Comparison of Agglomeration Behavior of Fine Particles in Liquid among Various Mixing Operations

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In order to compare the agglomeration and breakup behavior of fine particles in liquid by various mixing operations, model experiments were carried out and a mathematical model was developed and compared with the experimental results. Three kinds of mixing operations were examined: mechanical stirring by impeller (impeller), gas blow mixing (gas), and gas and liquid jet blow mixing by RH degasser (RH). Polymethylmethacrylate (PMMA) particles of 2.8×10^{-6} m in mean diameter and 3.0×10^3 mol·m⁻³ of KCl solution were used in the experiment as solid and liquid phases, respectively. Total number of PMMA particles at each mixing process decreased with the increasing time, although the agglomeration rate decreased. The PMMA agglomeration rate at the same mixing energy input in the liquid was in the following decreasing order: impeller, gas, and RH mixing. The experimental results of the impeller mixing were able to be explained by a turbulence agglomeration model. A breakup model of particles was newly developed assuming that the agglomerated PMMA particles adhered to the surface of bubbles during bubble floating in the liquid was divided into two pieces on the gas/liquid free surface at the moment of bubble bursting. By introducing this breakup model in addition to the agglomeration one, the calculation results for both of the gas and RH mixing agreed well with the experiment.

KEY WORDS: particle agglomeration; particles breakup; mechanical stirring; gas blow mixing; RH degassing.

1. Introduction

It is important to remove nonmetallic inclusions from molten steel by agglomeration, coalescence and flotation^{1–5)} for the purpose of obtaining clean steel. Many experimental and theoretical studies have been done on the agglomeration behavior of fine particles in liquid: agglomeration structure by using a fractal dimension,^{6–11)} wettability of particles,^{12,13)} heterogeneous agglomeration with a different surface electrification and size,^{14–18)} particle collision frequency in turbulent flow,^{19,20)} particle-size grouping.²¹⁾

However, there are few studies on comparison of the particle agglomeration behavior among various mixing operations, although it is indispensable for the optimal mixing operation and design. In this study, experiments were conducted for three kinds of mixing operations: mechanical stirring by impeller (impeller mixing), gas blow mixing (gas mixing), and gas and liquid mixing by RH degasser (RH mixing) in order to compare the agglomeration rate of fine particles among three mixing operations. The theoretical model on the particle agglomeration coexisting with the breakage of particles was next developed and evaluated. There were some theoretical and experimental studies on the particle breakup by particles collision^{22,23)} and fluid

shear stress,^{23,24)} however, the particle breakup of gas stirring was not treated there. Therefore, in this study, we developed a theoretical model considering not only particle agglomeration but also breakup of agglomerated particles adhered to bubbles during bubble bursting for the gas and RH mixing and compared with the experimental results. Here, RH degasser is a vacuum degassing device utilized at the secondary steel refining in steelmaking field. Molten steel in lower vessel is sucked in the upper vacuum vessel and circulated between the upper and lower vessels by the effect of bubble lift pump where argon gas is blown through a connected tube^{25,26)} as shown later in Fig. 1.

Fig. 1. Experimental apparatuses of impeller, gas and RH mixing.

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2. Experimental Setup

2.1. Experimental Apparatus

Figure 1 shows the experimental apparatuses of the impeller, gas and RH mixing. The same size of cylindrical and acrylic vessel (diameter, D_{vessel} ; 0.15 m, height, H_{vessel} ; 0.21 m) was used for the experiment. Volume, V , of KCl solution in the vessel was also fixed to $2.0 \times 10^{-3} \text{ m}^3$ and bath depth, H_{bd} , became 0.13 m. For the impeller mixing, paddle type of impeller (width, d_{im} ; $5.0 \times 10^{-2} \text{ m}$, thickness, b_{im} ; $2.0 \times 10^{-2} \text{ m}$, number of blade; 2) was used and the impeller immersion depth, H_{id} was kept to $8.0 \times 10^{-2} \text{ m}$. The gas blow mixing was made by a nozzle (inner diameter, D_{nozzle} ; $5.0 \times 10^{-3} \text{ m}$), and nozzle immersion depth, H_{id} , was $8.0 \times 10^{-2} \text{ m}$. For the RH mixing, chamber diameter, D_{chamber} , and height, H_{chamber} , were 0.114 and 0.080 m, respectively. Leg diameter, D_{leg} and height, H_{leg} , were 0.024 and 0.10 m, respectively. Nozzle immersion depth, H_{id} , between nozzle tip and chamber free surface was 0.11 m, and nozzle diameter, D_{nozzle} , of the leg was $1.0 \times 10^{-3} \text{ m}$. Volume of KCl solution including legs and chamber was $5.0 \times 10^{-4} \text{ m}^3$.

