Theoretical Study on Local Buckling of Steel Plate in Concrete-filled Tube Column under Axial Compression

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(Received on January 5, 2017; accepted on May 15, 2017; J-STAGE Advance published date: July 28, 2017)

Concrete filled tube (CFT) columns have been increasingly used as the load-bearing systems in engineering applications. The fact that the CFT column has steel plate restrained by infilled concrete and the availability of high strength structural steel leads to the application of thin steel plates in CFT columns. However, this gives rise to the local instability problem of thin steel plates under axial loadings. Furthermore, most of the studies on the local buckling of steel plates in contact with concrete reported in the literature were concerned with the cases where the four edges of the steel plate were assumed to be clamped or simply supported. This cannot reflect the real state of steel plate in CFT columns, where the unloaded edges of steel plate is more appropriate to be regarded as elastically restrained. This paper presents an analytical study of local buckling of steel plates in CFT columns. The steel plate, subjected to uniform axial compression, is assumed to be elastically rotationally restrained along loaded and unloaded edges. The approximate solution is obtained by Rayleigh-Ritz method. The solution of rotationally restrained plates is verified by comparing with available experimental and theoretical data in the literature. Good agreement is found between them for local buckling stress of the steel plate. This research provides basis for capacity design of CFT columns under axial compression.

KEY WORDS: steel plate analysis; Rayleigh-Ritz method; local buckling; rotationally restrained; axial compression; CFT column.

1. Introduction

CFT columns have been increasingly used in North America and Asia. A typical CFT column consists of four steel plates infilled with concrete. The composite action of the steel and concrete provides excellent strength and stiffness for the column. In a CFT column, steel plates under compression can only buckle locally outward owing to the restraints of the infilled concrete. This buckling mode results in a considerable increase in the critical local buckling strength of the steel plates as well as the load-bearing capacity of the column. Meanwhile, the steel plates offer confinement to the concrete, which enhances the concrete performance and thus, increasing the stability and strength of the column as a system.

The steel plate in CFT column can be modeled as an isotropic plate which is rotationally restrained along the two loaded and two unloaded edges and the rotational stiffness of the elastic restraint at the four edges is introduced to account for the flexibility of the conjunctions. As a result of different loading states and rotational restraints provided by concrete, the loaded and unloaded edges of the steel plate can be assumed as simply supported or clamped.

The fact that the CFT column has steel plates restrained by concrete and the availability of high strength structural steel leads to the application of thin steel plates in CFT columns. However, this gives rise to the local buckling issues of thin steel plates under axial loadings, which is common in CFT columns.

A number of research have been undertaken on the local buckling of plates with various edge restraints and under different loadings. It is well recognized that numerical methods such as the finite element method,^{1,2)} finite difference method,³⁾ and finite strip method⁴⁾ could provide explicit analytical solutions for instability analysis of plates. However, it is computationally complicated, time consuming and tedious for broad use in design as several iterations may be necessary to ensure the reasonable solution.

As an alternative to the numerical approach, the classical Rayleigh-Ritz method has been adopted by many researchers to obtain the local buckling solution of plates. In applying the variational formulation of this approach, Qiao and his research group⁵⁻⁸⁾ conducted extensive analyses to develop the explicit local buckling of composite plates with rotational restraint under uniaxial compression, uniform shear, and biaxial loading, and fully restrained under combined linearly varying axial and in-plane shear loadings. Kang and Leissa⁹⁾ proposed an exact buckling solution of plates subjected to linearly varying normal stresses and with

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DOI: <http://dx.doi.org/10.2355/isijinternational.ISIJINT-2016-755>

two opposite edges simply supported, and the other two edges may be clamped, simply supported, free, or elastically supported. Zhong and Gu⁽¹⁰⁾ also presented an exact buckling solution for simply supported symmetrical cross-ply rectangular plates subjected to linearly varying loadings. Mittelstedt⁽¹¹⁾ presented an energy-based approach to obtain the local buckling loads of blade-stringer-stiffened and compressively loaded orthotropic composite plates. Seif and Kabir⁽¹²⁾ developed an analytical approach for both symmetric and anti-symmetric local buckling of thin-plate in finite sizes and with a center crack under tension, the solution of which was based on the principle of minimum total potential energy. Recently, Galerkin method, which is a powerful numerical solution approach in solving differential equations, was used by Jaberzadeh and Azhari⁽¹³⁾ to analyze the elastic and inelastic local buckling of flat rectangular plates with centerline boundary conditions under non-uniform in-plane compression and shear stress.

