Analysis of Cohesive Particle Percolation in a Packed Bed Using Discrete Element Method

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Many granular materials are in cohesive or wet state in pyrometallurgy processes. The present work systematically studies the cohesive particle percolation behaviour in a packed bed by discrete element method (DEM). The results indicate that the vertical velocity of percolating particles increases with increasing the cohesive force from 0 to 2 m/$g$ (the gravity force of percolating particle, given by $f_e=g/4$). While for a higher cohesive force, e.g. $f_e=8m/g$, insufficient percolation occurs and percolating particles stick in the packed bed. Percolating particles in the packed bed shows a diffusivity for the cases of smaller cohesive force. The transverse dispersion of $f_e=2m/g$ is smaller than that of $f_e=0$, while the longitudinal dispersion becomes larger when the cohesive force changes from 0 to 2 m/$g$. In addition, the influence of other key variables, such as diameter ratio, damping coefficient and rolling friction coefficient on percolation behaviour is also discussed. The transverse dispersion coefficient increases with the diameter ratio, while the longitudinal dispersion coefficient decreases with the diameter ratio. With the increase of damping coefficient or rolling friction coefficient, the transverse dispersion coefficient decreases but the longitudinal dispersion increases. The study provides a fundamental understanding on percolation behaviour of cohesive particles in a packed bed, and is useful for processes understanding and optimization in cohesive particle handling and mixing.

KEY WORDS: particle percolation; cohesive force; percolation velocity; dispersion coefficient; discrete element method.

1. Introduction

Granular materials are commonly present in various processes in chemical engineering, food and metallurgical industries. If the grain sizes in a granular material are very different, one may observe that the smaller grains can drain through the piling of larger ones. This is usually termed spontaneous inter-particle percolation. The percolation is a common phenomenon in nature and industries.1) One specific industry example can be found in ironmaking blast furnace. When the smaller iron ore particles are loaded upon larger particles, the smaller particles may pass through the larger ones in descending motion under gravity.2) Besides, during burden charging in ironmaking blast furnace, the mixing and segregation behaviour can be observed,3–5) one of the main reasons is particle percolation. Moreover, the choking of gas slots in bustle pipe of COREX shaft furnace arises mainly from the insufficient percolation of dust particles in the burden.6) On the other hand, actually, in many industrial processes, granular materials may in cohesive or wet state.7–9) This is especially important in ironmaking processes. For example, the significant stickiness between iron ore particles was observed within the temperature range of 600 to 675°C.10) As an attractive force between particles, if the cohesion is significant, substantial difference from the free-flowing behaviour of particulate systems is evident. Hence, in the lump zone of blast furnace or COREX shaft furnace, the cohesion between particles, induced by reduced elastic modulus of relevant materials and in particular, softening-sticking bridge as a result of high temperature operation and reduction reaction of iron ore, may directly affect the percolation behaviour in the packed bed. Thus, it is necessary to understand the fundamentals in the percolation phenomenon in a packed bed, and moreover, explore the physics of cohesive particle percolation for understanding the mixing and segregation of multi-scale burdens in ironmaking processes.

In the past, the phenomena of cohesionless particle percolation have been investigated by means of various physical and numerical experiments. Bridgwater et al.11–13) made a pioneering research in experimentally studying the interparticle percolation including percolation velocity, residence time and radial distance distribution. Ippolito et al.14) experimentally investigated the dispersion of small spherical beads moving under the effect of gravity inside a packing of large spheres. Richard et al.15) used Monte Carlo method to analyse various properties relevant to percolation. Lomine

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and Oger\textsuperscript{16,17} conducted experiments and discrete element method (DEM) studies to analyse dispersion of particles through a porous structure. Rahman and Zhu\textsuperscript{18,19} and Li \textit{et al.}\textsuperscript{20} studied the effect of particle properties on particle percolation behaviour in a packed bed using DEM. Recently, Fukuda \textit{et al.}\textsuperscript{21} investigated the microscopic flow behaviour of powder in a simplified packing structure under various damping coefficients, diameter ratios and friction coefficients. These research works are useful for understanding the fundamentals about fine particle percolation behaviour in a packed bed. In addition, for particle percolating in ironmaking reactors, Yu \textit{et al.}\textsuperscript{22} studied inter-particle percolation of small particles (pellets) into large particles (coke) during burden descent in blast furnace through small-scale experiments and simulations. Then, Yu and Saxén\textsuperscript{23} investigated the effect of DEM parameters such as friction and restitution coefficients, shear modulus, as well as pellet diameter on the percolation velocity, residence time distribution, longitudinal and transverse dispersion are studied. The effects of some important phenomena have not been understood, for example, the effect of cohesion between particles on percolation behaviour was not studied in details.