2.2. Methods

Polymethylmethacrylate (PMMA) (Sekisui Plastics Co., Ltd.) particles of $2.8 \times 10^{-6} \text{ m}$ in mean diameter were used as a model particle. First, a given number of PMMP particles were put into a small amount of ion-exchanged water and almost thoroughly dispersed with an ultrasonic cleaner (Branson, Series 8500). $3.0 \times 10^3 \text{ mol} \cdot \text{m}^{-3}$ of KCl solution was used to obtain the rapid particle agglomeration condition by making thinner electric double layer around the particles and causing the negligible repulsion between particles. The experiment started when particles-including water was charged into the KCl solution. The sampling times were 0, 1, 2, 4, 10, 30, 60, 120 and 180 min, and the sampling point was set to 0.020 m apart from the center axis of the vessel and 0.05 m high from the bottom of the vessel. $1.0 \times 10^{-6} \text{ m}^3$ of particle-suspended liquid was taken in a single sampling. The temporal changes in number of agglomerated particles and size distribution were measured by a particle size distribution analyzer (Beckman Coulter, Inc., Multisizer 3). The particle size measured here was volume-equivalent diameter. The initial total number density of particle in KCl solution was $2.1 \times 10^{12} - 2.6 \times 10^{12} \text{ m}^{-3}$. Physical properties of PMMA particle and KCl solution are shown in **Table 1**.

2.3. Calculation of Energy Input Rate of Mixing Energy into Liquid Bath

The energy input rate, P [$\text{W} \cdot \text{kg}^{-1}$], of mixing into liquid bath was calculated to compare the agglomeration behavior between three kinds of mixing pattern at the same P value.

P of the impeller mixing is given by Nagata²⁷⁾ as follows:

$$P = N_p \rho_L R_s^3 d_{\text{im}}^5 \dots\dots\dots (1)$$

$$N_p = \frac{A}{Re} + B \left(\frac{10^3 + 1.2 Re^{0.66}}{10^3 + 3.2 Re^{0.66}} \right)^C \left(\frac{H_{\text{bd}}}{D_{\text{vessel}}} \right)^{0.35 + \frac{b_{\text{im}}}{D_{\text{vessel}}}} \dots\dots\dots (2)$$

Table 1. Physical properties of particle and liquid used for the experiment.

PMMA particle	Mean diameter [m]	2.8×10^{-6}
	Density, ρ_p [$\text{kg} \cdot \text{m}^{-3}$]	1.20×10^3
	Hamaker constant, A_{131} [J]	1.05×10^{-20}
Liquid	KCl concentration, C [$\text{mol} \cdot \text{m}^{-3}$]	3.0×10^3
	Density, ρ_L [$\text{kg} \cdot \text{m}^{-3}$]	1.13×10^3
	Viscosity, μ [$\text{Pa} \cdot \text{s}$]	9.27×10^{-4}

Table 2. Operating variables at the same energy input rate into liquid.

	Power [$\text{W} \cdot \text{kg}^{-1}$]	0.021	0.027	0.035
Impeller	R_m [rpm]	250	275	300
Gas	Q_g [$\text{L} \cdot \text{min}^{-1}$]	1.5	2.0	2.5
RH	Q_{RH} [$\text{L} \cdot \text{min}^{-1}$]	1	1.5	2

$$A = 14 \left(\frac{b_{\text{im}}}{D_{\text{vessel}}} \right) \left\{ 670 \left(\frac{d_{\text{im}}}{D_{\text{vessel}}} - 0.6 \right)^2 + 185 \right\} \dots\dots\dots (3)$$

