence of the mass transfer rate on the number of stirring plugs and the onset of droplet formation found in systems of different scale in the literature. The present paper summarizes the models and their results, while more details such as specifics of the computational methods, testing and validation against other available models and data are available in the open access theses repository, MacSphere alongside videos of the results of LES model for the interested reader. It is noteworthy that a parallel CFD study has been undertaken by Sulasalmi et al. to model slag entrainment and hence predict reduction rates in a CAS-OB process.

2. Mathematical Models

As mentioned, this complex problem is broken down into several sub models which are brought together for calculation of the interfacial mass transfer rate from slag to metal.

1. Drop by Drop Kinetic Model (DBD) to calculate the rate of overall mass transfer due to the various slag metal interfaces in Fig. 1 with values of entrainment parameters, interfacial area and mass transfer coefficients as inputs.
2. Multiphase Large Scale Eddy Simulations (LES) to calculate the entrainment rate, droplet size distribution and interfacial area at the slag eye.
3. Lagrangian Particle Tracking (LPT) to calculate the trajectory and residence time of droplets created in the LES.
4. Mass Transfer Rate Computations to calculate the overall slag metal interfacial mass transfer rate using the parameter values obtained in the LES and LPT models into the DBD model.

2.1. Drop by Drop Kinetic Model (DBD):

This kinetic model calculates mass transfer rate at various distinct interfaces namely, the bulk upper phase ($U_{Leye}$ and $U_{Lflat}$, see Fig. 1) and the entrained droplets of upper phase. Apart from the parameters for degree of entrainment, the other input parameters are 1) mass transfer coefficients $k_{flat}$, $k_{eye}$, and $k_{drop}$ – one for each mass transfer zone 2) partition coefficient, $L_S$, for distribution of a species $(S)$ between two phases and 3) system dimensions, namely vessel radius and heights of lower phase (LP) and upper phase (UP).

The important steps in the model, executed for every time step, are

1. Birth of droplets, decided by the entrainment rate and input size distribution.
2. Death of droplets (droplets rejoining their parent phase), decided by droplet residence times.
3. Mass transfer at every distinct interface computed from,

$$\frac{d(S)}{dt} = k_L A \left[ (S) - \frac{(S)}{L_S} \right] , \quad k_L = k_{flat}, k_{eye}, \text{ and } k_{drop}, \ldots (1)$$

assuming metal side mass transfer control.

4. Mass balance that accounts for the loss and gain of species at LP and UP.

In a parametric study conducted to assess the model capability, various trends of input parameters were considered, particularly how they would vary with gas injection rate, as shown in Fig. 3. For each combination of parameters and with fixed distributions for size and mean residence time of droplets (in Fig. 4), mass transfer rates were computed and plotted against trial numbers in Fig. 5. Three regimes of mass transfer rate are delineated: Regime I is before any entrainment, Regime II is due to the entrainment of UP and Regime III occurs when the entire bulk UP is entrained into the LP as observed in past work. These results demonstrate the capability of the model to calculate the mass transfer rate incorporating the degree of entrainment.

![Fig. 3. Parameter values considered for the parametric study of the DBD model.](image-url)
2.2. Multiphase Large Eddy Simulations (LES)

The aim of these simulations is to obtain estimates for entrainment rate and droplet size distribution for various gas injection rates and physical properties. Due to very large grid requirements, these simulations were performed in a confined thin slice domain in a gas stirred ladle as shown in Fig. 6. The inlet conditions to this thin slice domain were derived from the inexpensive RANS simulations of steel flow in the whole ladle, although in 2D (axisymmetric) coordinates due to the centric gas injection considered in the present work. These RANS simulations use the quasi single phase model (QSP) applied in earlier work\(^9\)–\(^{13}\) and the models of Krishnapisharody and Irons\(^{14}\) for gas fraction distribution in plume; therefore they are referred to as QSP-RANS hereafter. Although the QSP-RANS is essentially single phase, the presence of the slag phase and spout were incorporated into the 2D domain, as fixed boundaries but with a free surface boundary condition. The eddy viscosity of the RANS model was obtained using the SST \(k-\epsilon\) model of Menter\(^{15}\) with a free surface correction applied in the spout and slag eye regions. The resulting domain and flow field are shown in Fig. 11(a). Profiles of mean velocities were extracted from the QSP-RANS at locations corresponding to the location of inlet plane of the thin slice domain in the ladle.