This work studies the effect of cohesive force between particles on percolation behaviour in a packed bed by means of DEM approach. Percolation behaviours such as percolation velocity, residence time distribution, longitudinal and transverse dispersion are studied. The effects of some key variables such as size ratio of packing particles to percolation ones, damping coefficient, and rolling coefficient on percolation behaviours are investigated. This provides important fundamental understandings of percolation of cohesive particles in packed bed and thus beneficial for process understanding and optimization in ironmaking processes.

2. Model Details

2.1. DEM Model

Each single particle in a considered system undergoes both translational and rotational motion, described by Newton’s 2nd law of motion. The forces and torques considered include those originating from the particle’s contact with neighbouring particles, walls and surrounding fluids. The governing equations for translation and rotational motions of particle \(i\) with radius \(R_i\), mass \(m_i\) and moment of inertia \(I_i\), can be written as

\[
m_i \frac{d\mathbf{v}_i}{dt} = \sum_{j \neq i}^n \left( F_{a,i,j} + F_{n,i,j} + F_{t,i,j} + F_{c,i,j} + F_{f,i,j} + F_{g,i,j} + F_{d,i,j} \right) + m_ig \quad \ldots (1)
\]

\[
I_i \frac{d\mathbf{\omega}_i}{dt} = \sum_{j \neq i}^n \left( (T_j + M_j) \right) \quad \ldots (2)
\]

where, \(m_i\), \(I_i\), \(v_i\), and \(\omega_i\) represent mass, rotational inertia, translational velocity, and rotational velocity of particle \(i\), respectively; \(F_{a,i,j}\), \(F_{n,i,j}\), \(F_{t,i,j}\), \(F_{c,i,j}\), \(F_{f,i,j}\), \(F_{g,i,j}\), \(F_{d,i,j}\) represent normal contact force, normal damping force, tangential contact force, tangential damping force, cohesive force, and tangential and rolling friction torque of particle \(j\) acting on particle \(i\), respectively; \(g\) is the gravitational acceleration; \(k_i\) is the number of particles in contact with particle \(i\); and \(t\) is time. According to the literatures\textsuperscript{24–28} equations for contact force, damping force, friction force, and torque are listed in Table 1, where, \(R^2\) is equivalent radius; \(R\) is vector from the mass centre of the particle to the contact point; \(\dot{v}_i\) is equivalent Young’s modulus; \(E\) is Young’s modulus; \(\nu\) is Possion’s ratio; \(\delta_i\) is normal particle overlap; \(\mathbf{\hat{u}}\) is a unit vector from the centre of the particle to the contact point; \(m_j\) is equivalent mass; \(v_{n,i,j}\) is the normal relative velocity of particle \(i\) and \(j\); \(v_{t,i,j}\) is tangential relative velocity of particle \(i\) and \(j\); \(\delta_{ij}\) is particle tangential overlap; \(\delta_{ij,max}\) is maximum particle tangential overlap; \(\mathbf{\hat{\delta}}\) is unit vector of particle tangential overlap; \(\mu_{ij}\) is rolling friction coefficient; \(\mu_{ij}\) is sliding frictional coefficient; \(c_n\) is the normal damping coefficient; \(c_t\) is the tangential damping coefficient; \(\dot{\mathbf{\omega}}_{ij}\) is unit vector of particle angular velocity.

