
Zhongkai REN,1,2) Hong XIAO,1,2)* Xiao LIU1,2) and Zhichao YAN1,2)

1) College of Mechanical Engineering, Yanshan University, Qinhuangdao, Hebei, 066004 China.
2) National Engineering Research Center for Equipment and Technology of Cold Strip Rolling, Yanshan University, Qinhuangdao, Hebei, 066004 China.

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Strip shape control theory has been developed into a relatively accurate system. However, for ultra-thin strip rolling, strip shape control is still a bottleneck affecting production. The most important problem is that the transverse flow mechanism of the metal is not accurate. In this study, based on the theory of ultra-thin strip rolling proposed by Fleck, a new metal transverse displacement model is developed using the minimum energy principle. To verify the accuracy of the new model, experiments and finite element analyses were carried out. For the transverse flow of a thin strip, grids with a line thickness of 10 μm and clear boundaries were successfully manufactured on the strip surface using lithography. The transverse displacement for different working conditions was measured after rolling. For ultra-thin strip rolling, the distribution of the transverse flow is analyzed using FEA. Finally, a comparison shows that the calculations from the new model are more consistent with the measured value and the simulation results, verifying the accuracy of the new model.

KEY WORDS: ultra-thin strip rolling; strip shape control; metal transverse displacement; minimum energy principle; lithography technology.

1. Introduction

Strip shape control theory is one of the core control technologies involved in strip rolling. Over the years, strip shape control theory has been studied in depth and developed into a relatively accurate system. However, there are still some problems in the production process of ultra-thin strip, in which strip shape control is the production bottleneck. During cold rolling, the shape quality is usually determined by detecting the lateral tension distribution. The transverse tension distribution depends mainly on the transverse distribution of the roll gap and the metal transverse flow in the deformation zone. For ultra-thin strip rolling, the transverse flow mechanism for the roll gap and the metal transverse flow in the deformation zone is analyzed using FEA. Finally, a comparison shows that the calculations from the new model are more consistent with the measured value and the simulation results, verifying the accuracy of the new model.

KEY WORDS: ultra-thin strip rolling; strip shape control; metal transverse displacement; minimum energy principle; lithography technology.
exit is established by the minimum energy principle. Finally, the accuracy of the new model is verified via experimental studies and finite element simulations.

2. Theoretical Analysis

2.1. Characteristics of Ultra-thin Strip Rolling

The relationship between the strip and the roller is described by the Fleck theory, as shown in Fig. 1.\(^{15,16}\) The deformation zone is divided into the inlet elastic zone A, the inlet plastic zone B, the neutral zone C, the exit plastic zone D, and the exit recovery zone E. Plastic deformation occurs at the entry and exit regions, which are separated by a neutral zone where there is no significant plastic reduction.

From Fleck theory,\(^{15,16}\) the strip thickness \(h(x)\) through the bite is given by:

\[
h(x) = h_i(x) + 2b(x) \quad \text{(1)}
\]

\[
h_i(x) = h_i - \frac{x_a^2 - x^2}{R} \quad \text{(2)}
\]

\[
b(x) = -\frac{2(1-v_R^2)}{\pi E_R} \int_{0}^{x_s} p(s) \ln \left| \frac{s+x}{s-x} \right| ds \quad \text{(3)}
\]

where \(h_i(x)\) is the strip thickness variation for the undeformed roll, \(b(x)\) is the elastic displacement of the roll for the normal contact pressure, which is determined from the elastic half-space theory, \(h_i\) is an initial strip thickness, \(x_a\) is the x coordinate at the entry to the deformation zone, \(R\) is the radius of the work roll, \(E_R\) is the plane strain Young’s modulus of the roll, \(v_R\) is the Poisson’s ratio of the roll, \(x_s\) is the x coordinate at the exit of the deformation zone and \(p(s)\) is the contact pressure between the roll and the strip.

In the slip region between the strip and the rolls, the Coulomb friction is assumed,\(^{15,16}\) which is defined as:

\[
q_f(x) = \mu p(x) \quad \text{(4)}
\]

where \(\mu\) is the Coulomb friction coefficient and \(\alpha = 1\) when the strip is moving slower than the rolls and \(\alpha = -1\) when the strip is moving faster than the rolls.