Apart from the mean velocities the inlet conditions for LES domain also require the fluctuating velocities; these were generated using the synthetic eddy method (SEM) of Jarrin\(^{16}\) with the inputs to SEM derived from the QSP-RANS fields. The sum of the mean and fluctuating velocities gives the instantaneous velocity conditions to the inlet of LES domain.

Both QSP-RANS and LESs were performed using the open source CFD software, Gerris Flow Solver.\(^{17,18}\) It was primarily chosen for its ‘accurate surface tension model’ implemented within the Volume of Fluid framework to track multiphase interfaces precisely and its adaptive mesh refinement capability, which can reduce the number of grid points by an order of magnitude for multiphase flows. However Gerris does not have any turbulence model and must be implemented by the user, as required. Gerris being a generic Navier-Stokes equations solver, can also solve any number of scalar transport equations with various user defined source terms and diffusion coefficients. Thus RANS simulations were performed in Gerris by solving the transport equations of \(k\) and \(\epsilon\) of the SST \(k-\epsilon\) model, to supply the eddy viscosity to the momentum equations. Similarly for LES, the algebraic model of Smagorinsky-Lilly\(^{19}\) was implemented to calculate the eddy viscosity for the momentum equations. Moreover, SEM was implemented using various user defined functions in Gerris. The implementation of models in Gerris for QSP-RANS, LES and SEM were all verified against standard benchmark cases.\(^{2}\)

For the entrainment simulations, first the QPS-RANS simulations were performed for three gas injection rates (\(Q_{sp}\)), namely 1, 2 and 4 lpm(STP)/ton of steel, for a steel height and ladle diameter of 2 m each (44 ton capacity of liquid steel). Other parameters are tabulated in Table 1.

These parameters were used to estimate the slag eye radius using previous work\(^{20}\) and resulting slag thickness based on slag volume conservation, for each \(Q_{sp}\). Using these param-

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\(^{1}\) ‘ton’ used throughout this paper is an abbreviation of tonne, which is 1 000 kg
eters and the RANS profiles, LES cases were run in a thin-slice domain of 0.1 m width for three values of interfacial tension ($\sigma$), slag viscosity and $Q_{sp}$, all tabulated in Table 2.

Using Gerris commands the volume of entrained slag droplets were computed at regular time intervals. The resulting volumetric entrainment rates were scaled up from the thin slice domain to the cylindrical ladle (defined later in Eq. (3)). From LES trials for various combinations of parameters, the following conclusions were drawn:

1. The entrainment rate is predominantly affected by the gas injection rate with a smaller role of interfacial tension.
2. The droplet size statistics are mainly affected by interfacial tension alone.
3. Slag viscosity has only a minor effect on entrainment characteristics.

The topologies of the slag metal interface in Fig. 7 show increasing interface deformation with increasing $Q_{sp}$. The quantitative relation between $V_{PU}$ and $Q_{sp}$ will be shown in next section.

### 2.3. Entrainment Rate Correlation

Fluid velocities were ensemble averaged ($\langle U \rangle$, $\langle V \rangle$) in a volume of cuboid enclosing $UL_{eye}$, for various LES cases by post processing the simulation files at numerous time instances. Profiles of norm of ensemble averaged velocities ($v = \sqrt{(\langle U \rangle)^2 + (\langle V \rangle)^2}$) as a function of distance to $UL_{eye}$ are plotted in Fig. 8 for three $Q_{sp}$ values at $\sigma = 1.0 \text{ N/m}$. The aspect that the horizontal flow entering $UL_{eye}$ must be impeded by and directed downward at $UL_{eye}$ is well captured in these profiles, seen as vanishing horizontal velocities near $UL_{eye}$ and increasing vertical velocities in LP as $UL_{eye}$ is approached. The ensemble averaged velocities were further line averaged in the LP side ($UL_{eye}$) whose values are also shown in the labels of Fig. 8. Using the same procedure, values of $\beta_{UL}$ were obtained for various other LES cases performed with varying interfacial tension, gravity and slag density; the LES cases with varying gravity and slag density were performed at 2 lpm/ton and 1.0 N/m. A strong correlation for the entrainment rate was deduced as,