Cohesive force between particles can originate from several sources including van der Waals force, electrostatic force, and liquid bridges (capillary forces).\textsuperscript{29,30} This work is focused on a general understanding of percolation behav-

<table>
<thead>
<tr>
<th>Force and torque</th>
<th>Symbol</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Contact force</td>
<td>(F_{c,i,j})</td>
<td>(-\mu_i \left</td>
</tr>
<tr>
<td>Damping force</td>
<td>(F_{d,i,j})</td>
<td>(-c_n \sum_{j \neq i}^n \left( m_j E \sqrt{R^2 \delta_i^2} \right) v_{n,i,j})</td>
</tr>
<tr>
<td>Gravity</td>
<td>(F_{g,i})</td>
<td>(m_ig)</td>
</tr>
<tr>
<td>Tangential torque</td>
<td>(T_{ij})</td>
<td>(R_i \left( F_{a,i,j} + F_{c,i,j} \right) )</td>
</tr>
<tr>
<td>Rolling friction torque</td>
<td>(M_{ij})</td>
<td>(\mu_{ij} \left</td>
</tr>
</tbody>
</table>

Note: \[
\frac{1}{E} = \frac{1}{\mathbf{R}} - \frac{1}{\mathbf{R}} \frac{E}{\mathbf{R}} \\
\dot{v}_i = \dot{v}_j + \mathbf{a}_i \times \mathbf{R} - \mathbf{a}_j \times \mathbf{R}, v_{n,i,j} = (v_{n,i,j} - \mathbf{\hat{u}}) \times \mathbf{a}_i, v_{t,i,j} = (v_{t,i,j} - \mathbf{\hat{u}}) \times \mathbf{a}_i
\]
our of cohesive particle in a packed bed, thus a simplified model is employed to reduce computational requirement while reasonable and general results can be obtained. The cohesive force $f_{e,ij}$ is expressed as a multiple of the weight of a fine particle, i.e.

$$f_{e,ij} = K \rho g \pi d^3 / 6 \quad \cdots \cdots \cdots \cdots (3)$$

Note that the interparticle force expressed in Eq. (3) is artificially imposed. It is expressed as a multiple ($K$) of the weight of a single percolating particle. We assume that the cohesion force only exists when the gap between two particles is less than a critical value (1% of particle diameter), and the cohesion force disappears as soon as the gap exceeds the critical value. This force acts both between percolating-packed particles and percolating-percolating particles. The magnitude of the interparticle force can be easily varied by varying the value of $K$ in Eq. (3). A similar approach has been used in previous works for simplification.31–34)

2.2. Simulation Conditions

As shown in Fig. 1, the simulation setup is made of a cylindrical container of $\phi 15D \times 15D$ filled with a packing of monosize large particles of diameter $D$ (which are referred to as packing particles here). This packing is built by random gravity deposition of particles. This procedure gives a reproducible porosity around 0.4. Small particles (percolating particles) of diameter $d$ are put on the top of the packed bed. They are generated randomly at the centreline of the column in a circle of diameter of $1D$. These percolating particles pass through the packed bed towards the bottom of the column under gravity. Their dynamic details are recorded for analysis. The parameters used in the present simulations are listed in Table 2. In this work, the same simulation process is repeated three times and each packing is rebuilt for each simulation. Each numerical data which are presented in this paper are coming from a statistical mean of several simulations. The error bars are deduced from these replicate simulations.

3. Results and Discussion

3.1. Model Validity

In this work, the present DEM model is validated by the angle of repose for coarse spheres. The simulations were carried out under conditions similar to those used in previous physical experiments.35) The so-called discharging method is used to examine the angle of repose. The physical experiments were carried out in a rectangular container with a fixed middle plate and two side outlets. The width of the container was 400 mm, and the thickness can be scaled up or down with the particle size. In order to reduce the computational cost, a scaled down rectangular container was used in numerical simulation. Particles with diameter of 2 mm (particle density $= 2500$ kg/m$^3$, $\mu_s = 0.05$ mm and $\mu_r = 0.4$) were employed. Figure 2 shows the typical sandpiles constructed by physical experiments and numerical simulations.

