Effect of Particle Velocity on Penetration and Flotation Behavior

Akihiro MATSUZAWA,* Katsuhiro SASAI, Hiroshi HARADA and Mitsuhiro NUMATA

R & D Laboratories, Nippon Steel & Sumitomo Metal Corporation, 1 Oaza-Nishinosu, Oita, 870-0992 Japan.

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To improve the properties of steel, lower sulfur content is required. Therefore, powder blast is often used for desulfurization in secondary refining process, and increasing efficiency is important for desulfurization treatment. As a fundamental research, water model experiment under reduced pressure was carried out and behavior of a particle from penetration to flotation was analyzed. A polystyrene particle (3.2 mm in diameter) was blasted onto the water surface with Ar gas through a single-hole nozzle, and behavior of the particle was recorded by a high-speed camera. According to penetration of the particle, an air column was generated and a bubble remained on the particle after rupture of the air column. The particle with high velocity before penetration floated to the water surface rapidly even though they penetrated into water deeply as the residual bubble was easily generated. Therefore, increasing particle velocity before penetration does not result in increasing detention time of the particle, and it is suggested that the particle should be blasted by its own velocity without generating the air column to avoid remaining the bubble. In addition, flotation behavior of the particle was analyzed by a kinetic equation. As a result, when the residual bubble was greater than 2 mm in diameter, the apparent resistance coefficient increased with diameter of the residual bubble. This is because that increasing diameter of the residual bubble led to increasing projection cross section area of the particle including the bubble, and because the friction force between liquid and the particle was increased.

KEY WORDS: secondary refining process; water model experiment; velocity of a particle; penetration depth; residual bubble; detention time of a particle.

1. Introduction

Lower sulfur concentration of steel is required because demand for improvement of steel properties is increasing. Therefore, improving efficiency of dephosphorization or desulfurization treatment is one of the most important problems in the refining process. Desulfurization treatment for low sulfur steel is often operated in secondary refining processes, such as RH or LF, and powder blast is often carried out in RH process. In this process, refining reagent particles are blasted onto surface of molten steel in the vacuum vessel. These particles penetrate and are dispersed into the molten steel. Then, these particles transfer from down-leg to ladle and float in the ladle. After that, most of the particles are absorbed by ladle slag. It is considered that effective penetration and dispersion of refining reagent particles are important because surface tension of molten steel is large and difference of density between molten steel and refining reagent particles is also large.

Many researches related to penetration behavior of particle can be classified into two broad groups. One includes experiments using fine powder, while the other involves experiments using a single particle of relatively large diameter. The main purpose of the former group is clarifying penetration behavior from a macroscopic perspective. For example, Engh et al.1) suggested empirical formula of penetration depth by water model experiment, and Kimura2) reported that penetration mode was different according to the condition of powder blast. Also, estimation of mass transfer velocity in power blasting condition was reported.3) Furthermore, for powder injection process into liquid, Narita et al.4) reported penetration and diversion behavior, and Oda et al.5) evaluated effect of wettability (contact angle) and diameter of particle on the penetration ratio.

On the other hand, the main purpose of the latter group is clarifying penetration behavior in detail from microscopic perspective. For example, Ozawa et al.6) carried out experiments where a particle, such as glass, is dropped on mercury surface and suggested the critical condition of particle penetration as critical Weber number. Also, Lee et al.7) carried out water model experiment by blasting a polystyrene particle onto water surface, analyzed relation between particle penetrating velocity and penetration depth, and concluded that most of kinetic energy of the particle was lost by air column generation. Furthermore, Shimamoto et al.8) and Tanaka et al.9) carried out water model experiment which a single particle was dropped onto water surface, and phenomenon of particle penetration to liquid was analyzed. According to Shimamoto et al., an air column was generated by penetration of the particle and deformation of the water surface. Then, a part of the air
column was remained on the particle surface and became the residual bubble.

As mentioned above, many experimental results about critical penetrating condition or maximum penetration depth of the particle have been reported. However, changes of detention time of the particle with increasing maximum penetration depth have not been reported. Therefore, in this study, water model experiment blasting a single particle onto water surface was carried out in order to clarify the effect of particle velocity on penetration and flotation behavior. Then, relation between maximum penetration depth and detention time of the particle was analyzed. In addition, maximum penetration depth and flotation behavior of the particle was analyzed by kinetic equation.