$$V_{PU} = 81.16 \left[ \frac{v_{UL_{eye}} \sqrt{A_{UL_{eye}}^{nom}}}{(g\Delta \sigma)^{0.25}} \right] - 0.460, \quad R^2 = 0.985 \quad \ldots \quad (2)$$

where,

$$V_{PU}(\% \text{ / s}) = \left( \frac{1}{l_{UL_{eye}}} \sum_{t} \left[ \frac{100}{\pi R_{UL_{eye}} h_{UL_{eye}}^{nom}} \left| \frac{A_{UL_{eye}}^{nom}}{A_{UL_{eye},LES}} \right| \right] \right)^{\frac{1}{2}}$$

$$A_{UL_{eye}}^{nom} = \pi (2R_{eye} + h_{UL_{eye}}) \tan \theta_{UL_{eye}}) h_{UL_{eye}} l \cos \theta_{UL_{eye}} \quad \ldots \quad (4)$$

$$A_{UL_{eye},LES} = \pi (2R_{eye} + h_{UL_{eye}}) \tan \theta_{UL_{eye}}) h_{UL_{eye}} l \cos \theta_{UL_{eye}} \quad \ldots \quad (5)$$

<table>
<thead>
<tr>
<th>Table 2. Various values considered for physical parameters.</th>
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<tr>
<td>Gas injection rate, $Q_{sp}$, lpm/ton</td>
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<tr>
<td>Gas flow rate, lpm</td>
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<tr>
<td>Slag eye radius, $R_{eye}$, m</td>
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<td>Actual slag thickness, $h_{UL_{eye}}$, m</td>
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<td>Interfacial tension, $\sigma$, N/m</td>
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<td>Slag viscosity, Pa.s</td>
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<td>Slag density, $\rho_{U}$, kg/m$^3$</td>
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<td>Gravity, m/s$^2$</td>
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Fig. 8. Profiles of norm of ensemble averaged velocities, $v = \sqrt{(\langle U \rangle)^2 + (\langle V \rangle)^2}$ (left) and plot of $\langle U \rangle$ and $\langle V \rangle$ vectors (right) across $\eta$, the normal distance to $UL_{eye}$, $\eta < 0$ is UP and is $\eta > 0$ LP.
\[ \Delta \rho = \rho_L - \rho_U; \theta_{U_{\text{eye}}} \] is the inclination of \( U_{\text{eye}} \) with respect to the vertical axis and \( t_{\text{tot}} \) is the total simulation time.

It is interesting to note that the group, \( (g \Delta \rho \sigma)^{0.25} \) in Eq. (2) has occurred several times in the literature,\(^{7,21-23}\) modeling the critical conditions for onset of entrainment, as mostly directly proportional to a velocity scale, a feature that is also present in Eq. (2). The intercept in Eq. (2) indicates the existence of a critical velocity (or gas flow rate) for the onset of entrainment. Upon setting \( V_{\text{PU}} \) to zero and finding \( v_L \) for various values of \( \sigma \), it was found that the critical gas flow rate is approximately 90 lpm which is reasonable considering the range of gas flow rates tabulated in Table 2. Apart from these results, the strong correlation for \( V_{\text{PU}} \) shows that, despite the smaller LES domain the physics of the entrainment process is well captured, which supports the present LES approach.