<table>
<thead>
<tr>
<th>Variables</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of packed particle, $D$(m)</td>
<td>0.01</td>
</tr>
<tr>
<td>Percolating particle diameter, $d$</td>
<td>$1/14$–$1/8D$</td>
</tr>
<tr>
<td>Percolating particle number, $N$</td>
<td>500</td>
</tr>
<tr>
<td>Sliding frictional coefficient, $\mu_s$</td>
<td>0.3</td>
</tr>
<tr>
<td>Rolling frictional coefficient, $\mu_r$</td>
<td>0.001–0.02$D$</td>
</tr>
<tr>
<td>Young’s modulus, $E$(Pa)</td>
<td>$50000\pi D^2$</td>
</tr>
<tr>
<td>Poisson’s ratio, $\nu_p$</td>
<td>0.3</td>
</tr>
<tr>
<td>Damping coefficient, $c$</td>
<td>0.1–0.7</td>
</tr>
<tr>
<td>Time step, $\Delta t$(s)</td>
<td>$1.0 \times 10^{-7}$</td>
</tr>
</tbody>
</table>

Fig. 1. Geometry of the model used in this work.

Fig. 2. Sandpiles generated via physical and numerical experiments with different container thickness: (a) $w = 8$ mm; (b) $w = 24$ mm; (c) $w = 40$ mm.
with different container thicknesses. Note that the numerical and experimental results were obtained from different-sized containers although they appear to be of the same size in the two figures. Because of the relatively small number of particles, the numerical simulation sometimes does not produce a sandpile of smooth straight surface. Nevertheless, the results clearly indicated that the angle of repose decreases with increasing container thickness, and the numerical simulations and physical experiments are comparable. Therefore, the present model is suitable to carry out further simulation.

3.2. Effect of Cohesive Force on Percolation Behaviour

When a smaller percolating particle is put on a packed bed of larger particles, it may move down through the bed in longitudinal and transverse directions under gravity and interactions with the packing particles. The longitudinal direction is referred to the flow direction, and the transverse one is referred to the direction perpendicular to the flow direction. The percolation velocity reflects, to a degree, the dispersion property of percolating particles. Therefore, the percolation velocity and dispersion are two important percolation indicators.

3.2.1. Percolation Velocity

Figure 3 shows the evolution of normalized mean height and normalized mean vertical velocity of percolating particles under different cohesive force conditions when \( D/d = 8, \ c = 0.3 \) and \( \mu = 0.001D \). The time is set to be proportional to the free fall time to past a single large sphere diameter and the velocity is in units of free fall velocity reached after falling over one large particle. It is indicated that the normalized height and mean velocity decreases rapidly once percolating particles collide to the packed particles. Then the percolating particles would move toward to the opening of the orifice and percolate among packed particles. For those percolating particles with cohesionless force, the normalized height decreases gradually and the mean velocity decreases progressively towards a steady value. This phenomenon can be related to previous results: for the case of particles falling down in a random packed bed of larger particles, the vertical velocity is a constant.\(^{11,12,16,18}\) For the cohesive force \( f_e = 2 \text{ mg} \), the variation trends of the normalized height and mean velocity are similar to that of cohesionless case, except the mean vertical velocity larger than that of \( f_e = 0 \text{ mg} \). The reason will be discussed in next section by combining the dispersion behaviour. While for the case with \( f_e = 8 \text{ mg} \), the normalized height decreases progressively towards to a steady value and the mean velocity is reduced to zero. This is because cohesive force is strong enough to resist the inertia motion of percolating particles and the insufficient percolation can occur. Percolating particles will finally adhere to the packed particles and blockage can be observed in this condition.

Figure 4 presents the statistic distributions of residence time for different cohesive forces when \( D/d = 8, \ c = 0.3 \) and \( \mu = 0.001D \). As the percolating particles directly stick on the surface of packed particles when the cohesive force \( f_e = 8 \text{ mg} \), the residence time for this condition is not considered. It has been obtained in the previous experimental studies that the residence time distribution is roughly similar to a normal distribution.\(^{12}\) Similar trend can also be observed in the present simulation. Besides, with the cohesive force increasing from 0 to 2 mg, the distribution curve shifts to the left. It is because the larger the cohesive force, the higher the particle percolation velocity, and then the less time for the particles to reach the bottom of the packed bed.
3.2.2. Dispersion Behaviour

The dispersion of percolating particles is a random walk process. The percolation flow of the small particle is subject to stochastic motion, which is caused by the interaction between percolating and packing particles. Such stochastic motion leads to dispersion of percolating particle within a region in the packed bed. Figure 5 illustrates the distribution function of particle positions at the exit of packed bed for different cohesive forces. It can be seen that the smaller the cohesive force, the more off-centre the dispersion distance. Especially for the case \( f_e = 8 \text{ m}g \), the insufficient percolation occurs and the percolating particles almost stick in the centre part of the packed bed. In order to quantitatively describe the dispersion behaviour, detailed information, such as the transverse and longitudinal dispersion coefficients, will be discussed.