2. Experimental Procedures

A schematic view of experimental apparatus is shown in Fig. 1. It was made of transparent acrylic resin and the vacuum vessel (inner size is 280 mm in height, 180 mm in width and 90 mm in depth) is rectangular to avoid distortion caused by refraction. This apparatus has close-packed structure including water bath to reduce pressure in the vessel. Pressure in the vessel was measured by the pressure gauge. A predetermined amount of Ar gas was inserted into the vessel through a single-hole nozzle (5 mm in inner diameter) which was set at center of upper plate of the vessel. The nozzle had 70 mm or 140 mm in length, and water depth in the vessel before reducing pressure was 80 mm.

Experimental procedures are as follows. Pressure in the vessel was reduced by the vacuum pump and maintained at 52 kPa by the vacuum regulator. Water depth in the vessel was increased to 84 mm by reducing pressure. Then, cock-B was closed and a single particle (polypropylene, 910 kg/m³ in density, 3.2 mm in diameter) was kept between cock-A and cock-B. Next, cock-B was opened after closing cock-A. The particle was dropped through the nozzle and blasted onto water surface with Ar gas. Behavior of the particle was recorded by a high speed camera (250 frames/s) and change of penetration depth of the particle over time was analyzed by the equation which converts length on the monitor of the camera to actual length. The converting equation was obtained by preliminary experiments.

Experimental conditions are shown in Table 1. Ar flow rate, Q_Ar, and nozzle gap, h (distance from static water surface to tip of the nozzle) was varied. Experiments were carried out 3 times in each condition.

Particle velocity before penetration to the water surface was measured in another experiment. The particle discharging from nozzle was recorded by the high speed camera (1 000 frames/s), and particle velocity was calculated from transport distance between 2 frames (0.001 s) just before penetration to the water surface. Particle velocities before penetration are listed in Table 1.

3. Experimental Results and Discussion

3.1. Change of Penetration Depth over Time of a Particle

Recorded pictures in case D2 (particle velocity before penetration: 6.7 m/s) is shown in Fig. 2 as an example of penetration and flotation behavior. The water surface was deformed by particle penetration, and an air column was generated. When the air column was ruptured, a part of the air column remained on the particle and became a residual bubble. After the rupture of air column, the particle was penetrated until maximum depth and floated to the water surface.

On the other hand, particle penetrated without generating an air column nor a residual bubble in some conditions which particle velocity before penetration is low, such as case A2 (particle velocity before penetration: 1.1 m/s) shown in Fig. 3.

Changes of penetration depth over time are shown in

![Fig. 1. Experimental apparatus.](image-url)
Fig. 2. Penetration and flotation behavior of a particle (D2-2).

Fig. 3. Penetration and flotation behavior of a particle (A2-1).

Fig. 4. Though penetration was deepest in condition E2, the time from penetration to flotation to the water surface became longest in condition B1.

Measured values from pictures of high speed camera are shown in Fig. 5. (a) Particle velocity after penetration was calculated by transfer distance between 2 frames (0.004 s) just after penetration to water surface. (b) Maximum air column length, \( H_{\text{max}} \), was defined as distance from base point to upper side of the particle just before air column rupture, and (c) maximum penetration depth, \( L_{\text{max}} \), was defined as distance from base point to center of the particle at maximum depth. Base point means cavity depth of the water surface before penetration. (d) Diameter of residual bubble, \( d_b \), was measured at the moment when the shape of the bubble became almost sphere. (e) Average flotation velocity was obtained by liner regression calculation of particle depth over time during the period that particle flotation velocity became almost constant. (f) Detention time of the particle was defined as the period from air column rupture to flotation to the water surface. In some conditions, particle was hidden behind the cavity before reaching the water surface. Therefore, detention time was calculated by extrapolating particle depth to the water surface during flotation.

Experimental data obtained in this experiment are shown in Table 2. Data with “-” indicate that air column was not generated. It was found that more than half of maximum penetration depth consisted of air column by comparison of maximum air column length, \( H_{\text{max}} \), with maximum penetration depth, \( L_{\text{max}} \). In Table 2, \( t_b \) is the time from air column rupture to reaching maximum penetration depth, and \( t_c \) is the flotation time from maximum penetration depth to the water surface. Detention time of a particle consisted of \( t_b \) and \( t_c \), and ratio of \( t_c \) to detention time was larger than that of \( t_b \).