$$B = 10^{\left\{ 1.3 - 4 \left(\frac{b_{\text{im}}}{D_{\text{vessel}}} - 0.5 \right)^2 - 1.14 \left(\frac{d_{\text{im}}}{D_{\text{vessel}}} \right) \right\}} \dots\dots\dots (4)$$

$$C = 1.1 + 4 \left(\frac{b_{\text{im}}}{D_{\text{vessel}}} \right) - 2.5 \left(\frac{d_{\text{im}}}{D_{\text{vessel}}} - 0.5 \right)^2 - 7 \left(\frac{b_{\text{im}}}{D_{\text{vessel}}} \right)^4 \dots\dots\dots (5)$$

where N_p is number of power [-], R_s is rotation speed [s^{-1}] and Re is Reynolds number [-]. The other symbols are shown in Fig. 1.

P of gas and RH mixing is the energy rate given by buoyancy force of bubble during floatation and expressed by Eq. (6).²⁸⁾

$$P = 3.97 \times 10^{-3} T \log \left(1 + \frac{H_{\text{id}}}{10.3} \right) \dots\dots\dots (6)$$

where Q_B : injected gas flow rate [$\text{m}^3 \cdot \text{s}^{-1}$], T : liquid temperature [K].

Table 2 shows the operating variables at the same energy input rate into liquid for three kinds of mixing operations. P value was changed to 0.021, 0.027 and $0.035 \text{ W} \cdot \text{kg}^{-1}$. Gas flow rate of RH is less than that of gas mixing due to longer immersion depth, H_{id} .

3. Experimental Results

3.1. Change in Particle Distribution with Time

Figure 2 shows the temporal change in number distribution of particles normalized by the initial total number of particles. The mixing pattern was mechanical stirring and rotation speed was fixed to 250 rpm. The peak of number of particles in agglomerated cluster of 1, that is no agglomeration, was largest in all. However, particles agglomerated with each other and the peaks of agglomerated particles appeared with time.

Fig. 2. Change in particle size distribution with time for impeller mixing.

3.2. Change in Number of Particles with Time for Each Mixing Pattern

The temporal change in number of total particles with and without agglomeration normalized by the initial particle number for impeller, gas and RH mixing is shown in Fig. 3. The number of particles for all mixing practices decreased with time, which indicated the agglomeration of particles. Now, we focused on the agglomeration rate defined by the decreasing number of particles per unit time. For the impeller and RH mixing, the larger energy input rate caused the more rapid agglomeration rate until about 60 min, although there was no marked change in gas mixing. The number of particles in RH mixing reversed after about 120 min, that is, the agglomeration rate decreased with the increase in the energy input.

Fig. 3. Change in number of particles with time for impeller, gas and RH mixing.

Fig. 4. Change in number of particles with time at the same energy input rate into liquid.

3.3. Change in Number of Particles with Time at the Same Energy Input Rate into Liquid

Figure 4 shows the comparison of the normalized number of total particles by the initial one among the impeller, gas and RH mixing. The energy input rate was changed to 0.021, 0.027 and 0.035 W·kg⁻¹. The agglomeration rate at the same mixing energy input was in the following decreasing order: impeller, gas, and RH mixing. We assumed that the effective agglomeration rate of the gas and RH mixing was smaller than that of impeller mixing due to breakup of a part of agglomerated particles. The breakup is estimated to be attributable to gas bubbles which do not exist for the impeller mixing.

4. Theoretical Agglomeration Model and its Comparison with Experimental Results

4.1. Agglomeration and Breakup Model

4.1.1. Agglomeration Rate

The agglomeration behavior of particles is explained by a discrete population balance and collision frequency function between two particles.²⁹⁾ Assuming that the agglomeration occurs by the collision of *i*- and *j*- agglomerated particles, the number of collisions per second and per cubic meter is expressed by the product of collision frequency function, $\alpha(a_i, a_j)$ [m³·s⁻¹], of clusters of *i*- and *j*- particles, number density, N_i [m⁻³], and N_j [m⁻³], of cluster of *i*- and *j*- particles. Here, a_i and a_j are volumetric equivalent radii of cluster of *i*- and *j*- particles [m], respectively.