In the sticking region where no-slip conditions apply, the difference between the longitudinal strain between the rolls and the strip is a constant that is defined as:\(^{15,16}\)

\[
\varepsilon_R(x) - \varepsilon_s(x) = \frac{V_R - V_f}{V_f} = \eta \quad \text{(5)}
\]

This Equation \((d\varepsilon_R(x)/dx - d\varepsilon_s(x)/dx = 0)\) is given by derivation from Eq. (5), and from the incompressibility condition \((\varepsilon^s_z(x) + \varepsilon^s_y(y) = 0)\) of the plastic deformation, the strip strain in the x direction is given by:

\[
\varepsilon_s(x) = \varepsilon^s_z(x) + \varepsilon^s_y(x) = \frac{(1+\nu_y)(1-2\nu_y)}{E} \left( p + \sigma_s \right) - \ln \left( 1 + \frac{h - h_i}{h_i} \right) \quad \text{(6)}
\]

The longitudinal strain of the roll is given by the half-space theory:\(^{19}\)

\[
\varepsilon_R(x) = \frac{\partial u_R}{\partial x} = -\frac{(1-2\nu_y)(1+\nu_y) \rho p(x) - 2}{\pi E_R} \int_{x_a}^{x_s} q_x - q_y \, ds \quad \text{(7)}
\]

And, based on equilibrium equation and contact pressure expression, the friction force \(q_f(x)\) in the neutral zone is given

\[
q_f(x) = \frac{E_R}{2(1-4\nu_y)/(1-\nu_y) - (1-2\nu_y)/(1-\nu_y) E_R / E} \left( \frac{dh(x)}{dx} \right) \quad \text{(8)}
\]

where \(\eta\) is a constant, \(V_f\) is the entry speed of the strip, \(V_R\) is the peripheral speed of the undeformed roll, \(\varepsilon_s(x)\) is the longitudinal strain at the surface of the rolls, and \(\varepsilon_R(x)\) is the longitudinal strain at the surface of the strip.

Based on this theory, the variables obtained at the neutral surface include the coordinate \(x_a\) at the entry, the starting coordinate \(x_b\) at the adhesive zone, the ending coordinate \(x_c\) at the adhesive zone, the coordinate \(x_d\) at the exit, the coordinate \(x_e\) at the neutral point, and the strip thickness \(h_e\).

2.2. The Metal Transverse Flow for Thin Strip Rolling

Using the minimum energy principle, Lian\(^{10}\) calculate the deformation rate and deformation power in the deformation zone. The transverse displacement distribution is solved via a variational method.

2.2.1. Deformation Velocity of the Thin Strip in the Deformation Zone

The lateral displacement of the metal in the deformation zone is a function of \(x\) and \(y\), which is expressed by:\(^{10}\)

\[
w(x, y) = u(y) \left( 1 - \frac{h(x)-h_i}{\Delta h} \right) \quad \text{(9)}
\]

where \(u(y)\) is the lateral displacement at the exit and \(\Delta h\) is the amount of reduction achieved in a single rolling pass.

The transverse flow velocity of the metal in the deformation zone is expressed as:\(^{10}\)

\[
v_y = \frac{dw(x, y)}{dt} = -\frac{u(y) \, dh(x)}{\Delta h \, dt} \quad \text{(10)}
\]

The transverse deformation rate of the metal is expressed as:\(^{10}\)

\[
\varepsilon_y = \frac{\partial v_y}{\partial y} = \frac{u'(y) \, dh(x)}{\Delta h \, dt} \quad \text{(11)}
\]

The deformation rate of the strip in the height direction is expressed as:\(^{10}\)

\[
\varepsilon_z = \frac{\partial v_z}{\partial y} = \frac{1}{h_e} \frac{dh(x)}{dt} \quad \text{(12)}
\]
Based on the volume incompressibility condition, the deformation rate in the rolling direction is expressed as:\textsuperscript{(10)}

\[ \xi_t = -\xi_s(\xi_y + \xi_z) \left[ \frac{u(y)}{\Delta h} - \frac{1}{h_c} \right] \frac{dh(x)}{dt} \]............ (13)

2.2.2. Using the Variational Method to Solve for the Transverse Displacement

In the deformation zone, the total power is equal to the sum of the power from plastic deformation, the power from friction and the elastic power from the tension force.