In this experiment, there were some cases where the particle did not penetrate in vertical direction and its orbit bent widely to horizontal direction after penetration. In these cases, maximum penetration depth became smaller than the case that particle penetrated in vertical direction. The cases of B2-1, E1-2, and E1-3 in Table 2 were eliminated from analysis because maximum depth position deviated more than 20° to horizontal direction from penetration position at the water surface.

3.2. Particle Velocity after Penetration

Comparison of particle velocity after penetration with that before penetration is shown in Fig. 6. Particle velocity after penetration was 0.7–3.4 m/s though particle velocity before penetration was 1.1–10.9 m/s, and it did not increase so much as particle velocity before penetration increased. It means that kinetic energy of the particle is largely lost by penetration to the water surface.

3.3. Maximum Length of Air Column and Maximum Penetration Depth

Effect of penetration on maximum air column length, \( H_{\text{max}} \), is shown in Fig. 7. \( H_{\text{max}} = 0 \) mm means that air column was not generated, and it was caused by lower particle velocity before penetration than 3.4 m/s. Also, \( H_{\text{max}} \) increased with increasing particle velocity before penetration in the case that particle velocity before penetration was larger than 3.4 m/s.

With regards to collision of a particle to liquid surface, a hypothesis is suggested \(^{10,11}\) that liquid film spreads along particle surface (progress of wetting) and air column is...
Fig. 4. Change of penetration depth of a particle from water surface with time.

Fig. 5. Schematic diagram of a particle behavior and measured values in present work.
Table 2. Experimental results.

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<th>Particle velocity after penetration (m/s)</th>
<th>Maximum length of air column, H_{max} (mm)</th>
<th>Maximum penetration depth, L_{max} (mm)</th>
<th>Diameter of residual bubble, d_{B} (mm)</th>
<th>Average flotation velocity (m/s)</th>
<th>Average penetration angle (°)</th>
<th>Rupture time of air column (s)</th>
<th>t_{b}</th>
<th>t_{c}</th>
<th>Detention time of a particle (s)</th>
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*means unavailable result for analysis because average penetration angle is larger than 20°.

$t_{b}$: Time from rupture of air column to maximum penetration depth, $t_{c}$: Time from maximum penetration depth to water surface
generated in the case that particle velocity is larger than moving velocity of liquid film. In this experiment, moving velocity of liquid film is not different among all conditions because wettability between particle and liquid is the same. Therefore, air column is generated in the conditions where particle velocity before penetration is larger than the criterion velocity.

Effect of particle velocity before penetration on maximum penetration depth, \( L_{\text{max}} \), is shown in Fig. 8. \( L_{\text{max}} \) was increased with increasing particle velocity before penetration. Possible reasons of the scatter of maximum penetration depth are estimated that friction between the particle and inner wall of nozzle during passing through the nozzle, penetration angle of the particle to water surface, consumption of a part of kinetic energy to rotation of the particle, and so on.

According to Ozawa et al., a kinetic equation of a particle passing through gas-liquid interface vertically is given by Eq. (1). First term of the right side is liquid drag force, second term is gravity force, third term is buoyancy force, and forth term is force caused by interfacial tension.

\[
-\frac{dv_P}{dt} = \frac{4}{3} \pi r_P^3 \rho_P \frac{dv_P}{dt} + \frac{4}{3} \pi r_P^3 \rho_L \frac{dv_P}{dt} C_D \cdot \frac{x}{r_P} = \frac{2}{3} (\rho' + \alpha) \frac{dv_P}{dt} + \frac{1}{2} C_D \phi (x') \cdot v_P^2 - \frac{4}{3} \pi r_P^3 \rho g + \frac{4}{3} \pi r_P^3 \rho_L g \cdot \frac{x}{r_P} + 2 \pi r_P \sigma_{\text{GL}} \cdot \phi (\frac{x}{r_P})
\]

Where, \( v_P \) is particle velocity (m/s), \( t \) is time (s), \( r_P \) is radius of a particle (m), \( \rho_P \) is density of a particle (kg/m\(^3\)), \( C_D \) is drag coefficient (\(-\)), \( x \) is penetration depth of a particle into liquid (m), \( g \) is gravitational acceleration (m/s\(^2\)), and \( \sigma_{\text{GL}} \) is surface tension of liquid (N/m). Also, \( \alpha \) is coefficient related to imaginary mass (\(-\)), \( \phi (x/r_P) \), \( \phi (x/r_P') \), and \( \phi (x/r_P) \) are coefficient related to drag coefficient, buoyancy force and surface tension, respectively.