Then, the time-derivative of number density, N_k [m⁻³], of *k*- particles is given by Eq. (7) where the first and second items of the right side represent the formation and disappearance rates of cluster of *k*- particles by the collision of *i* and *j*- particles, and *i* and *k*- particles, respectively.

$$\frac{dN_k}{dt} = \frac{1}{2} \sum_{i=1, i+j=k}^{i=k-1} \alpha(a_i, a_j) N_i N_j - \sum_{i=1}^{\text{Max}} \alpha(a_i, a_k) N_i N_k \dots (7)$$

The collision frequency function, $\alpha(a_i, a_j)$, of fine particles in Eq. (7) under turbulent flow was introduced by Saffman-Turner.³⁰⁾

$$\alpha(a_i, a_j) = 1.3(a_i + a_j)^3 \left(\frac{\varepsilon}{\nu} \right)^{\frac{1}{2}} \dots (8)$$

where ε is dissipation rate of turbulence energy [m²·s⁻³] and ν is coefficient of kinetic viscosity [m²·s⁻¹]. Considering the effect of London-van der Waals force based on liquid viscosity and electric polarization between particles,³¹⁾ Eq. (8) is corrected by agglomeration efficiency, Z , as follows.

$$\alpha(a_i, a_j) = 1.3Z(a_i + a_j)^3 \left(\frac{\varepsilon}{\nu} \right)^{\frac{1}{2}} \dots (8')$$

$$Z = 0.727 \left[\frac{\mu a_1^3 \left(\frac{\varepsilon}{\nu} \right)^{\frac{1}{2}}}{A_{131}} \right]^{-0.242} \dots (9)$$

where μ is viscosity and A_{131} is Hamaker constant [J] when suffix 1 means particle and 3 liquid.

Substituting Eqs. (8') and (9) into (7), number of a given cluster of *k*- particles is calculated under the initial condition.

4.1.2. Introduction of Breakup Item

As described in chapter 3, the agglomeration rate of the gas and RH mixing was less than that of the impeller mixing at the same energy input rate into liquid. Thus, we assumed that bubbles of the gas and RH mixing divided partly the agglomerated particles and reduced the net agglomeration rate. **Figure 5** shows the schematic image of particle breakup in the liquid. Various clusters of particles adhere to gas bubbles during floating in the liquid^{32,33)} and they are divided into smaller particles in connection with bubble bursting at the free surface. Here, the bubble adhesion was not observed. On the other hand, we assumed that the impeller mixing in this study had no influence on the breakup of agglomerated particles because the rotation speed was not so high.

Introducing the breakup items for the gas and RH mixing, the time-derivative of number density, N_k [m⁻³], of cluster of *k*- particles is expressed as Eq. (10) where the first and second terms of the right side result from the gross agglomeration as shown in Eq. (7) and the third and fourth terms indicate the breakup of agglomerated particles. As shown in Eq. (10), each particle is assumed to be divided into two parts at bubble bursting.

$$\begin{aligned} \frac{dN_k}{dt} = & \frac{1}{2} \sum_{i=1, i+j=k}^{i=k-1} \alpha(a_i, a_j) N_i N_j - \sum_{i=1}^{\text{Max}} \alpha(a_i, a_k) N_i N_k \\ & + \sum_{i=k+1}^{\text{Max}} \chi_{kli} \beta(a_i, d_B) N_i N_B - \beta(a_i, d_B) N_k N_B \end{aligned} \dots (10)$$

where $\beta(a_i, d_B)$ is breakup frequency function by bubble bursting [m³·s⁻¹], d_B is mean bubble diameter [m], N_B is number density of bubbles [m⁻³], and χ_{kli} is breakup distribution function defined as volumetric ratio of cluster of *k*- particles obtained from the breakup of cluster of *i*- particles. The d_B and N_B values in Eq. (10) are obtained from the following section.