The power from plastic deformation is expressed as:\textsuperscript{(10)}

\[ P_p = k \int_{0}^{h/2} \int_{0}^{h/3} Hdydz \]............. (14)

where \( k \) is the shear yield stress of the strip, \( k = 0.577 \sigma_s \), where \( \sigma_s \) is yield strength of the strip after rolling and \( b \) is width of the strip.

The power from friction is expressed as:\textsuperscript{(10)}

\[ P_f = 2 \int_{0}^{b/2} dy \int_{0}^{h/3} \left[ \sqrt{v_x^2 + \frac{v_l^2}{v_t^2}} \right] F(y) dy \]............. (15)

where \( v_t \) is the average friction force in the deformation zone and \( v_l \) is the longitudinal sliding speed of the metal relative to the roller surface.

The elastic power from the tension force is expressed as:\textsuperscript{(10)}

\[ P_e = \frac{1}{2} v_l h_c \left[ 1 - v_s^2 \right] \int_{0}^{h/2} \sigma_s(y) dy \]............. (16)

where \( \overline{v_l} \) is the average speed at the exit of the deformation zone, \( h_c \) is the strip thickness at the exit, \( E_s \) is the elastic modulus of the strip, \( v_l \) is the Poisson ratio of the strip and \( \sigma_s(y) \) is the unit front tension.

Substituting Eqs. (14), (15) and (16) into Eq. (13) gives:\textsuperscript{(10)}

\[ F(y,u,y,u') = 2k \int_{0}^{h/2} \int_{0}^{h/3} Hdydz \int_{0}^{h/2} \left[ \sqrt{v_x^2 + \frac{v_l^2}{v_t^2}} \right] F(y) dy \int_{0}^{h/2} \sigma_s(y) dy \]............. (17)

According to the minimum energy principle, the total deformation power is a minimum during rolling. This is the simplest variational problem with an unknown function \( u(y) \). The variable \( F(y,u,u') \) is the basic function and \( u(y) \) must satisfy the following Euler differential equation:\textsuperscript{(10)}

\[ \frac{d}{dy} \left( \frac{\partial F}{\partial u'} \right) - \left( \frac{\partial F}{\partial u} \right) = 0 \]............ (19)

\[ \frac{\partial F}{\partial u} = 8tu(y)I(u) \]............. (20)

\[ \frac{d}{dy} \left( \frac{\partial F}{\partial u'} \right) = \frac{3kh_c}{2Ah} \left( \frac{E}{1 - v_s^2} \right) u(y) + \frac{E}{1 - v_s^2} \]............ (21)

\[ I(u) = \int_{0}^{1} \frac{z^2}{h + \Delta h c^2} dz \]............. (22)

where \( z = x/l, s_o = x_o/l, h_c = (h_c + h_o)/2, c = \Delta hl/h_o, l_o \) is the length of the adhesive zone, \( \Delta h \) is the thickness difference in the strip, \( h_s \) is the thickness at the neutral point and \( \Delta b \) is the lateral spread of the strip.

Substituting Eqs. (20), (21) and (22) into Eq. (19) gives

\[ \frac{d}{dy} \left( \frac{\partial F}{\partial u'} \right) = \frac{8(1 - v_s^2)}{Ech_s} I(u) u - \frac{1}{\xi} \left[ \frac{h'}{h} - \frac{H'(y)}{H} - \frac{L'(y)}{L} \right] \]............. (23)

\[ \xi = 1 + \frac{3(1 - v_s^2) h_s}{2E \Delta h} \]............. (24)

2.3. The Metal Transverse Flow for Ultra-thin Strip Rolling

In Lian theory, the conventional cold rolling theory is used. Thus, Lian theory is not suitable for the ultra-thin strip rolling. In this study, based on the theory of ultra-thin strip rolling proposed by Fleck, a new metal transverse flow model is developed. Therefore, the basic function \( F(y,u,u') \) is as follows:

\[ F(y,u,y,u') = 2k \int_{0}^{h/2} \int_{0}^{h/3} Hdydz \int_{0}^{h/2} \left[ \sqrt{v_x^2 + \frac{v_l^2}{v_t^2}} \right] F(y) dy \int_{0}^{h/2} \sigma_s(y) dy \]............. (25)