Equation (2) is obtained by deformation of Eq. (1), replacing \( x/r_P \) to \( x' \) and \( \rho_P/\rho_L \) to \( \rho' \).

\[
-\frac{2}{3} (\rho' + \alpha) \frac{dv_P}{dt} + \frac{1}{2} C_D \phi (x') \cdot v_P^2 - \frac{4}{3} \pi r_P^3 \rho g \left( \rho' - \phi (x') \right) + \frac{2 \sigma_{\text{GL}}}{\rho_P} \phi (x') \quad \ldots \ldots (2)
\]

Then, as same as Ozawa et al., Eq. (3) is obtained by replacing \( \phi (x') \) to constant \( \phi_1 \), \( \phi (x) \) to constant \( \phi_2 \), and \( \phi (x) \) to \( A(x - 1 - \cos \theta) \), \( \rho_0 \) to \( \rho + \alpha \), and \( \phi_2 \) to \( \phi - \phi_2 \); \( \theta \) is contact angle (\(^\circ\)), \( A \) is proportional coefficient of interfacial tension by generating cavity on liquid surface.

\[
-\frac{dv_P}{dt} + \frac{3 C_D \phi}{4 \rho_a} \frac{L_{\text{max}}}{v_P^2} = -\frac{3 C_D \phi}{4 \rho_a} \frac{L_{\text{max}}}{v_P^2} + \frac{2 \rho g \rho_0}{\rho_a} + \frac{3 \sigma_{\text{GL}}}{\rho_P \rho_0} (1 + \cos \theta) \ldots \ldots (3)
\]

Defining particle velocity before penetration as \( v_{\text{pen}} \), it can be defined as \( v_{\text{pen}} = v_P \) at \( x' = 0 \). Therefore, Eq. (4) is obtained by solving Eq. (3) about \( v_{\text{pen}} \) at this critical condition and inserting \( v_{\text{pen}} = 0 \) at \( x = L_{\text{max}} (= L_{\text{max}} \cdot r_P) \).

\[
v_{\text{pen}} = \exp \left( \frac{3 C_D \phi}{4 \rho_a} \frac{L_{\text{max}}}{v_{\text{pen}}} - \frac{4 A \sigma_{\text{GL}}}{\rho_0 \rho_2 C_D \phi} \right) + \frac{4 A \sigma_{\text{GL}}}{\rho_0 \rho_2 C_D \phi} \left( 1 + \cos \theta \right)^3 \left( 1 \right) \ldots \ldots (4)
\]

As mentioned above, more than half of maximum penetration depth consisted of air column. Therefore, as same as Ozawa et al., they were defined that \( a \) as 0.25, \( \phi_1 \) as 1, \( \phi_2 \) as 0.5, and \( A \) as 2.5, based on the assumption that a half part of particle was dipped into liquid. Also, they were defined that drag coefficient \( C_D \) as 0.44, gravitational acceleration \( g \) as 9.8 m/s\(^2\), density of water \( \rho_0 \) as 1 000 kg/m\(^3\), surface tension of water \( \sigma_{\text{GL}} \) as 0.073 N/m, and contact angle between polypropylene and water \( \theta \) as 95\(^\circ\).\(^{13}\)

Calculation results of maximum penetration depth \( L_{\text{max}} \) (= \( L_{\text{max}} \cdot r_P \)) by Eq. (4) is shown in Fig. 8 by a solid line. It was less than observed values of present work. Even though surface tension of water is lower than that of mercury used in experiments by Ozawa et al.,\(^{10}\) same value of A was used in this calculation. Therefore, it is estimated that influence of interfacial tension is over-evaluated.

