Additional Information on “Simple Tundish Mixing Model of Continuous Casting during a Grade Transition” by Cho and Kim

Bernardo Martins BRAGA* and Roberto Parreiras TAVARES
Department of Metallurgical and Materials Engineering, School of Engineering, Federal University of Minas Gerais, Ave. Presidente Antônio Carlos, No. 6627, Belo Horizonte, 31270-901 Brazil.

(Received on January 24, 2018; accepted on March 7, 2018)

Cho and Kim1) proposed a simple model for predicting the amount of mix steel generated during ladle changes involving different steel grades in the same tundish. The authors solved the equations of the model numerically and validated their model for two different tundishes. This model has the advantage of having only one parameter, which should be calibrated for each tundish. In the present work, general analytical solutions were deduced for the Cho and Kim model. The solutions obtained are dependent on a single variable. This variable allows easy identification of the optimum operating parameters for the ladle change as it is illustrated by an application example. This work also investigated the meaning of the parameter of the Cho and Kim model in terms of chemical reactor theory.

KEY WORDS: grade transition; continuous casting; tundish mixing; analytical solution; optimization.

1. Introduction

In order to meet the production mix without interrupting the continuous casting process, several steel plants perform ladle changes involving different steel grades using the same tundish. As a result, some steel of mixed composition is cast. Since this “mix steel” is disqualified or scrapped, it is important to estimate its amount for both process control and cost calculation.

To this end, Cho and Kim1) proposed a simple tundish mixing model. The authors1) validated their model for two different tundishes. This model has the advantage of having only one parameter, which should be calibrated for each tundish. As a comparison, the model proposed by Huang and Thomas2) has six parameters. Equations of both models were solved numerically.

In the present work, general analytical solutions were deduced for the Cho and Kim model.1) The solutions obtained are dependent on a single variable. This variable allows easy identification of the optimum operating parameters for the ladle change as it is illustrated by an application example. This work also investigated the meaning of the parameter of the Cho and Kim model1) in terms of chemical reactor theory.

2. Model Description

In general, the mass balance equation of liquid steel within a tundish is:

\[ \frac{dM_i}{dt} = \Gamma_{in} - \Gamma_{out} \] ........................ (1)

Where \( M_i \) is the mass of liquid steel within the tundish (kg), \( t \) is the time (s), \( \Gamma_{in} \) is the mass flow rate of liquid steel entering the tundish (kg/s), \( \Gamma_{out} \) is the total mass flow rate of liquid steel leaving the tundish:

\[ \Gamma_{out} = \sum \Gamma_{out}^{f} \] ........................ (2)

Where \( \Gamma_{out}^{f} \) is the mass flow rate of liquid steel through the \( f \) strand of the tundish (kg/s) and \( n \) is the total number of strands of the tundish.

In accordance with Cho and Kim model1), the mass balance equation for a solute dissolved in liquid steel is:

\[ \frac{dM_{si}}{dt} = M_{si} \] ........................ (3)

Where \( w_{ave} \) is the average mass fraction of the solute in the liquid steel entering the tundish and \( w_{out} \) is the mass fraction of the solute in the liquid steel leaving the tundish. According to Cho and Kim model1), \( w_{out} \) is calculated as:

\[ w_{out} = f'w_{ave} \] ........................ (4)

Where \( f' \) is a model parameter \((f' > 0)\) that should be adjusted for each tundish. It is noteworthy that the tundish behaves like an unsteady CSTR3) (continuous-stirred tank reactor) when \( f' = 1 \).

A general analytical solution for the system formed by Eqs. (1), (3) and (4) can be obtained using the method employed by Zenger and Niemi4) for treating the perfect mixer model (CSTR). Nonetheless, some caution is necessary in order to avoid the prediction of negative outlet concentrations by Eq. (3) when \( f' > 1 \). Hence, the cases \( 0 < f' \leq 1 \) and \( f' > 1 \) were considered separately.

2.1. Analytical Solution for \( 0 < f' \leq 1 \)

The combination of Eqs. (1), (3) and (4) leads to:

\[ \frac{dw_{ave}}{dt} = \frac{\Gamma_{in} (1-f') - \Gamma_{out} (w_{ave} - w_{ave})}{M_{si}} \] ........................ (5)

The dimensionless variable \( z \) is defined as:

\[ z = \int \left( \frac{\Gamma_{in} (1-f') - \Gamma_{out} (w_{ave} - w_{ave})}{M_{si}} \right) dt \] ........................ (6)

Where \( t_0 \) is the time instant when a new ladle is opened (s) and \( f' \) is a variable of integration (s). Hereinafter, it will be assumed that \( t_0 = 0 \). By means of the variable \( z \), Eq. (5) can be rewritten as:

\[ \frac{dw_{ave}}{dz} = w_{ave} - w_{ave} \] ........................ (7)

During a ladle transition, \( w_{ave} \) is constant. So, the following solution can be readily obtained:

\[ w_{ave} = \frac{w_{ave} - w_{ave} \cdot e^{-f'z}}{w_{ave} - w_{ave}} \] ........................ (8)

Where \( w_{ave} \) is the initial solute mass fraction.