Similar to the local buckling problems of plates, a special case about the local buckling of steel plates in steel-concrete composite members has been studied by a few researchers. Many of the studies were based on the numerical approach. Uy and Bradford⁽¹⁴⁾ proposed a modified semi-analytical finite strip method to study the elastic local buckling behavior of steel plates in composite steel-concrete members. Liang and Uy⁽¹⁵⁾ studied the local and post-local buckling of steel plates in concrete-filled tubular columns under axial compression by finite element method and proposed effective width formulas for the steel plates. Furthermore, Liang *et al.*^(16,17) studied the local buckling strength of steel plates in double skin composite panels and concrete-filled thin-walled steel tubular beam-columns by using finite element method. On the other hand, some were based on energy method. Wright⁽¹⁸⁾ described an analytical model for evaluating local buckling of a variety of sections where the steel plates was in contact with a rigid medium. Recently, Cai and Long⁽¹⁹⁾ and Long *et al.*⁽²⁰⁾ applied Rayleigh-Ritz method to obtain the solution of elastic buckling of steel plates in rectangular concrete-filled steel tubular columns with binding bars under axial and eccentric compressions. Arabzade *et al.*⁽²¹⁾ used theoretical models to obtain the elastic buckling coefficients of steel plates in composite steel plate walls with various aspect ratios under shear loading based on Rayleigh-Ritz method.

However, most of the studies on the local buckling of steel plates in contact with concrete reported in the literature were concerned with the cases where the four edges of the steel plate were assumed to be clamped or simply supported. It should be acknowledged that the four edges of the steel plate in CFT columns were restrained to some extent by the surrounded components, but the restraints are normally not stiff enough to be regarded as clamped. The partial restraint has a pronounced impact on the local buckling behavior of CFT columns under out-of-plane compression. Therefore, the analytical solution of the CFT columns elastically restrained along four edges is needed in order to predict accurately the local buckling response of steel plates in CFT columns.

This paper presents a theoretical study on the local buckling response of steel plates in CFT columns based on the Rayleigh-Ritz method. Two opposite edges are subjected to

uniform axial compression and all four edges (both loaded and unloaded) are assumed to be rotationally-restrained. The solution of rotationally restrained plates is verified by comparing with available experimental and theoretical data in the literature.

2. Analytical Formulation

2.1. Yield Line Model for Orthotropic Problem

The Rayleigh-Ritz method was first adopted by Timoshenko and Gere⁽²²⁾ to obtain the solution of stability problems of the plate. The analysis in this paper is based on this method, which also resembles the method used by several previous researchers such as Mittelstedt⁽¹¹⁾ and Cai and Long.⁽¹⁹⁾ Figure 1 shows a steel plate in CFT column subjected to axial compression N_x along the x -direction.

According to classical theory of elastic stability,⁽²²⁾ in the calculation of critical values of forces applied in the middle plane of a plate at which the flat form of equilibrium becomes unstable, the governing differential equation of the deflection surface for the buckled plate is given by the following equation

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{1}{D} \left(N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2 N_{xy} \frac{\partial^2 w}{\partial x \partial y} \right) \quad (1)$$

where D is called the flexural rigidity of the plate and is given as,

$$D = \frac{Et^3}{12(1-\nu^2)} \quad (2)$$

Using the energy method to investigate the local buckling of the elastically restrained steel plate subjected to one axial in-plane compression, the total potential energy of the steel plate (Π) is the summation of the strain energy stored in the plate associate with the thin plate deforming (U), the strain energy associate with the elastic restraint along the rotationally restrained boundary of the plate (V), and the energy associate with the external loads (U_T). It can be mathematically expressed by,

Fig. 1. Analytical model of elastically rotationally restrained steel plate.

$$\Pi = U + V + U_{\Gamma} \dots\dots\dots (3)$$

The strain energy stored in the steel plate U is given as

$$U = \frac{D}{2} \iint_{\Omega} \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu) \left[\frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dx dy \dots\dots\dots (4)$$

where Ω is the area of the steel plate.