Then, another calculation was carried out on the assumption that influence of interfacial tension is nothing. In this calculation, \( \phi (x/r_P) \) equals zero in Eq. (1). Therefore, particle movement can be expressed by Eq. (5).

\[
-\frac{dv_P}{dt} + \frac{3 C_D \phi}{4 \rho_a} \frac{L_{\text{max}}}{v_P^2} = \frac{2 \rho g \rho_0}{\rho_a} \ldots \ldots (5)
\]

Equation (6) can be obtained by solving Eq. (5) in same critical condition as Eq. (3).

\[
L_{\text{max}} = \frac{4 \rho_a \rho_0}{3 C_D \phi} \ln \left( 1 - \frac{3 C_D \phi v_{\text{pen}}^2}{8 \rho g \rho_0} \right) \ldots \ldots (6)
\]
Maximum penetration depth calculated by Eq. (6) is shown in Fig. 8 by a broken line. It was larger than observed values and did not agree with them. However, it was nearer to observed value than calculated value by Eq. (4). It would mean that influence of interfacial tension is small.

Besides, defining particle velocity after penetration as \( v_{p0} \) (m/s), maximum penetration depth was calculated by Eq. (6). In this calculation, they were defined that \( \alpha \) as 0.25, \( \varphi_1 \) as 1, \( \varphi_2 \) as 0.5 and \( C_D \) as 0.44. As shown in Fig. 9, calculated value almost agreed with observed value. It is estimated that maximum penetration depth is almost governed by particle velocity after penetration and is not influenced by rupture of air column nor residual bubble.

### 3.4. Diameter of Residual Bubble and Average Flotation Velocity

Effect of particle velocity before penetration on diameter of residual bubble is shown in Fig. 10. It had tendency that the diameter of residual bubble increased with increasing particle velocity before penetration. Also, as shown in Fig. 11, the diameter of residual bubble had correlation with maximum air column length, \( H_{\text{max}} \), except for experiment C1-3. It is assumed that the reason why the diameter of residual bubble becomes large is due to increasing particle velocity before penetration causing generation of air column and increasing gas volume involved into water.

Effect of the diameter of residual bubble on average flotation velocity of the particle is shown in Fig. 12. The particle with large residual bubble floated rapidly. The reason is estimated that apparent density of the particle is decreased by the residual bubble. Apparent density of a particle with a residual bubble, \( \rho'_P \) (kg/m³), is given by Eq. (7).

![Fig. 9. Comparison of experimental results with calculated value about maximum penetration depth.](image1)

![Fig. 10. Effect of particle velocity before penetration on diameter of residual bubble.](image2)

![Fig. 11. Relation between maximum length of air column and diameter of residual bubble.](image3)

![Fig. 12. Effect of diameter of residual bubble on average flotation velocity of a particle.](image4)
\[
\rho_v = \frac{d^3 \rho_r + d_b^3 \rho_b}{d^3 + d_b^3} \quad \text{............... (7)}
\]

Where, \(d_p\) is diameter of a particle (m) and \(\rho_b\) is density of a residual bubble (kg/m³).

Terminal velocity of a particle moving in liquid, \(v_t\), is expressed by Eqs. (8)–(10) corresponding to Reynolds Number, \(Re\).

\[
Re < 6 \quad v_t = \frac{g \Delta \rho d_p^2}{18 \mu_L} \quad \text{(Stoke’s law) .......... (8)}
\]

\[
6 < Re < 500 \quad v_t = \left( \frac{4 \Delta \rho g d_p^4}{225 \mu_r \rho_r} \right)^{\frac{1}{5}} d_p \quad \text{(Allen’s law) ... (9)}
\]

\[
500 < Re < 10^5 \quad v_t = \left( \frac{3 g \Delta \rho d_p^4}{\rho_L} \right)^{\frac{1}{5}} \quad \text{(Newton’s law) ... (10)}
\]

Where, \(\Delta \rho (= \rho_L - \rho_v)\) is difference of density between liquid and particle with residual bubble (kg/m³), and \(\mu_L\) is viscosity of liquid (Pa s).

Terminal velocity calculated by Eq. (9), Allen’s law, and Eq. (10), Newton’s law, are also shown in Fig. 12 by a broken line and a solid line, respectively. Terminal velocity calculated by Eq. (8), Stokes’s law, was not indicated because it was 0.5–4.5 m/s and \(Re (1 \times 600–14 \times 000)\) was drastically exceeded coverage of Stokes’s law, \(Re < 6\). Density of a residual bubble \(\rho_b\) is 0.85 kg/m³ (density of Ar at 52 kPa, 20°C), viscosity of water \(\mu_L\) is 1.0 × 10⁻³ Pa s. Average flotation velocity of the particle followed Allen’s law in the case that the diameter of the residual bubble was smaller than 2 mm, and followed Newton’s law in the case that the diameter of the residual bubble was larger than 2 mm. Reynolds number is indicated at vertical axis of right side in Fig. 12. Reynolds number of average flotation velocity with 2 mm residual bubble in diameter is approximately 500, and it corresponds to boundary of Allen’s region and Newton’s region. Therefore, average flotation velocity measured in this investigation is valid.