2.4. Lagrangian Particle Tracking (LPT)

In the LES model the slag droplets were removed shortly after their births to facilitate counting. To calculate their residence times this LPT model was developed to follow the droplets path through the steady state flow field (RANS) of metal phase. The motion of a large number of slag droplets of various sizes was tracked based on the forces acting on the droplets and their interaction with the flow field in which they are entrained. The time span between the onset of entrainment and when the droplets rejoin the bulk slag phase at \( U_{\text{flat}} \) was defined as the residence time. From the average of residence times for each size the mean residence time was obtained. The forces on the droplets considered were buoyancy, drag, added mass and force due to pressure gradient in the flow field. The 2D (axisymmetric) LPT model, developed using C programming language, was verified against benchmark cases of Maxey.\(^{24}\) The influence of stochastic nature of fluid turbulence on droplet motion was accounted by a ‘varied eddy interaction model’ of Wang and James.\(^{25}\)

The steady state flow field for LPT should correspond to that used to supply input conditions for the LES. However, it was later realized that, unlike in LES, the corresponding QSP-RANS field did not have flow separation in the downstream steel-flow past the \( U_{\text{eye}} \). To achieve flow separation in the RANS field a small protruding ‘lip’ structure was introduced at the intersection of fixed boundaries for \( U_{\text{eye}} \) and \( U_{\text{flat}} \) (see Fig. 10) and QSP-RANS simulations were repeated for three gas injection rates. Moreover in these new simulations, a 100% turbulence intensity was imposed in the nodes along \( U_{\text{eye}} \); analyzing the LES velocity profiles revealed that the large fluctuations in the slag-metal interface at \( U_{\text{eye}} \) lead to apparent turbulence intensities as large as 100% as compared to 20% that is usually obtained in the QSP-RANS near \( U_{\text{eye}} \). The flow field for 4 lpm/ton after these modifications is shown in Fig. 11(b) demonstrating clear flow separation as compared to that without the lip modification.

The mean residence time of droplets of diameter 0.8 to 44 mm were computed using the modified RANS fields and are plotted in Fig. 12 for various values of \( Q_{\sigma} \) and \( \sigma \). Generally residence times are seen to increase with \( Q_{\sigma} \) and decrease with \( \sigma \) and droplet size.

In the next step the mean residence times from the current step (LPT) will be combined with entrainment rates and size distribution from LES step to calculate mass transfer rates. However, the LPT has used modified RANS fields that are different from those used for LES inputs, which apparently
decouples the results from these two steps. To clarify, various RANS profiles, at the locations corresponding to LES inlet, were compared before and after the ‘lip’ modification. It was found that the mean velocities were largely unchanged whereas the Reynolds stresses have significantly changed.

The distribution of turbulence length scale is one of the input parameters for SEM that generates inlet conditions for LES domain. By varying this parameter by a factor of 4 for three \( Q_{sp} \) at \( \sigma = 1.0 \), it was found that entrainment rate and droplet size distribution were practically unaltered. This shows that the role of inlet turbulence is limited to triggering disturbances in slag metal interface at \( U_{leye} \) while the mean flow properties play the major role in affecting the entrainment rate. Since the mean velocities before and after ‘lip’ modification were largely unchanged, the LPT and LES results may still be considered as coherent.

2.5. Mass Transfer Computations

In this step the entrainment parameters from the previous models were used in the DBD model to compute mass transfer rates for the 44 ton ladle. The model inputs are as follows:

1. Entrainment rate and size distribution from LES for each \( Q_{sp} - \sigma \) combination
2. Mean residence time for various droplet sizes from LPT for each \( Q_{sp} - \sigma \) combination
3. LP side mass transfer coefficient for each interface, namely \( U_{leye} \), \( U_{leye} \), and \( U_{drop} \), based on Higbie’s penetration theory,

\[
k_L = 2\sqrt{\frac{DU}{\pi L}} \tag{6}
\]

where, the diffusion coefficient \( D = 4.8\times10^{-9} \text{ m}^2/\text{s} \) for diffusion of sulphur in liquid iron at 1 873 K,\(^{26}\) \( L \) and \( U \) are the length and velocity scales, respectively, associated with an interface. For \( U_{flat} \) and \( U_{leye} \), \( L \) is the physical length of the interface and \( U \) is the average velocity along the interface computed from QSP-RANS. For \( U_{drop} \), \( L \) is the droplet diameter and \( U \) was obtained from LPT as the average of velocity of all drops for a give size, throughout their lifetime. The computed mean mass transfer coefficients are plotted in Figs. 13(a) and 13(b). Values of \( k_L \) at \( U_{leye} \) are the smallest among all interfaces, due to the weak flow around that interface. For droplets, \( k_L \) values are more or less independent of \( Q_{sp} \) and \( \sigma \).