Assuming that the steel plate is elastically restrained along four edges with the elastic rotational restraint stiffness coefficients k_x along the two opposite loaded edges and k_y along the two opposite unloaded edges, the analytical model of the steel plate in CFT column is indicated in Fig. 1. For steel plate with rotational restraints distributed along the four edges, the strain energy U_{Γ} stored in the equivalent elastic rotational springs is expressed as

$$U_{\Gamma} = \frac{1}{2} \int_{\Gamma_x} k_y \left(\frac{\partial w}{\partial y} \bigg|_{y=0} \right)^2 dx + \frac{1}{2} \int_{\Gamma_x} k_y \left(\frac{\partial w}{\partial y} \bigg|_{y=b} \right)^2 dx \dots\dots (5) \\ + \frac{1}{2} \int_{\Gamma_y} k_x \left(\frac{\partial w}{\partial x} \bigg|_{x=0} \right)^2 dy + \frac{1}{2} \int_{\Gamma_y} k_x \left(\frac{\partial w}{\partial x} \bigg|_{x=a} \right)^2 dy$$

where k_x is the rotational stiffness of the elastic restraint at the loaded edges of $x = 0$ and a (see Fig. 1) and Γ_y is along the width of the plate ($\Gamma_y = 0$ to b); k_y is the rotational stiffness of the elastic restraint at the unloaded edges of $y = 0$ and b (see Fig. 1) and Γ_x is along the length of the plate ($\Gamma_x = 0$ to a).

The energy associated with the external loads U_{Γ} can be written as

$$V = \frac{1}{2} \iint_{\Omega} \left[N_x \left(\frac{\partial w}{\partial x} \right)^2 + N_y \left(\frac{\partial w}{\partial y} \right)^2 + 2N_{xy} \frac{\partial w}{\partial x} \frac{\partial w}{\partial y} \right] dx dy \dots\dots (6)$$

where N_x is the axial force per unit length at the boundary of $x = 0$ and a ; N_y is the axial force per unit length at the boundary of $y = 0$ and b ; and N_{xy} is the force per unit in the transverse direction.

For the CFT columns subjected to axial compression, the boundary stress of the steel plate is $\sigma_y = \tau_{xy} = 0$, and thus

$$N_x = -\sigma_x t \dots\dots\dots (7)$$

$$N_y = -\sigma_y t = 0 \dots\dots\dots (8)$$

$$N_{xy} = -\tau_{xy} t = 0 \dots\dots\dots (9)$$

2.2. Displacement Shape Function

It is of significant importance to choose the suitable form of the plate out-of-plane buckling displacement function w in order to solve the eigenvalue problem. In this research, a unique plate buckled displacement shape was adopted to obtain the explicit analytical solution for local buckling of the steel plate in CFT column under uniform in-plane compression in the x direction, as shown in Fig. 1.

The steel plate in composite wall only buckles outwards due to the restraint of the infilled concrete which prevents

the plate buckling inwards. Assuming that the displacement shape in the x direction is a combine sine and cosine function and the deflection in the y direction is a biquadratic function, the displacement shape function can be expressed as

$$w(x, y) = C \left[\frac{y}{b} + \phi_1 \left(\frac{y}{b} \right)^2 + \phi_2 \left(\frac{y}{b} \right)^3 + \phi_3 \left(\frac{y}{b} \right)^4 \right] \times \left[(1-\omega) \sin \frac{\pi x}{a} + \omega \left(1 - \cos \frac{2\pi x}{a} \right) \right] \dots\dots (10)$$

where C is a constant, and ϕ_1 , ϕ_2 , ϕ_3 and ω are the constants to be determined which should satisfy both the boundary conditions and the requirement of compatibility.