3.5. Detention Time of a Particle

Relation between particle velocity before penetration and detention time of a particle is shown in Fig. 13. Detention time of the particle in the cases that particle velocity before penetration was higher than 3.4 m/s had a tendency to become shorter than in the cases that particle velocity before penetration was lower than 3.4 m/s. It is because that a residual bubble with larger than 2 mm in diameter is easy to generate and the particle floats rapidly in the case that particle velocity before penetration is higher than 3.4 m/s.

Relation between maximum penetration depth and detention time of a particle is shown in Fig. 14. Experimental data were separated into two groups. One is indicated by triangular mark in Fig. 14, corresponding to smaller than 1 mm in diameter of residual bubble. While the other is indicated by circular mark, corresponding to larger than 2 mm in diameter of it. It suggests that effect of increasing detention time by increasing maximum penetration depth is hardly obtained in the case of the particle with a residual bubble larger than 2 mm.

Previous studies have mainly focused on maximum penetration depth. However, by results in present work, it is considered that blasting particle without a residual bubble is more important than increasing maximum penetration depth by higher particle velocity before penetration. Factors able to affect an air column and a residual bubble are considered wettability between a particle and liquid, pressure, particle diameter, and so on. More investigation is necessary into how these factors affect an air column and a residual bubble.

3.6. Flotation Behavior of a Particle

As shown in Table 2, ratio of \(t_c\) to detention time was larger than that of \(t_b\). Therefore, theoretical analysis about flotation behavior of a particle was carried out with considering a residual bubble. Kinetic equation of a particle floating in liquid is expressed by Eq. (11). First term of the right side is drag force, second term is force of gravity, and third term is buoyancy force.

\[
\frac{dv_r}{dt} \left\{ \frac{4}{3} \pi n_r \rho_r + \frac{4}{3} \pi n_b \rho_b + \alpha \right\} = -\frac{1}{2} \rho_L v_r^2 C_p - \frac{4}{3} \pi \left( n_r^3 + n_b^3 \right) \rho_L g \quad \text{............... (11)}
\]

Where, \(r_b\) is radius of residual bubble (m). Equation (11)
can be deformed to Eq. (12) with definitions of \( \rho_c^* \) as Eq. (13) and \( \rho_d^* \) as Eq. (14).

\[
\frac{dv_P}{dt} = -\frac{3C_D}{8\rho_P\rho_c} v_P^2 - g \cdot \frac{\rho_d^*}{\rho_c} \tag{12}
\]

\[
\rho_c^* = \frac{\rho_T}{\rho_c} + \left( \frac{m}{n} \right)^3 \cdot \frac{\rho_T}{\rho_c} + \alpha \cdot \left( 1 + \left( \frac{m}{n} \right)^3 \right) \tag{13}
\]

\[
\rho_d^* = \frac{\rho_T}{\rho_c} + \left( \frac{m}{n} \right)^3 \cdot \frac{\rho_T}{\rho_c} + \alpha \cdot \left( 1 + \left( \frac{m}{n} \right)^3 \right) \tag{14}
\]

Particle velocity \( v_P \) is expressed by Eq. (15) and \( \frac{dv_P}{dt} \) is expressed by Eq. (16), with definition minute period of time as \( \Delta(t) \) and flotation distance from maximum penetration depth as \( X(t) \) (m).

\[
v_P = \frac{X(t + \Delta t) - X(t)}{\Delta t} \tag{15}
\]

\[
\frac{dv_P}{dt} = \frac{X(t + \Delta t) - 2X(t) + X(t - \Delta t)}{\Delta t^2} \tag{16}
\]