\[ + 2 \int_{0}^{1} \left[ dF \right]_{h + \Delta h c^2} dz \]............. (26)

\[ + \int_{0}^{1} \left[ dF \right]_{h + \Delta h c^2} dz \]............. (27)

\[ \begin{array}{l}
\int_{0}^{1} \left[ dF \right]_{h + \Delta h c^2} dz \\
\int_{0}^{1} \left[ dF \right]_{h + \Delta h c^2} dz \\
\int_{0}^{1} \left[ dF \right]_{h + \Delta h c^2} dz \\
\int_{0}^{1} \left[ dF \right]_{h + \Delta h c^2} dz \\
\end{array} 
\]............. (28)

where \( z_i = z_o - n(2l), z_o = z + d/l, s(u) = (z^2 - z_o^2)^2 + 4z^2 \left( \frac{u}{c} \right)^2 \),

\[ s_t(u) = \left( z^2 - z_t^2 \right)^2 + 4z^2 \left( \frac{u}{c} \right)^2 \]

\[ I(u) = \int_{0}^{1} 4\sqrt{\frac{z_i (z^2 - z_i^2)}{\pi z_t^2 s_t(u)}} dz \]............. (29)

\[ + \int_{0}^{1} 4\sqrt{\frac{z_i (z^2 - z_i^2)}{\pi z_t^2 s_t(u)}} dz \]............. (30)

\[ + \int_{0}^{1} 4\sqrt{\frac{z_i (z^2 - z_i^2)}{\pi z_t^2 s_t(u)}} dz \]............. (31)

where \( z_i = z_o - n(2l), z_o = z + d/l, s(u) = (z^2 - z_o^2)^2 + 4z^2 \left( \frac{u}{c} \right)^2 \),

\[ s_t(u) = \left( z^2 - z_t^2 \right)^2 + 4z^2 \left( \frac{u}{c} \right)^2 \]
Substituting Eq. (26) into Eq. (23) gives

\[
u(y) = \left[ \frac{\Delta B}{2} - \sum_{i=1}^{m} c_{2i} \right] / \left( k_B / 2 \right) * shK_y
+ \sum_{i=1}^{m} c_{2i} \left( \frac{2y}{B} \right)^{2i-1} \cdots (27)
\]

where \( B \) is the strip width, \( b_{2j} \) is fitting coefficient for the strip thickness along the width at the entry and \( m \) is the number of terms used during fitting polynomial equations.

Since the lateral spread \( \Delta b \) introduced by the boundary condition is an unknown, it must be solved for solving the lateral displacement function. Thus, \( u(y) \) is a family of curves, in which only one minimizes the total deformation power. Finally, substituting Eq. (27) into Eq. (17), and the lateral spread \( \Delta b \) is obtained by \( dP/d\Delta b = 0 \).

### 3. Experiment to Determine the Transverse Displacement of a Thin Strip

To verify the accuracy of the theoretical model, it is necessary to accurately measure the transverse displacement of the strip surface. Therefore, a grid with a line thickness of 10 \( \mu m \) and clear boundaries was successfully manufactured on the surface of 304-stainless steel raw material using lithography. Then, various rolling tests were carried out at different conditions using a 20-high mill in the laboratory, and the 20-high mill rolls system arrangement is shown in Fig. 2, its primary technological parameters are listed in Table 1.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Work roll (mm)</td>
<td>( \phi 11.5 \times 160 )</td>
</tr>
<tr>
<td>First intermediate roll (mm)</td>
<td>( \phi 22 \times 170 )</td>
</tr>
<tr>
<td>Second intermediate roll (mm)</td>
<td>( \phi 36 \times 160 )</td>
</tr>
<tr>
<td>Back up roll (mm)</td>
<td>( \phi 60 \times 40 \times 3 )</td>
</tr>
<tr>
<td>Taper angle of first intermediate roll</td>
<td>0.001, 0.002, 0.003</td>
</tr>
<tr>
<td>Taper length of first intermediate roll (mm)</td>
<td>50, 60</td>
</tr>
</tbody>
</table>

![Fig. 2. 20-high mill rolls system arrangement.](image)