The DEM model is possible to access individual particle positions, anywhere at any time, inside the packing of larger spheres. The position of particle \( k \) in the horizontal plane can be described as \( r_k^2 = x_k^2 + y_k^2 \), where \( x_k \) and \( y_k \) are the particle positions. So, the variance of the position distributions of the \( N \) moving particles in this plane is

\[
\langle (\Delta r)^2 \rangle = \frac{1}{N} \sum_{k=1}^{N} (r_k - \langle r \rangle)^2
\]  \quad (4)

where \( \langle r \rangle = \frac{1}{N} \sum_{k=1}^{N} r_k \). In the same manner, if the particle position in the flow direction is denoted by \( z_k \), it can be written as

\[
\langle (\Delta z)^2 \rangle = \frac{1}{N} \sum_{k=1}^{N} (z_k - \langle z \rangle)^2
\]  \quad (5)

where \( \langle z \rangle = \frac{1}{N} \sum_{k=1}^{N} z_k \).

Then, the transverse and axial dispersion coefficients \( D_{\perp} \) and \( D_{\parallel} \) can be defined from the time evolution of \( \langle (\Delta r)^2 \rangle \) and \( \langle (\Delta z)^2 \rangle \) with Einstein-Smoluchowski equation as follows.

\[
\langle (\Delta r)^2 \rangle = 2D_{\perp}t \quad \text{and} \quad \langle (\Delta z)^2 \rangle = 2D_{\parallel}t
\]  \quad (6)

Figure 6 presents the variances, \( \langle (\Delta r)^2 \rangle \) and \( \langle (\Delta z)^2 \rangle \) calculated with Eqs. (4) and (5), of particle position distribution for different cohesive forces versus time. Two main features can be observed from Fig. 6. The first is that a transition state occurs in the initial period. When particles are launched on top of the packed bed, a quantity of them rebounces on the upper packing surface. Moreover, due to the presence of other small particles, some particles cannot enter in the porous space since pores are already filled with other particles. These effects can lead to a penetration delay. Thus, the percolating particles need time to reach a diffusive stage. The more probable value of the transition time is close to a value of \( 5.7(D/4 \text{ g})^{1/2} \) when \( f_e = 0 \). This value is corresponding to the free fall time over a distance of \( 2D \), integrating the distance between the two initial packing...
positions and the first grain thickness. With increasing the cohesive force, the transition time slightly increases. The other feature is the linear evolution of \( \langle (\Delta r)^2 \rangle \) and \( \langle (\Delta z)^2 \rangle \) are observed after the transition state. Linear fits obtained with unweighted least-squares method are presented for cohesive force \( f_e = 0 \) and \( f_e = 2 \) mg. We can notice that the linear evolution of the two variances with time is a typical proof of a diffusive property. It should be pointed out that for the larger values of \( t \), the deviation from the fits in Fig. 6 can be explained by the finite size of our simulation and some percolating particles have already reached the bottom of the packed bed. Diffusive motion of an isolated particle and a blob of particles were found by Ippolito et al.\(^{14}\) and Lomine et al.\(^{16}\) respectively. Our simulations prove the same behaviour for smaller cohesive force. However, as mentioned above, the insufficient percolation occurs for the case \( f_e = 8 \) mg, and no diffusive motion can be observed. From the Fig. 6, it also can be seen that the transverse dispersion of \( f_e = 2 \) mg is smaller than that of \( f_e = 0 \), while the longitudinal dispersion becomes larger when cohesive force changes from 0 to 2 mg. The main reason can be explained as: When percolating particles meet packing particles, they would move downwards and experience multiple collisions. All the contact between the percolating and packing particles would directly decrease the bouncing due to the cohesive force acting as an attractive force. Under the effect of cohesive force, percolating particles would explore laterally the packed bed more difficulty and have a greater probability of passing through the vertical pore. Therefore, the transverse dispersion decreases while the longitudinal dispersion and normalized mean vertical velocity increase with the cohesive force changing from 0 to 2 mg.