Inserting Eqs. (15) and (16) to (14), a quadratic equation about \( X(t+\Delta t) \) can be described as Eq. (17)

\[
\frac{3C_D}{8\rho_P\rho_c} X(t + \Delta t)^2 + \left( -2 \cdot \frac{3C_D}{8\rho_P\rho_c} X(t) + 1 \right) \cdot X(t + \Delta t) + \frac{3C_D}{8\rho_P\rho_c} X(t)^2 - 2X(t) + X(t - \Delta t) + g \cdot \frac{\rho_d^*}{\rho_c} \Delta t^2 = 0 \tag{17}
\]

\( X(t+\Delta t) \) was obtained by solving Eq. (17) and sequential calculation was operated under the condition; \( \Delta t = 0.04 \) s. The plus sign was used for radical sign involved in formula of solution of the quadratic equation because solution was diverged in the case of the minus sign. Initial condition of the calculation is that the particle reaches maximum penetration depth \( X = 0 \) at \( t = 0 \) and \( v_P \) was obtained by transfer distance of a particle during 0–0.04 s. Also, \( C_D \) is calculated by Stokes’s law in the case of \( Re < 6 \), by Allen’s law in the case of \( Re = 6–500 \) and by Newton’s law in the case of \( Re > 500 \). Also, \( \alpha \) is 0.5 because the particle is completely dipped into liquid during flotation.

Calculation examples are shown in Fig. 15. Calculated value in the cases that diameter of residual bubble \( d_B \) is (a) 0 mm and (b) 1.6 mm almost agreed with observed values. On the other hand, calculated value in the cases that residual bubble is (c) 2.4 mm and (d) 3.4 mm (dashed line) had deviation against observed values. In these cases, flotation
time by calculation was shorter than that by observation. In observation of the particle during flotation, the residual bubble was not always adhered above center of the particle. It suggests that increasing diameter of the residual bubble is led to increase projection cross section area of the particle including the bubble like case (c) or case (d), and friction force between liquid and the particle is increased. Then, drag coefficient $C_D$ in Eq. (11) was assumed to apparently increase to $C_{D'}$, correlation parameter $f$ ($C_{D'} = f·C_D$) meaning ratio of $C_{D'}$ to $C_D$ was defined, and parameter $f$ was adjusted to agree calculated value with observed values. As a result, flotation behavior of the particle was reproduced as shown in Fig. 15 with a solid line by $f = 2.1$ in case (c) and $f = 3.2$ in case (d).

The same analysis was carried out under other conditions. Relation between diameter of the residual bubble and correlation parameter $f$ is shown in Fig. 16. Though $f$ is approximately 1 in the case that diameter of the residual bubble is smaller than 2 mm, $f$ increases in the case that it is larger than 2 mm. It can be concluded that increasing diameter of the residual bubble led to increasing of projection cross section area of the particle including the bubble and because the friction force between liquid and the particle was increased.

In present work, it is assumed that influence of macroscopic flow of liquid by air column generation and rupture of an air column is small because the particle floated up to the water surface in almost constant velocity. More investigation is necessary how liquid flow affects to penetration and flotation behavior.

4. Conclusions

In order to clarify the effect of particle velocity on penetration and flotation behavior, water model experiment blasting a single particle (3.2 mm in diameter) onto water surface was carried out. It was observed that generation and rupture of an air column occurred, a part of ruptured air column remained on the particle and became a residual bubble. Effect of particle penetration velocity on maximum penetration depth, diameter of the residual bubble and detention time of the particle were analyzed. Results obtained in this investigation are following:

(1) Maximum penetration depth increased with increasing particle velocity before penetration. Observed values of maximum penetration depth almost agreed with calculated value of a kinetic equation which particle velocity after penetration was used as the initial condition.

(2) The particle with high velocity before penetration floated to the water surface rapidly, even though maximum penetration depth was large, because the residual bubble was easily generated. It is considered that blasting particle without residual bubble is more important than increasing maximum penetration depth by higher particle velocity before penetration.

(3) Diameter of the residual bubble had correlated to maximum air column length. To avoid generating the residual bubble, the particle has to be penetrated with enough velocity so not as to generate the air column.

(4) Flotation behavior of the particle without a residual bubble corresponded to calculated value by a kinetic equation. On the other hand, flotation time of the particle with a residual bubble larger than 2 mm in diameter was larger than calculated value. This is because that increasing diameter of the residual bubble leads to increasing projection cross section area of the particle including the bubble, and because the friction force between liquid and the particle is increased.

REFERENCES