We assumed that $\beta(a_i, d_B)$ was composed of the product of two items: one was the frequency function, γ [m³·s⁻¹], of particles adherence to bubbles and the other was the breakup

Fig. 5. Schematic image of particle adhesion to bubble during floating and agglomerated particle breakup at bubble bursting.

efficiency, δ [-], of adhering particles at the bubble bursting, as expressed by Eq. (11).

$$\beta(a_i, d_B) = \gamma \cdot \delta \quad \dots\dots\dots (11)$$

Equation (12), which was introduced by Arai *et al.*,³⁴⁾ was used as the γ item, whereas we expressed δ as the function of non-dimensional particle number, bubble diameter and bubble number so as to obtain the constant parameter, K , by Eq. (13).

$$\gamma = \frac{\pi(a_i + d_B)^2}{4} u_B \cdot A \left(\frac{a_i}{d_B} \right) \varepsilon^{-B} \frac{1}{1 + \exp[-L(\theta - \theta_i)]} \quad \dots\dots\dots (12)$$

$$\delta = K \cdot i^{\frac{5}{3}} \left(\frac{d_B}{a_i} \right)^5 \left(\frac{N_B}{N_{t0}} \right)^2 \quad \dots\dots\dots (13)$$

where u_B is bubble floating velocity [m/s], θ is contact angle, $A(=0.57)$, $B(=0.21)$, $L(=3.5)$ and $\theta_i(=88)$ are constant. As described in 4.2.2, breakup factor, K becomes = 150.

The assumption of binary breakup of agglomerated particles leads to χ_{kli} expressed by Eq. (14)

$$\chi_{kli} = \frac{1}{i-1} \quad \dots\dots\dots (14)$$

4.1.3. Bubble Diameter and Number Density

Bubble frequency, f_B [s^{-1}], of the gas and RH mixing was observed by a video camera and the mean bubble diameter was calculated from Eq. (15).³⁵⁾

$$d_B = \left(\frac{6Q_B}{\pi f_B} \right)^{\frac{1}{3}} \quad \dots\dots\dots (15)$$

where, Q_B is gas flow rate [$m^3 \cdot s^{-1}$].

Figure 6 shows the relationship between d_B and gas flow rate, Q_g [$L \cdot min^{-1}$], of the gas mixing or gas flow rate, Q_{RH} [$L \cdot min^{-1}$], of the RH mixing in addition to the empirical Eq. (16).^{36,37)}

$$d_B = 0.54(Q_B D_{nozzle}^{0.5})^{0.289} \quad \dots\dots\dots (16)$$

where D_{nozzle} is nozzle diameter [mm], and 5 mm for the gas mixing and 1 mm for the RH mixing in this study. As the results agreed well with them, their formula was applied for the calculation of d_B .

On the other hand, as f_B is obtained from Eqs. (15) and (16) as follows,

$$f_B = 12.1 Q_B^{0.133} D_{nozzle}^{-0.434} \quad \dots\dots\dots (17)$$

Number density, N_B , of bubbles is expressed by Eq. (18).

$$N_B = \frac{f_B H_{id}}{V u_B} \quad \dots\dots\dots (18)$$

where V is water volume [m^3] and H_{id} is floating distance of bubble [m] as shown in Fig. 1. Bubble floating velocity, u_B [$m \cdot s^{-1}$], is given by Eq. (19) on condition that buoyancy and drag are equal.³⁸⁾ In practice, we can use Eq. (19') due to $\rho_{Liquid} \gg \rho_{Air}$, where ρ_{Liquid} and ρ_{Air} are liquid and air density [$kg \cdot m^{-3}$], respectively.

Fig. 6. Relationship between bubble diameter and gas flow rate for gas and RH mixing.

$$u_B = \sqrt{\frac{4g(\rho_{Liquid} - \rho_{Air})d_B}{3C_D \rho_{Liquid}}} \quad \dots\dots\dots (19)$$

$$u_B = \sqrt{\frac{4gd_B}{3C_D}} \quad \dots\dots\dots (19')$$

where g is acceleration of gravity [$m \cdot s^{-2}$], C_D is drag coefficient [-] of which value is 0.44 at $500 < Re < 10^5$.

Thus, substituting Eqs. (16) and (18) into Eq. (10), temporal change in N_k for the gas and RH mixing can be calculated.