As shown in Fig. 1, the boundary conditions of the plate elastically restrained against rotation at loaded and unloaded edges are given by

$$w(0, y) = w(a, y) = 0 \dots\dots\dots (11a)$$

$$M_x(0, y) = -D \left(\frac{\partial^2 w}{\partial x^2} \right)_{x=0} = -k_x \left(\frac{\partial w}{\partial x} \right)_{x=0} \dots\dots (11b)$$

$$M_x(a, y) = -D \left(\frac{\partial^2 w}{\partial x^2} \right)_{x=a} = +k_x \left(\frac{\partial w}{\partial x} \right)_{x=a} \dots\dots (11c)$$

$$w(x, 0) = w(x, b) = 0 \dots\dots\dots (12a)$$

$$M_y(x, 0) = -D \left(\frac{\partial^2 w}{\partial y^2} \right)_{y=0} = -k_y \left(\frac{\partial w}{\partial y} \right)_{y=0} \dots\dots (12b)$$

$$M_y(x, b) = -D \left(\frac{\partial^2 w}{\partial y^2} \right)_{y=b} = +k_y \left(\frac{\partial w}{\partial y} \right)_{y=b} \dots\dots (12c)$$

Substituting the derivative function of Eq. (10) with respect to y into Eq. (11) gives the weight constant ω in terms of the rotational stiffness of the elastic restraint (k_x) at the loaded edges:

$$\omega = \frac{k_x a}{k_x a + 4\pi D} \dots\dots\dots (13)$$

Substituting the first-order partial derivative and the second-order partial derivative of Eq. (10) with respect to y into Eq. (12), the unknown constants ϕ_1 , ϕ_2 , ϕ_3 can be solved in terms of the elastic rotational restraint stiffness (k_y) along the two opposite unloaded edges as

$$\phi_1 = \frac{k_y b}{2D} \dots\dots\dots (14a)$$

$$\phi_2 = -\frac{2D + k_y b}{D} \dots\dots\dots (14b)$$

$$\phi_3 = \frac{2D + k_y b}{2D} \dots\dots\dots (14c)$$

By substituting Eq. (10) into Eqs. (4), (5), and (6), integrating and summing them according to Eq. (3), the total

potential energy of the steel plate is given as

$$\begin{aligned} \Pi = & \frac{D}{2} \left[\frac{b\pi^3 C^2}{a^3} A_1 B_1 + \frac{aC^2}{b^3} A_2 B_2 \right. \\ & \left. + 2(1-\nu) \frac{\pi C^2}{ab} A_3 B_3 - 2\nu \frac{\pi C^2}{ab} A_4 B_4 \right] \\ & - \frac{\sigma_x t b \pi C^2}{2a} A_1 B_4 + \frac{k_y a C^2}{2b^2} (1+A_5^2) B_5 + \frac{k_x b \pi^2 C^2 (1-\omega)^2}{a^2} A_6 \end{aligned} \quad (15)$$

where $A_1, A_2, A_3, A_4, A_5, A_6, B_1, B_2, B_3, B_4$, and B_5 are defined as

$$A_1 = \frac{1}{3} + \frac{\phi_1}{2} + \frac{\phi_3 + \phi_1 \phi_2}{3} + \frac{\phi_2 \phi_3}{4} + \frac{\phi_1^2 + 2\phi_2}{5} + \frac{\phi_2^2 + 2\phi_1 \phi_3}{7} + \frac{\phi_3^2}{9} \quad (16a)$$

$$A_2 = 4\phi_1^2 + 12\phi_2^2 + \frac{144}{5}\phi_3^2 + 12\phi_1 \phi_2 + 16\phi_1 \phi_3 + 36\phi_2 \phi_3 \quad (16b)$$

$$\begin{aligned} A_3 = & 1 + 2\phi_1 + 2\phi_2 + 2\phi_3 + \frac{4}{3}\phi_1^2 + \frac{9}{5}\phi_2^2 + \frac{16}{7}\phi_3^2 \\ & + 3\phi_1 \phi_2 + \frac{16}{5}\phi_1 \phi_3 + 4\phi_2 \phi_3 \end{aligned} \quad (16c)$$

$$\begin{aligned} A_4 = & \phi_1 + 2\phi_2 + 3\phi_3 + \frac{2}{3}\phi_1^2 + \frac{6}{5}\phi_2^2 + \frac{12}{7}\phi_3^2 \\ & + 2\phi_1 \phi_2 + \frac{14}{5}\phi_1 \phi_3 + 3\phi_2 \phi_3 \end{aligned} \quad (16d)$$