4. Interfacial area at \( U_{leye} \) obtained from LES. It was observed in the LES results that the large fluctuations at \( U_{leye} \) led to a large slag metal interfacial area, as also seen in Fig. 7. For each case, numerous LES output files were post-processed to extract the interfacial area at \( U_{leye} \) in the region marked in Fig. 14, for example, averaged and scaled up for the cylindrical ladle. The scaled up areas normalized by the ladle cross section area for each \( Q_{sp} - \sigma \) combination are plotted in Fig. 15; expectedly, the area increases with increasing \( Q_{sp} \) and decreasing \( \sigma \); moreover at higher gas injection rates, the area at \( U_{leye} \) is much larger than at \( U_{leye} \).
The parameters were fed into the DBD model and mass transfer rates were computed. For the present discussion, results from $0.5 \text{ N/m}$ are omitted. The mass transfer rate constants ($\beta$) for various $Q_{sp}-\sigma$ combination are plotted in Fig. 16 against specific stirring rate, defined as,

$$
\varepsilon (W/\text{ton}) = \frac{RTQ(STP \text{ m}^3/\text{s})}{m_L \text{ton} V_N} \ln \left(1 + \frac{\rho g h_L}{101.325} \right) \quad \text{(7)}
$$

where, $m_L$ is mass of steel, $V_N = 22.4 \text{ liter per mole of gas}$ and $R = 8.314 \text{ J/mol/K}$. The results show trends similar to the correlation of Graham,

$$
\beta = 6 \times 10^{-6} \varepsilon^{1.4}, \quad \text{.......................... (8)}
$$

measured in a 165 ton ladle with two bottom plugs for gas injection.

The role of interfacial tension is such that the lowering $\sigma$ increases entrainment rate (Eq. (2)), increases droplet residence time (Fig. 12) and increases interfacial area at $UL_{eye}$ (Fig. 15) altogether increasing the mass transfer rate ($\beta$).

The contribution of droplets to mass transfer calculated in the DBD model from

$$
\% \text{Droplet Contribution} = \frac{\text{Num. moles transferred at droplets}}{\text{Total num. of moles transferred}} \times 100 \quad \text{......... (9)}
$$

is plotted in Fig. 17. The droplet contribution is most significant only at higher stirring rate. Since the values of area and $k_L$ at $UL_{eye}$ are larger than those at $UL_{flat}$, it is clear that the contribution from $UL_{eye}$ is significant even at higher flow rates and hence cannot be disregarded as assumed by Sulasalmi et al.\textsuperscript{6)}

3. Discussion and Application to Ladle Metallurgy Operations

The present model is very complicated and certainly not a tool that would be used in day-to-day operations; however, the results are informative and adaptable to process conditions. In this section issues of scale-up, mass transfer in Regime I (gentle stirring without droplet formation), the break-point between Regime I and II and the effect of multiple plugs are addressed.

3.1. Scale Up

A 44 ton ladle was selected for the previous calculations which are very time-consuming because of the LES calculations. To reduce the complexity of scale up, scaling relationships were developed so that the LES calculations of the 44 ton ladle could be reused, although performing separate QSP-RANS and LPT models. A ladle of 150 ton capacity, geometrically similar to the 44 ton ladle with the linear scaling factor ($\lambda$) of 1.5 was considered.

It has been established by Krishnapisharody and Irons\textsuperscript{25)} that the dynamic similarity for plume and melt flow in the ladle is ensured through the dimensionless gas flow rate,

$$
Q^* = \frac{Q}{g^{0.5} h^2_L} \quad \text{.......................... (10)}
$$

The resulting similarity is characteristic of Froude similarity criterion, for which the velocity ratio between two geometrically similar systems, the 44 and 150 ton ladles, at corresponding locations follow the ratio,

$$
U_{150}/U_{44} = \lambda^{0.5} \quad \text{...................... (11)}
$$

Based on Eq. (10) three gas flow rates at half bath height were chosen corresponding to the 44 ton ladle, namely 500, 1000 and 2000 lpm. The entrainment rates were scaled up from 44 ton according to,