### 3.3. Effect of Key Variables on Percolation Behaviour

As the percolating particles would stick in the packed bed when \( f_e = 8 \) mg, the percolation behaviour, such as percolation velocity, transverse dispersion and longitudinal dispersion, will not be discussed in the following part. Detailed studies will focus on the effect of other key variables, e.g. diameter ratio, damping coefficient and rolling coefficient, on percolation behaviour. In this section, the fitted lines obtained by the unweighted least-squares method are represented in the main diffusion stage, and all the dispersion coefficients are determined by the slope of the fitted lines.

#### 3.3.1. Diameter Ratio \( D/d \)

**Figure 7** shows the variation of percolation velocity with diameter ratio of packing particle to percolating particle for different cohesive force. It can be seen that with increasing the particle diameter ratio, the percolation velocity increases. This can be easily explained by considering a single particle falling down toward to a pore. The particle trajectory must be aligned with the pore hole to pass through it without bouncing around for big ratio of particle size. If not, the smaller the size ratio is, the more important is the collisions of particles. In such a case, the downward motion of the percolating particle is more difficult and leads to a decrease in the percolation velocity.

**Figure 8** shows the variations of transverse dispersion and longitudinal dispersion coefficients with the particle diameter ratio. Figure 8(a) demonstrates that the slope of the
fitted line increases with the diameter ratio. With the diameter ratio $D/d$ increases from 8 to 14, the transverse dispersion coefficient of $f_e=0$ and $f_e=2 \, \text{mg}$ increases from 0.785 to 1.21 cm$^2$/s and 0.476 to 0.902 cm$^2$/s, respectively, as shown in the up left region of Fig. 8(a). The smaller the percolating particles are, the longer distance they can laterally move. On the other hand, Fig. 8(b) shows that, the longitudinal dispersion $D_\parallel$ decreases with the increasing of the diameter ratio. The longitudinal dispersion coefficient of $f_e=0$ and $f_e=2 \, \text{mg}$ decreases from 4.674 to 2.525 cm$^2$/s and 9.12 to 3.105 cm$^2$/s respectively, with increasing the diameter ratio. The transverse dispersion coefficient varies in opposition to the longitudinal dispersion. For the smaller diameter ratio, the pore throats acting like gates in packed bed would create a “gate or valve effect” which could lead to an impediment on the transverse motion of percolating particle, hence reduces the transverse dispersion. On the other hand, when $D/d$ is small, the particles have a smaller probability of moving outside a pore in the horizontal direction and thus a greater probability of passing through the vertical pore.

### 3.3.2. Damping Coefficient

Damping force is modelled as a dashpot that dissipates a proportion of the relative kinetic energy. It is related to the relative velocity of the contacting particles. So, the damping coefficient has a significant effect on the percolation. The variation of percolation velocity with damping coefficient is shown in Fig. 9. It can be observed that for higher damping coefficient between percolating and packing particles, the percolation velocity is higher. This is because, for higher damping coefficient, the percolating particles would easily pass through the pore hole without bouncing around. Although increasing the damping coefficient could dissipate the relative kinetic energy, the effective percolation path in the packed bed is shorten. Under the action of gravity force, the mean velocity in vertical direction will increase with the increasing of damping coefficient.

Figure 10 shows the variations of the transverse dispersion coefficient and longitudinal dispersion coefficient with damping coefficient. Figure 10(a) reveals that the transverse dispersion coefficient decreases with the damping coefficient. The smaller the damping coefficient is, the longer distance they can laterally move. From the Fig. 10(b), it can be seen that the longitudinal dispersion $D_\parallel$ increases with the increasing of the damping coefficient. For example, when the cohesive force $f_e=2 \, \text{mg}$, with increasing of damping coefficient, the transverse dispersion coefficient decreases from 0.853 to 0.542 cm$^2$/s while the longitudinal dispersion increases from 3.041 to 4.095 cm$^2$/s. The main reason can be explained as: Damping coefficient is related to restitution coefficient. For a fixed Young’s modulus, increasing damping coefficient decreases restitution coefficient. Thus, with a smaller restitution coefficient, the percolating particles bounce less and can more difficulty find their way into the radial space between packing particles, giving a decrease in transverse dispersion coefficient. This way, longitudinal crossing of individual pore is easier and leads to an increase in the longitudinal dispersion coefficient.