$$A_5 = 1 + 2\phi_1 + 3\phi_2 + 4\phi_3 \quad (16e)$$

$$\begin{aligned} A_6 = & \frac{1}{3} + \frac{1}{2}\phi_1 + \frac{2}{5}\phi_2 + \frac{1}{3}\phi_3 + \frac{1}{3}\phi_1 \phi_2 + \frac{2}{7}\phi_1 \phi_3 \\ & + \frac{1}{4}\phi_2 \phi_3 + \frac{1}{5}\phi_1^2 + \frac{1}{7}\phi_2^2 + \frac{1}{9}\phi_3^2 \end{aligned} \quad (16f)$$

$$B_1 = \frac{\pi(1-\omega)^2}{2} + 8\pi\omega^2 - \frac{16\omega(1-\omega)}{3} \quad (16g)$$

$$B_2 = \frac{(1-\omega)^2}{2} + \frac{3\omega^2}{2} + \frac{14\omega(1-\omega)}{3} \quad (16h)$$

$$B_3 = \frac{\pi(1-\omega)^2}{2} + 2\pi\omega^2 + \frac{8\omega(1-\omega)}{3} \quad (16i)$$

$$B_4 = \frac{\pi(1-\omega)^2}{2} + 2\pi\omega^2 + \frac{16\omega(1-\omega)}{3} \quad (16j)$$

$$B_5 = \frac{(1-\omega)^2}{2} + \frac{3\omega^2}{2} + \frac{4\omega(1-\omega)}{\pi} \quad (16k)$$

The first polynomial in Eq. (15) represents the change in strain energy due to the deforming of the plate, the second represents the work done by the applied loads, and the third and the fourth represent the energy associated with

the elastic rotational restraint at the unloaded and loaded edges, respectively. Note that $k_x = 0$ or $k_y = 0$ stands for the loaded or unloaded edges being simply supported; while $k_x \rightarrow \infty$ or $k_y \rightarrow \infty$ represents loaded or unloaded edges being clamped. Any values of k_x or k_y between the two extreme conditions corresponds to the boundary edges being elastically restrained.

2.3. Explicit Solution

According to the principle of minimum potential energy, the value of $\sigma_x t$ can be obtained by the condition of minimizing the value of Π by taking a partial derivative of Eq. (15) with respect to C and setting the derivative equal to zero:

$$\frac{\partial \Pi}{\partial C} = 0 \quad (17)$$

Substituting Eq. (15) into (17) and Rearranging gives

$$\begin{aligned} \sigma_x t = & \frac{\pi^2 D}{b^2} \left[\frac{B_1}{\gamma^2 B_4} + \frac{\gamma^2 A_2 B_2}{\pi^3 A_1 B_4} + \frac{2(1-\nu) A_3 B_3 - 2\nu A_4 B_4}{\pi^2 A_1 B_4} \right. \\ & \left. + \frac{2\lambda_y \gamma^2 (1+A_5^2) B_5}{\pi^3 A_1 B_4} + \frac{4\lambda_x (1-\omega)^2 A_6}{\gamma^2 \pi A_1 B_4} \right] \\ & = \frac{k\pi^2 D}{b^2} \end{aligned} \quad (18)$$

where γ is the half-wave parameter or so called aspect ratio and is defined as $\gamma = a/b$; k is elastic local buckling coefficient of the steel plate as given in Eq. (19); λ_x and λ_y are elastically restraining factors of loaded and unloaded edges, respectively, as defined in Eqs. (20)–(21).

$$\begin{aligned} k = & \frac{B_1}{\gamma^2 B_4} + \frac{\gamma^2 A_2 B_2}{\pi^3 A_1 B_4} + \frac{2(1-\nu) A_3 B_3 - 2\nu A_4 B_4}{\pi^2 A_1 B_4} \\ & + \frac{2\lambda_y \gamma^2 (1+A_5^2) B_5}{\pi^3 A_1 B_4} + \frac{4\lambda_x (1-\omega)^2 A_6}{\gamma^2 \pi A_1 B_4} \end{aligned} \quad (19)$$

$$\lambda_x = \frac{k_x a}{2D} \quad (20)$$

$$\lambda_y = \frac{k_y b}{2D} \quad (21)$$

By taking a partial derivative of Eq. (19) with respect to γ , the minimum of Eq. (19) can be found when the critical aspect ratio γ_{cr} is given by

$$\gamma_{cr} = \left[\frac{\pi^3 A_1 B_1 + 4\lambda_x \pi^2 (1-\omega)^2 A_6}{A_2 B_2 + 2\lambda_y (1+A_5^2) B_5} \right]^{\frac{1}{4}} \quad (22)$$