$$
VPU_{150} = \lambda^{3/2} (VPU_{44} + 0.46) - 0.46 \quad \text{......... (12)}
$$

using Eqs. (2) and (11). Since droplet size distributions are mainly affected by the interfacial tension, the same droplet size distribution parameters of the 44 ton cases were used. The interfacial areas at $UL_{eye}$ were simply scaled up from the 44 ton cases as,

$$
A_{UL_{eye,150}} = \lambda^2 A_{UL_{eye,44}} \quad \text{..................... (13)}
$$

although strictly it may not be true. For droplet residence time, QSP-RANS for the three flow rates and subsequent LPT for $\sigma = 1.0$ and $2.0 \text{ N/m}$ were performed for each flow rate. The mass transfer coefficients were obtained from RANS fields using the same procedure for the 44 ton cases.

The results of the DBD model plotted in Fig. 18 show a remarkable similarity with Graham’s correlation. Compar-
ing Figs. 16 and 18 we see that $\beta - \varepsilon$ curves of 150 ton cases lie above that of 44 ton cases; this is because, for a given $\varepsilon$, the gas flow rate is higher in the larger vessel leading to higher plume velocity and velocity near $U_{\text{Leye}}$ that results in higher area at $U_{\text{Leye}}$, entrainment rate, droplet residence time and leading to higher mass transfer rate. These considerations show that the model results of the 44 and 150 ton cases are self consistent.

3.2. Regime I Mass Transfer:
As noted earlier the critical gas flow rate for the 44 ton ladle is approximately 90 lpm. So QSP-RANS for flow rates of 90, 60 and 30 lpm for the 44 ton ladle and 200, 100 and 50 lpm for the 150 ton ladle were performed. Since droplets are absent, mass transfer rates were calculated directly from,

$$\beta = \sum \left( \frac{k_i A}{V_{\text{L}}} \right), i = U_{\text{Lflat}}, U_{\text{Leye}} \quad \text{(14)}$$

using the values for $k_i$ and $A$ obtained from the RANS simulations. Figures 16 and 18 are replotted with the $\beta - \varepsilon$ values of Regime I in Fig. 19. It also contains values of exponents to $\varepsilon$ in the $\beta \propto \varepsilon$ relation, obtained by fitting power law type models for various $\beta - \varepsilon$ curves. Exponent values of 0.31 and 0.32 for Regime I are within the range of 0.25 to 0.4 as summarized by Asai et al.\textsuperscript{21} Moreover, the illustrative lines drawn to connect the $\beta - \varepsilon$ curves of Regimes I and II for the 44 ton case indicate that the curves of two regimes intersect at around $\varepsilon = 5$ W/ton that corresponds to 90 lpm, the critical gas flow rate (as noted in the end of Sec. 2.3). These results support the predictions of both the Regime I and II mass transfer rates.

3.3. Multiple Bottom Plugs
In the water modeling studies of Kim and Freuhan\textsuperscript{29} it was shown that the exponent to $\varepsilon$ decreased from 2.5 to 2.1 for multiple gas injection tuyeres in Regime II. This aspect is captured by the present models as higher exponents to $\varepsilon$ for the 150 ton cases (labeled in Fig. 19) with single plug configuration as compared to that of the 165 ton–double plug ladle of Graham’s trials. It can be shown that the reason for decrease in the exponent with increase in number of plugs is primarily due to the division of gas flow rate between multiple plumes, which decreases the velocity near $U_{\text{Leye}}$, in turn decreasing the total entrainment rate, droplet residence times and mass transfer coefficients and thereby decreasing the mass transfer rate, as compared to a single plug configuration.

3.4. Breakpoint between Regimes I and II
$\beta - \varepsilon$ curves of Regime I, Regime II for $\sigma = 1.0$ and that of Ishida et al.’s 2.5 ton trials are plotted in Fig. 20(a). It can be seen that as the ladle size increases the break point

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**Fig. 18.** Comparisons similar to Fig. 16 but for 150 ton cases.

**Fig. 19.** Figures 16 and 18 replotted including $\beta - \varepsilon$ plots of Regime I, labeled with exponents to $\varepsilon$.