3.1. Preparation of Specimen

A grid with line thickness of 10 \( \mu m \) and clear boundaries was successfully manufactured on the surface of 304-stainless samples via lithography. The grid sections were shown to be very clear at a magnification of 50x, as shown in Fig. 3. The manufacturing process for this is as follows:

1. Preparation of the strip: Each strip was cut into a 250 mm long specimen.
2. Spin coating treatment: BP-212 series positive photoresist was coated to the surface of the strip using a KW-4A spin coating machine.
3. Solidification treatment: Each specimen was solidified in a drum wind drying oven.
4. Exposure processing: A mask with a diameter of 10 \( \mu m \) and grids of 2 \( \times 0.3 \) mm was installed on a BG-401A exposure machine prior to exposure for 20 seconds at room temperature.
5. Development processing: Development was carried out using TMAH with a concentration of 0.26 N.
6. Solidification strengthening: To improve the adhesion of the photoresist on the surface of the thin strip, the specimen was again placed in the drum wind drying oven.

3.2. Measurement of the Transverse Displacement

The lateral displacement was measured in two steps, the displacement of the edge was measured directly, and the remaining positions were measured indirectly. Before and after rolling, the distance from the edge of the strip to the nearest grid line was measured. Afterwards, the variation in the lateral displacement at the edge was obtained by determining the difference between the two measurements. The remaining positions were measured as follows. A picture was taken every 2 mm using an electron microscope at a magnification of 50x. These pictures were pieced together with Photoshop software. Afterwards, as shown in Fig. 4(a), the spliced picture was measured using the AutoCAD software. Specifically, as shown in Fig. 4(b), two borderlines were drawn at the required position to ensure that they coincide with the boundaries of the grid line. Then, the symmetry line of the boundaries was taken as the center line of the grid line. The centerline of the strip was used as a datum. Based on the datum, the distance was measured every 4 mm. Finally, the actual value of the lateral displacement was obtained from the measured value and the magnification times.

![Fig. 3. Finished specimen with with line thickness of 10 \( \mu m \).](image)
3.3. Comparison between the Theoretical Calculations and Measured Results

To carry out the theoretical analysis, it is necessary to establish a strip shape control model. Based on the new transverse displacement model and the Lian model, the elastic deformation model for the roll system proposed by Yuan\(^{20}\) is used. Then, the strip shape control models for the 20-high mill are established.

The transverse displacements for all three cases were calculated using the new model and the Lian model and are presented in Fig. 4 for comparison. In this paper, we assume the outside along the width direction of the strip is the positive direction and the inside is the negative direction. In Fig. 5(a), the strip entry and exit thicknesses are 0.3 mm and 0.25 mm, the strip width is 85 mm, the front and back tensions are 118 MPa/118 MPa and the taper angle and taper length for the first intermediate roll are 0.003 and 50 mm, respectively. In Fig. 5(b), the strip entry and exit thicknesses are 0.2 mm and 0.165 mm, respectively, the strip width is 85 mm, the front and back tensions are 88 MPa/88 MPa and the taper angle and taper length for the first intermediate roll are 0.002 and 50 mm. In Fig. 5(c), the strip entry and exit thicknesses are 0.3 mm and 0.25 mm, respectively, the strip width is 85 mm, the front and back tensions are 78 MPa/78 MPa and the taper angle and taper length for the first intermediate roll are 0.002 and 50 mm, respectively. The comparisons show that the calculations from the new model are more consistent with the measured values, validating the accuracy of the new model.

4. Finite Element Analysis for Transverse Displacement of Ultra-thin Strip

For ultra-thin strip rolling, the accuracy of the new model is verified by finite element analysis (FEA). A finite element model is created based on the technological parameters for the 20-high mill in the Table 1. Since the roller system is symmetrical, a quarter symmetry model, as shown in Fig. 6, is used for the analysis. The monoblock housing is simplified to be a rigid structure in this model, as it normally has
the highest mill modulus. The roller system and strip are modeled as elastic and elastic-plastic bodies, respectively. The elastic modulus and Poisson’s ratio of the work roll are 236 GPa and 0.3, respectively. The elastic modulus and Poisson’s ratio of the other roll are 207 GPa and 0.25, respectively. In the model, the true stress-strain curve of the strip is transformed according to the measured data via a tensile test. The yield criterion is modeled using the von Mises yield criterion. Friction between the work roll and the workpiece is modeled using the Coulomb friction law. As lubricants are used in the experiments, the friction coefficient is assumed to be 0.08.