4.1.4. Calculation Procedure

The following non-dimensional Eqs. (20) and (21) were obtained from dividing Eq. (10) by N_k^* , N_B^* and t^* defined as Eqs. (22), (23) and (24), respectively.

$$\begin{aligned} \frac{dN_k^*}{dt^*} = & \frac{1}{2} \sum_{i=1, i+j=k}^{i=k-1} \left(\frac{1}{i^3} + \frac{1}{j^3} \right)^3 N_i^* N_j^* - \sum_{i=1}^{\text{Max}} \left(\frac{1}{i^3} + \frac{1}{k^3} \right)^3 N_i^* N_k^* \\ & + \sum_{i=k+1}^{\text{Max}} \chi_{kli} S \cdot i^2 \left[\frac{a_i}{a_1} + \frac{d_B}{a_1} \right]^2 N_i^* N_B^* - S \cdot k^2 \left[\frac{a_k}{a_1} + \frac{d_B}{a_1} \right]^2 N_k^* N_B^* \end{aligned} \quad \dots\dots\dots (20)$$

$$S = 51.6 \cdot \frac{d_B^4 N_B^2 u_B v^{0.5}}{a_1^5 N_{t0}^2 \alpha \varepsilon^{0.71}} \cdot \frac{1}{1 + \exp[-L(\theta - \theta_i)]} \quad \dots\dots\dots (21)$$

$$N_k^* = \frac{N_k}{N_{t0}} \quad \dots\dots\dots (22)$$

$$N_B^* = \frac{N_B}{N_{t0}} \dots\dots\dots (23)$$

$$t^* = 1.3\alpha a_1^3 \left(\frac{\varepsilon}{v} \right)^{\frac{1}{2}} N_{t0} t \dots\dots\dots (24)$$

where N_{t0} is the initial total number density of particles.

As seen in Fig. 2, the particle diameter of 10×10^{-6} m which corresponded to the agglomerated particle number of $[(10 \times 10^{-6}) / (2.8 \times 10^{-6})]^3 = 45.6$ was not almost observed in this study. Thus, we set MAX as 100 and Δt^* as 0.0005 or 0.001. Under the initial condition of Eq. (25) where $N_{k,0}$ indicates the initial number density of k-particles [m^{-3}], the temporal change in N_k (k: 1 to MAX) was calculated numerically by Runge-Kutta method. The values of $N_{k,0}$ were obtained from Fig. 2.

$$\begin{cases} N_1 = N_{1,0} = 1.05 \times 10^7, N_2 = N_{2,0} = 1.58 \times 10^6, \\ N_3 = N_{3,0} = 4.56 \times 10^5, N_4 = N_{4,0} = 2.00 \times 10^5, \dots (25) \\ N_5 = N_6 = \dots = N_{99} = N_{100} = 0 \end{cases}$$

4.2. Comparison between Calculation and Experimental Results

4.2.1. Temporal Change in Number of Particles for Impeller Mixing

Figure 7 shows the comparison of the temporal change in number from one to four clusters of particles between the calculated and experimental results for the impeller mixing. Rotation speed was 250 rpm. The calculated results agreed with the experimental ones. The change in number of clusters of all particles with time is shown in **Fig. 8**. The calculation also explained the experiment successfully. Thus, the

agglomeration behavior of the impeller mixing in this study proceeded without breakup of particles.

4.2.2. Effect of Breakup Factor on Temporal Change in Agglomerated Particles

We show that K of 150 described in Section 4.1.2 is the optimal value. **Figure 9** indicates the change in number of 1 particle, clusters of 2, 3, and 4 particles, and total cluster of all particles with time for the gas mixing. Gas flow rate was $1.5 \text{ L} \cdot \text{min}^{-1}$. When the breakup factor, K , was 150, the calculation fitted well to the experimental results. In the case of the RH mixing of $Q_{RH} = 1.0 \text{ L} \cdot \text{min}^{-1}$, the optimal K value was also 150 as shown in **Fig. 10**. Thus, the breakup behavior of agglomerated particles for the gas and RH mixing was able to be explained by the breakup model that the adhering particles to the floating bubbles were divided at

Fig. 8. Comparison of temporal change in number of clusters of all particles between calculated and experimental impeller mixing.

Fig. 7. Comparison of temporal change in number of particles in clusters between calculated and experimental impeller mixing.

Fig. 9. Comparison of temporal change in number of particles in clusters between calculated and experimental gas mixing as a parameter of breakup factor.