Substituting Eq. (22) into Eq. (19), the critical elastic local buckling coefficient k_{cr} can be obtained. When the steel plate is relatively long (e.g., $\gamma \geq \gamma_{cr}$), which is generally the case for steel plate in CFT column, the critical local buckling stress of the steel plate (σ_{cr}) elastically restrained against rotation at loaded and unloaded edges can be determined by substituting k_{cr} and D (Eq. (2)) into Eq. (18) as

$$\sigma_{cr} = \frac{k_{cr}\pi^2 E}{12(1-\nu^2)(b/t)^2} \dots\dots\dots (23)$$

While for some rare case with relatively short-span plates (e.g., $\gamma < \gamma_{cr}$), the critical local buckling stress should be calculated directly by employing Eq. (18).

3. Validity of the Explicit Solution

In this section, the explicit formulas for two special cases which are commonly used in the practical CFT column design and analysis are first discussed and compared with previous theory and data. A more general case was then presented and the accuracy of the explicit local buckling solution based on the energy method was validated against experimental results.

3.1. Case 1: Clamped at Loaded Edge and Simply-supported at Unloaded Edges

When the elastic rotational restraint stiffness $k_y = 0$, and $k_x \rightarrow \infty$, it results in $\omega = 1$ from Eq. (13), $\lambda_x = 0$ from Eq. (20), $\lambda_y = 0$ from Eq. (21), and $\gamma_{cr} = 1.519$ from Eq. (22). This case is equivalent to the condition that the steel plate is clamped at the loaded edges and simply supported at the unloaded edges. Substituting $\gamma_{cr} = 1.519$ into Eq. (19) gives the value $k_{cr} = 5.467$. Then the critical local buckling stress is

$$\sigma_{cr} = \frac{5.467\pi^2 E}{12(1-\nu^2)(b/t)^2} \dots\dots\dots (24)$$

This result is close to $k_{cr} = 5.6$ suggested and used by Uy and Bradford¹⁴⁾ based on finite strip method.

3.2. Case 2: Clamped at All Edges

When the elastic rotational restraint stiffness $k_y \rightarrow \infty$, and $k_x \rightarrow \infty$, it results in $\omega = 1$, $\lambda_x \rightarrow \infty$, $\lambda_y \rightarrow \infty$, and $\gamma_{cr} = 1.008$. This case is equivalent to the condition that the steel plate is clamped at the loaded and unloaded edges. Substituting $\gamma_{cr} = 1.008$ into Eq. (19) gives the value $k_{cr} = 10.311$. Then the critical local buckling stress is

$$\sigma_{cr} = \frac{10.311\pi^2 E}{12(1-\nu^2)(b/t)^2} \dots\dots\dots (25)$$

This result is close to $k_{cr} = 10.31$ which was proposed by Uy and Bradford,¹⁴⁾ $k_{cr} = 9.99$ recommended by Bridge and O'Shea²³⁾ based on finite strip analysis, and $k_{cr} = 9.81$ obtained by Liang and Uy¹⁵⁾ and Liang *et al.*¹⁷⁾ by a linear finite element buckling analysis of plates based on the bifurcation buckling theory.

3.3. Case 3: Clamped at Loaded Edges and Elastically-restrained at Unloaded Edges

For a more general case where the CFT column is under axial compression in practical engineering application, the loaded edges of the steel plate can be regarded as be clamped while the unloaded edges is more exactly to consider elastically rotationally restraint, *i.e.*, it can be assumed that $k_x \rightarrow \infty$. In this case and referring back to Eq. (19), the

most important issue to deal with is to find out the most suitable solutions for λ_y . Bleich²⁴⁾ proposed an equation to predict the elastically restraining factor for steel plate in box steel tube without infilled concrete as shown in Eqs. (26)–(28).