Two conditions are simulated using the FEA: 0.05 mm and 0.04 mm foil. In Fig. 7(a), the strip entry and exit thicknesses are 0.05 mm and 0.037 mm, the strip width is 65 mm, the front and back tensions are 118 MPa/118 MPa and the taper angle and taper length of the first intermediate roll are 0.002 and 60 mm, respectively. In Fig. 7(b), the strip entry and exit thicknesses are 0.04 mm and 0.033 mm, the strip width is 65 mm, the front and back tensions are 90 MPa/90 MPa and the taper angle and taper length of the first intermediate roll are 0.002 and 60 mm, respectively. As shown in Fig. 7, when the strip thickness is very thin, the transverse displacements in the middle part of the strip are very small, lateral metal flow occurs mainly at the edges of the strip and the transverse displacement is negative around the edge. The comparisons show that the Lian model is not suitable for ultra-thin strip rolling. However, the calculations based on the new model are consistent with the FEA results, validating the accuracy of the new model.

5. Conclusions

The transverse flow of the metal in the roll gap is the basis for studying the transverse tension distribution in strip rolling. However, the traditional transverse displacement model is not suitable for ultra-thin strip rolling. To address this problem, a new accurate model is developed based on the Fleck theory in this study. For thin strip rolling, the transverse displacement distribution is examined via experiments. Grids with a 10 μm line thickness and clear boundaries were successfully manufactured on the surface of 304-stainless steel samples via lithography. Then, the transverse displacement distribution for different processing conditions was measured after rolling. By comparing the new model and the Lian model with the measured results, we see that the calculations from the new model are more consistent with the measured values, proving that the new model is more accurate. For ultra-thin strip rolling, the transverse displacement distribution is analyzed using FEA. As the strip thickness decreases, the metal lateral flow occurs mainly at the edges of the strip and the transverse displacement is negative around the edges. Comparing the new model and the Lian model with the simulations, we see that the Lian model is not suitable for ultra-thin strip rolling. However, the calculations from the new model are consistent with the FEA results, verifying the accuracy of the new model.
Acknowledgements
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Nomenclature
\( h_i(x) \): Strip thickness variation for the undeformed roll
\( b(x) \): Elastic displacement of the roll for the normal contact pressure
\( h_i \): Initial strip thickness
\( x_c \): The x coordinate at the entry to the deformation zone
\( R \): Radius of the work roll
\( E_R \): Plane strain Young’s modulus of the roll
\( v_R \): Poisson’s ratio of the roll
\( x_d \): The x coordinate at the exit of the deformation zone
\( p(s) \): Contact pressure between the roll and the strip
\( q_1(x) \): Coulomb friction
\( \mu \): Coulomb friction coefficient
\( \eta \): A constant
\( v_S \): Entry speed of the strip
\( v_R \): Peripheral speed of the undeformed roll
\( \varepsilon_L(x) \): Longitudinal strain at the surface of the rolls
\( \varepsilon_T(x) \): Longitudinal strain at the surface of the strip
\( x_b \): Starting coordinate at the adhesive zone
\( x_c \): Ending coordinate at the adhesive zone
\( h_o \): Strip thickness at the neutral point
\( u(y) \): Lateral displacement at the exit
\( \Delta b \): Amount of reduction achieved in a single rolling pass
\( v_T \): Transverse flow velocity of the metal
\( \xi_T \): Transverse deformation rate of the metal
\( \xi_S \): Deformation rate of the strip in the height direction
\( \xi_L \): Deformation rate in the rolling direction
\( P_p \): Power from plastic deformation
\( P_m \): Power from friction
\( \bar{v}_T \): Average speed at the exit of the deformation zone
\( h_o \): Strip thickness at the exit
\( E_S \): Elastic modulus of the strip
\( v_S \): Poisson ratio of the strip
\( \sigma_0(y) \): Unit front tension
\( l_o \): Length of the adhesive zone
\( b_2 \): Lateral spread of the strip
\( B \): Strip width
\( m \): Number of terms used during fitting polynomial equations

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