Fig. 10. Comparison of temporal change in number of particles in clusters between calculated and experimental RH mixing as a parameter of breakup factor.

the bubble bursting according to the function of Eq. (13). Figures 9 and 10, the agglomeration rate of particles was predicted to be decreased at $t^* > 1$, although it was not verified by the experiment.

4.2.3. Prediction of Agglomeration Behavior for Three Kinds of Mixing Process

As shown in Sections 4.2.1 and 4.2.2, the calculated results agreed with experimental ones at $P=0.021 \text{ W} \cdot \text{kg}^{-1}$ as shown in Table 2. By using the same K value ($=150$), the temporal change in number, N_i , of all particles was calculated and compared with the experiment under the other conditions. **Figures 11 and 12** show the change in N_i with time at P of 0.027 and $0.035 \text{ W} \cdot \text{kg}^{-1}$, respectively. For three kinds of mixing operation practices such as the impeller, gas and RH, the calculation agreed well with the experiment at the different P values. Compared with the impeller mixing,

the smaller agglomeration rates were reproduced for the gas and RH mixing by using the same breakup factor of $K=150$.

The agglomeration rate for the RH mixing was smaller than that for the gas mixing due to the larger breakup of agglomerated particles. Substituting Eqs. (11)–(13) and (16)–(19) into Eq. (11), $\beta(a_i, d_B)$ can be expressed by d_{nozzle} as follows:

$$\beta(a_i, d_B) \propto d_{\text{nozzle}}^{-0.073} \quad (26)$$

As the nozzle diameters of the RH and gas mixing in this study are 0.001 and 0.005 m, respectively, Eq. (27) is given

$$\beta(a_i, d_B)|_{\text{RH}} / \beta(a_i, d_B)|_{\text{gas}} \propto \left(\frac{0.001}{0.005} \right)^{-0.073} = 1.12 \dots (27)$$

where $\beta(a_i, d_B)|_i$ is $\beta(a_i, d_B)$ of the i ($i=\text{RH, gas}$) mixing. The smaller agglomeration rate of the RH mixing resulted

Fig. 11. Prediction of temporal change in number of clusters of all particles for impeller, gas and RH mixing at energy input rate of $0.027 \text{ W} \cdot \text{kg}^{-1}$.

Fig. 12. Prediction of temporal change in number of clusters of all particles for impeller, gas and RH mixing at energy input rate of $0.035 \text{ W} \cdot \text{kg}^{-1}$.

from the smaller diameter and larger gas velocity through the nozzle.

The experimental results of the impeller mixing were explained only by the particle agglomeration model as described in Section 4.2.1, whereas those of the gas and RH mixings were interpreted by the addition of the breakup model of agglomerated particles. Thus, as shown in Fig. 4, the slightly less agglomeration rate of the impeller mixing than that of the gas mixing or the same level was attributed to the smaller breakup rate of agglomerated particles for

the gas mixing. On the other hand, the overall agglomeration rate of the RH mixing was smaller than that of the gas mixing due to the larger $\beta(a_i, d_B)$ value, that is, the greater breakup rate of the agglomerated particles of the RH condition in this study.

Thus, the agglomeration behavior of fine particles was predicted by the newly developed mathematical model including not only the agglomeration but also the breakup of particles under the different mixing patterns.

5. Conclusions

Model experiments on the agglomeration and breakup behavior of fine particles in liquid among different mixing operations, that is impeller (impeller), gas blow mixing (gas), and gas and liquid jet blow mixing by RH degasser (RH) mixing, were carried out and a mathematical model was developed and compared with them. Results obtained in this study were as follows.

(1) Total number of particles at each mixing process decreased with the increasing time, although the agglomeration rate decreased.

(2) The agglomeration rate of particles at the same mixing energy input in the liquid was in the following decreasing order: impeller > gas > RH mixing.

(3) The experimental results of the impeller mixing were able to be explained by turbulence agglomeration model.

(4) A breakup model of particles was developed assuming that the agglomerated particles adhered to the surface of bubbles during bubble floating in the liquid was divided into two pieces on the gas/liquid free surface at the moment of bubble bursting.

(5) By introducing this breakup model in addition to the agglomeration one, the calculation for both of the gas and RH mixing agreed well with the experimental results.

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