**Fig. 20.** (a) Comparison of $\beta - \varepsilon$ plots of 44 ton and 150 ton of present work, corresponding to $\sigma = 1.0$ for Regime-II, with that of Ishida et al. (2.5 ton). (b) $\beta - \varepsilon$ plots similar to (a) but with those of Graham and Lachmund et al. data from industrial ladles; the lack of break points indicating the transition from Regimes I to II in the latter cases is illustrated by the extending dashed lines.
for transition from Regimes I to II decreases both in $\beta$ and $\epsilon$. Excluding Ishida et al.’s case and including the $\beta-\epsilon$ plots of Graham$^{25}$ and Lachmund et al.$^{30}$ Fig. 20(a) is replotted in Fig. 20(b); these two latter cases represent the trials from industrial ladles of 165 ton with two bottom plugs and 185 ton with three bottom plugs, respectively. The increased ladle capacity and the number of bottom plug has resulted in smaller slope of $\ln(\beta)-\ln(\epsilon)$ plots that has pushed the break point to very small values of $\beta$ and $\epsilon$ that the break point is virtually absent. The missing break point has been noted by the corresponding authors$^{25,30}$ although without a definite explanation. Thus, these analyses clarify that the point of transition between Regimes I and II will lower both in $\beta$ and $\epsilon$ as the ladle capacity and number of bottom plugs increases.

3.5. Comparison with Previous Work

Recently Sulasalmi et al.$^6$ have proposed a comprehensive model for the reduction stage in CAS-OB process that involves reduction of metallic oxides governed by slag metal interfacial mass transfer. The slag entrainment was simulated using a multiphase LES model in a reduced domain located in the slag eye region similar to the present work. However LESs were performed only to estimate the averaged droplet size.$^{31}$ The entrainment rate was estimated using the models of Oeters$^{31,32}$ (also found elsewhere$^{32,33}$) for a given average droplet size. For droplet residence time a value of 45 $s$ was obtained as a fitting parameter to their overall model so as to yield an ‘emulsification fraction’ of 45 to 100%. However, the present estimates in Fig. 12 show that such high residence times are possible only for fine droplets ($< 1$ mm) while their average droplet sizes were between 6 to 8 mm. On the other hand, Lachmund et al.$^{30}$ have also used the Oeters’ model for entrainment rate and predicted an average residence time of 5 $s$ for a ladle of 185 ton. Given their measured droplet sizes to be smaller than 2.5 $mm$, their predicted residence time is quite low. The models of Oeters$^{31,32}$ have not been adequately validated and in the light of these contradictions its use is questionable. In contrast, the present work has dedicated models developed for each entrainment parameter leading to mass transfer rate predictions that are self-consistent and agree with literature correlation; the overall model is not simple due to the complexity of the slag entrainment phenomena.

4. Conclusions

A novel model to study the role that slag droplet entrainment has on slag-metal reactions in ladle metallurgy was developed. The overall model has several new sub-models. The important conclusions are:

(1) The large scale eddy model simulations revealed that the gas flow rate had the largest effect on the rate of droplet formation; the slag metal interfacial tension has small impeding effect, while the slag viscosity had almost no effect on droplet formation.

(2) The droplet contribution to interfacial mass transfer increased with gas flow rate, but the interfacial area at the slag eye also contributed significantly to mass transfer.

(3) Scale up procedure was developed to assess the effects of ladle size on the mass transfer rate. This allowed comparison to be made with available information on mass transfer in full-scale operations. The calculated rate of mass transfer was in good accord with full-scale results.

(4) The model demonstrates that increasing the number of bottom plugs decreases the dependence of interfacial mass transfer rate on total gas flow rate which is consistent with industrial practice.

(5) The breakpoint for the transition from Regime I to II is well-predicted in the model for the 44 ton ladle. Using the model results the variation in breakpoints found in literature was rationalized as due to varying ladle sizes and number of bottom plugs. In large scale operation the flow rates are well above the breakpoint.

Acknowledgement

The authors thank the financial support from Natural Sciences and Engineering Research Council of Canada for this work.

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