$$\lambda_y = \left(\frac{t_w}{t_f} \right)^3 \frac{r}{\rho} \dots\dots\dots (26)$$

$$r = 1 - \left(\frac{t_f b_w}{t_w b_f} \right)^2 \dots\dots\dots (27)$$

$$\rho = \frac{1}{\pi} \tanh \left(\frac{\pi b_w}{4b_f} \right) \left[1 + \frac{\pi b_w / 2b_f}{\sinh(\pi b_w / 2b_f)} \right] \dots\dots\dots (28)$$

where b_f and t_f are the width and thickness of the calculated steel plate, respectively; b_w and t_w are the width and thickness of the adjacent steel plate, respectively. Bleich²⁴⁾ suggested values of $k_{cr} = 4.0$ and $k_{cr} = 6.97$ for steel plate with loaded edges clamped and unloaded edges simply supported and clamped, respectively. Note that in Bleich's research, the steel plate was not restrained by the concrete. Comparing these results with the critical elastic local buckling coefficient for steel plate infilled by concrete (case 1 and case 2 in this paper), it can be observed that the critical local buckling stress increases by around 37% to 50% due to the presence of concrete. Therefore, in order to account for this effect, a reduction factor of 0.5 is considered to reduce r . It should be mentioned that a larger value (0.5 rather than 0.37) was selected due to the fact that this leads to the predicted value of σ_{cr} on the conservative side as can be seen from Eqs. (19), (23) and (26). Consequently, Eq. (27) is modified as

$$r' = 1 - \beta_r \left(\frac{t_f b_w}{t_w b_f} \right)^2 \dots\dots\dots (29)$$

where $\beta_r = 0.5$ is the reduction factor used for considering the beneficial restraining effects offered by concrete.

In order to validate the accuracy of the proposed formulas, the theoretical results from the equations proposed above are compared with the experimental data obtained by Uy^{25,26)} and Mo *et al.*,²⁷⁾ as shown in **Table 1**. The theoretical value of σ_{cr} was obtained according to Eq. (23), where k_{cr} was calculated from Eq. (19) and λ_y was determined from Bleich's formula of Eq. (26) including modified r' . It should be note that the data of cases where the yielding occurs before the local buckling of steel plate have been excluded in Table 1.

Uy²⁵⁾ conducted an extensive set of experiments for the local buckling response of steel plates restrained by infilled concrete. The tests included five specimens (with concrete) which are identified in Table 1. The experiments were undertaken in a 5 000 kN capacity compression testing machine. The composite section was designed to bear the load uniformly. The local buckling half-wavelengths were measured throughout the tests. The results demonstrated that the measured half wavelengths for all the tested specimens were initially equal to the width of the steel plate

Table 1. Comparison of experiments by other researchers and theoretical results.

Specimen No.	b (mm)	t_1 (mm)	t_2 (mm)	$\sigma_{cr,e}$ (MPa)	$\sigma_{cr,p}$ (MPa)	$\sigma_{cr,p}/\sigma_{cr,e}$	Reference
LB7	240	3	3	200	197	0.985	Uy (1998)
LB9	300	3	3	120	126	1.050	
FB1	360	3	3	93.4	88	0.942	Uy (2001)
FB2	420	3	3	79.9	64	0.801	
FB3	480	3	3	43.7	49	1.121	
FB4	540	3	3	38.8	39	1.005	
SCC2	200	3	3	246	283	1.150	Mo <i>et al.</i> (2004)
SCC3	200	3	2	191	221	1.157	
SCC4	200	2	5	166	183	1.102	
SCC6	200	2	2	118	126	1.068	
Average						1.038	
Standard deviation						0.104	

*Note: t_1 and t_2 = thickness of steel plate at the loaded and unloaded sides, respectively; $\sigma_{cr,e}$ = local buckling strength from experimental results; $\sigma_{cr,p}$ = local buckling strength determined from analytical model.

as expected from the theoretical assumptions. The half-wavelength reduced in all the specimens due to the yielding and shortening. The calculated local buckling stress $\sigma_{cr,e}$ was based on the relation $\sigma_{cr,e} = E\epsilon_e$ for the specimens that buckled in the elastic region.

Uy²⁶⁾ tested five concrete-filled steel sections. The load was applied only on the steel by virtue of a 20 mm recess produced at each end. Meanwhile, the tested specimens were pretreated with grease between the concrete and steel to ensure that no bond or load transfer to the concrete would be developed. The range of slenderness varied from 120 to 180.