### 3.3.3. Rolling Friction Coefficient

Rolling friction provides an effective mechanism to control the translational and rotational motions and largely...
determine the energy dissipation at the contact point. It can also be considered as a shape factor in DEM model.\textsuperscript{36,37} The effect of rolling friction coefficient on percolation velocity is shown in Fig. 11. It can be observed that for higher rolling friction coefficient, the percolation velocity is lower. The main reason is that the contact between the percolating and packing particles would result in a rolling resistance due to elastic hysteresis losses or viscous dissipation. Therefore, the larger rolling friction coefficient would lead to smaller percolation velocity.

Figure 12 shows the variations of the transverse dispersion coefficient and longitudinal dispersion coefficient with $\mu_c$. From the Fig. 12(a), it can be seen that the larger the rolling friction coefficient is, the less is the particles transverse dispersion. With increasing of rolling friction coefficient, the transverse dispersion coefficient of $f_e=0$ and $f_e=2 \text{ mg}$ decreases from 0.948 to 0.672 cm\(^2\)/s and 0.675 to 0.356 cm\(^2\)/s, respectively. Besides, Fig. 12(b) shows that, the longitudinal dispersion coefficient $D_{\parallel}$ increases with the increasing of the rolling friction coefficient. The longitudinal dispersion coefficient of $f_e=0$ and $f_e=2 \text{ mg}$ increases from 2.842 to 5.337 cm\(^2\)/s and 3.502 to 11.749 cm\(^2\)/s, respectively. The main is that, for smaller rolling friction coefficient, the energy dissipation is reduced and the particles can explore laterally the porous medium more easily due to the more chance to bounce around the packing particles. This observation is consistent with the results of the study of Yu and Saxén,\textsuperscript{23} where a low rolling friction between pellets promotes the percolation in ironmaking blast furnace. On the other hand, when the rolling friction coefficient is increased, more relative kinetic energy can be dissipated, and the gravity force is more important. Hence, the longitudinal crossing of individual pore is easier and leads to an increase in the longitudinal dispersion coefficient.

4. Conclusions

The percolation behaviour plays an important role in many pyrometallurgy processes. The key phenomena including percolation velocity, residence time distribution, longitudinal and transverse dispersion, of cohesive fine particles in a packed bed is studied numerically by means of DEM approach. The effects of key variables such as the diameter ratio of packing particle to percolating particle, damping coefficient, and rolling friction coefficient on percolation behaviours are examined. The following conclusions are obtained:

(1) The vertical velocity of percolating particles moving down through a random packed bed of larger particles is constant, but it increases with increasing the cohesive force from 0 to 2 mg. While for a higher cohesive force, e.g. $f_e=8 \text{ mg}$, insufficient percolation occurs and percolating particles may stick in the packed bed.

(2) The percolating particles moving in the packed bed shows a diffusive property for smaller cohesive force. The transverse dispersion of $f_e=2 \text{ mg}$ is smaller than that of $f_e=0$, while the longitudinal dispersion becomes larger when cohesive force changes from 0 to 2 mg.

(3) With the increase of diameter ratio of packing particles to percolating ones, the percolation velocity increases. The transverse dispersion coefficient increases with the
diameter ratio, while the longitudinal dispersion coefficient decreases with the diameter ratio.

(4) Damping coefficient affects the percolation behaviour. For higher damping coefficient, the percolation velocity is larger. Increasing the damping coefficient reduces the transverse dispersion coefficient, but increases the longitudinal dispersion coefficient.

(5) Increasing the rolling friction coefficient reduces the percolation velocity. The larger the rolling friction coefficient is, the less is the particles transverse dispersion. On the other hand, the longitudinal dispersion increases with the increasing of the rolling friction coefficient.

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