Mo *et al.*²⁷⁾ experimentally investigated the local buckling of steel plates in concrete-filled steel tube column. Six specimens with the cross-sectional area of 200×200 mm was tested under axial compression. Both steel plate and concrete were designed to bear the axial load. Different thicknesses ranging from 2 mm to 5 mm were selected as the variables. The result of Specimen SCC5 was excluded from Table 1 as the recorded experimental data was not valid.

It can be found that, in general, the formulas proposed in the present study has good accuracy with the experimental values. The ratio of the predicted values ($\sigma_{cr,p}$) to the experimental ones ($\sigma_{cr,e}$) range from 0.801 to 1.157 with a mean of 1.027 and a standard deviation of 0.089. These values closely corresponded. Meanwhile, it is obvious that an increase of the b/t values leads to a decrease in the critical local buckling stress.

4. Summary and Conclusions

In this paper, an approximate classical solution, based on Rayleigh-Rize method, was provided for local buckling problem of steel plate subjected to uniform axial compression and with elastically-rotational restraint at loaded and unloaded edges. The resulting solution is obtained based on the principle of minimum total potential energy. The proposed solution was first discussed based on two extreme

cases where the steel plate was clamped at the loaded edges and simply supported or clamped at the unloaded edges. The predicted values of elastic local buckling coefficient agrees well with the numerical results by various researchers. A more general case was then presented and comparisons made between the proposed formulas and extensive experimental data show that the model provides remarkably accurate predictions of the local buckling stresses that are suitable for use. Furthermore, the increase in the width to thickness ratio results in a reduction in the critical buckling stress. The impact of thickness to width ratio of the steel plate becomes significant for the cases where the local buckling occurs before the yielding. The work in this paper provide basis for the capacity design of CFT column under axial compression

Acknowledgments

This work is sponsored by Jiangsu Provincial Natural Science Foundation of China (Grant No. BK20170685), A Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD). The authors also appreciate the idea on development of displacement function by Dr. Luyang Shan and Prof. Pizhong Qiao at Washington State University, which provides solid basis for the development of this paper.

Notation

- a : a half buckling wave length
- b : width of steel plate
- b_f, b_w : width of the calculated steel plate and adjacent steel plate, respectively
- C : A constant
- D : flexural rigidity of the plate
- E : elastic modulus of steel plate
- k : elastic local buckling coefficient of the steel plate
- k_{cr} : critical elastic local buckling coefficient of the steel plate
- k_x : rotational stiffness of the elastic restraint at the

loaded edges of $x = 0$ and a
 k_y : rotational stiffness of the elastic restraint at the unloaded edges of $y = 0$ and b
 N_x : surface force in the normal direction along x -axis
 N_y : surface force in the normal direction along y -axis
 N_{xy} : surface force in the transverse direction
 t : thickness of steel plate
 t_1, t_2 : thickness of steel plate at the loaded and unloaded sides, respectively
 t_f, t_w : thickness of the calculated steel plate and adjacent steel plate, respectively
 U : strain energy stored in the plate associate with the thin plate deforming
 U_Γ : work done by the external loads
 V : strain energy stored in elastic restraint edges
 σ_{cr} : critical local buckling stress of the steel plate
 $\sigma_{cr,e}$: measured local buckling stress from the experiments
 $\sigma_{cr,p}$: predicted local buckling stress based on the proposed formulas
 σ_x, σ_y : normal stress in the x and y direction, respectively
 τ_{xy} : shear stress in the steel plate
 w : deflection function
 ν : Poison's ratio of steel ($\nu = 0.3$)
 Π : total potential energy of the steel plate
 Ω : area of the plate
 Γ_x : along the length of the plate ($\Gamma_x = 0$ to a)
 Γ_y : along the width of the plate (Γ_y to b)
 ϕ_1, ϕ_2, ϕ_3 : unkown constants representing the deflection along the y direction
 ω : weight constant representing the deflection shape along the x direction

ϵ_e : measured local buckling strain from the tests
 β_r : reduction factor used for considering the beneficial restraining effects offered by concrete
 γ : aspect ratio ($\gamma = a/b$)
 γ_{cr} : critical aspect ratio
 λ_x, λ_y : elastically restraining factors of loaded and unloaded edges, respectively

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