Substituting Eq. (8) in Eq. (4), one gets:

\[ w_{ave} = \frac{w_{ave} - w_{ave} \cdot e^{-f'z}}{w_{ave} - w_{ave}} \] ........................ (9)

Therefore, the dimensionless concentration, \( f' \), predicted by the model is:

\[ f' = \frac{w_{ave} - w_{ave}}{w_{ave} - w_{ave}} \] ........................ (10)

2.2. Approximated Analytical Solution for \( f' > 1 \)

When \( f' > 1 \), Eq. (10) is clearly invalid, since it predicts \( F < 0 \) at \( z = 0 \). However, one could propose the following approximated solution for this condition:

\[ F = \max(0, 1 - f' \cdot e^{-z}) \] ........................ (11)

In order to evaluate the adequacy of this approximation, the mean residence time of the tundish operating at steady state, \( T_{res} \) (s), was calculated using the approximated solution5):

\[ T_{res} = \int \left( 1 - F \right) dt = \frac{1 + \ln f'}{f'} \frac{M_{si}}{f'} \] ........................ (12)

The relative error of the mean residence time is:
Where $\bar{T}$ is the mean residence time calculated using the exact solution (s), which assumes the theoretical value $M_d/\bar{T}_0$. The relative error is lower than 1% when $f \leq 1.160$ and it is lower than 5% when $f \leq 1.426$. These results may be taken as validity ranges for the approximated solution. Hence, the approximate solution is valid for the tundishes studied by Cho and Kim.  

2.3. Exact Analytical Solution for $f > 1$

When $f > 1$, two steps of different tundish behaviors should be considered. These steps are separated by the transition time, $t_{trn}$ (8).

During the first step, $0 \leq t \leq t_{trn}$, steel with initial solute mass fraction $w_0$ leaves the tundish while new steel with solute mass fraction $w_{in}$ enters in the equipment. So, during the first step, $w_{out} = w_0$ and $F = 0$. The mass balance equations of steel and solute for this step, Eqs. (1) and (3), lead to:

$$\frac{d}{dt}(w_{in}M_d - w_0M_d) = (w_0 - w_0)F_{ww}$$  \hspace{1cm}  (14)

Thus, for the first step:

$$w_{in} - w_0 = \frac{1}{M_d}\left[\Gamma w_{ww}dt\right]$$  \hspace{1cm}  (15)

During the second step, $t \geq t_{trn}$, Eq. (4) is valid. At the transition instant, $t = t_{trn}$:

$$w_0 = f \cdot w_{in} + (1-f) \cdot w_0$$  \hspace{1cm}  (16)

Hence, the transition time satisfies the following relation obtained from Eqs. (15) and (16):

$$\frac{1}{M_d} \int_{0}^{t_{trn}} \Gamma w_{ww}dt = \frac{-(1-f)}{f}$$  \hspace{1cm}  (17)

During the second step, Eq. (7) holds. However, its solution is now:

$$w_{in} - w_{in} = \frac{w_0}{M_d} - \frac{w_0}{M_d}e^{-t/(1-f)}$$  \hspace{1cm}  (18)

Where $w_{in}$ is the average mass fraction of solute in liquid steel at the transition time, which satisfies:

$$w_{in} - w_0 = \frac{-(1-f)}{f}$$  \hspace{1cm}  (19)

And $z_{in}$ is the value of the dimensionless variable $z$ evaluated at the transition time:

$$z_{in} = \int_{0}^{t_{trn}} \Gamma \frac{-(1-f)}{f} \cdot \Gamma w_{ww}dt$$  \hspace{1cm}  (20)

Substituting Eq. (18) in Eq. (4), one gets:

$$\frac{w_{in}}{w_0} = \frac{w_0}{w_0} - \frac{w_0}{w_0} e^{-z_{in}}$$  \hspace{1cm}  (21)

After some algebraic manipulations, Eq. (21) becomes:

$$F = \frac{w_{in} - w_0}{w_0 - w_0} = \left(1 - e^{-z_{in}}\right)$$  \hspace{1cm}  (22)

The final solution for the second step is obtained by the substitution of Eq. (19) in Eq. (22):

$$F = \frac{w_{in} - w_0}{w_0 - w_0} = 1 - e^{-t/(1-f)}$$  \hspace{1cm}  (23)

3. Interpretation of the Model Parameter

The functional form of the analytical solutions deduced, Eqs. (10) and (23), suggests that the Cho and Kim model \(^1\) is related to compartment models used in chemical reaction engineering. \(^5\) This issue, which clarifies the meaning of the model parameter ($f$), is examined in this section. It is considered a simplified (hypothetical) condition in which the tundish operates with constant volume and flow rate during ladle transition.

3.1. Interpretation when $f \leq 1$

In the condition considered, the dimensionless variable $z$ is:

$$z = f \cdot \frac{\Gamma w_{ww}}{M_d}$$  \hspace{1cm}  (24)

So, the analytical solution for $f \leq 1$, Eq. (10), becomes:

$$w_{out} = \left[1 - e^{-t/(1-f)}\right]w_0$$  \hspace{1cm}  (25)

Equation (25) corresponds to the accumulative residence time distribution (RTD) of a system composed of a well-mixed volume in parallel with a bypass flow \(^3\) (short circuit), provided that:

$$f = \frac{Q_{b}}{Q} = 1 - \frac{Q_{b}}{Q}$$  \hspace{1cm}  (26)

Where $Q_b/Q$ is the fraction of the total flow rate that passes through the active region (well-mixed volume) of the tundish and $Q_b/Q$ is the fraction of the total flow rate that passes through the bypass. Therefore, $(1-f)$ is the fraction of bypass flow when $f \leq 1$.

3.2. Interpretation when $f > 1$

For the simplified case, the transition time, $t_{trn}$, is:

$$t_{trn} = \frac{(1-f)M_d}{\Gamma w_{ww}}$$  \hspace{1cm}  (27)

Thus, the analytical solution for $f > 1$ becomes:

$$F = \left\{\begin{array}{ll}
0, & t_{trn}/M_d < -f/(1-f) \\
1 - e^{-t/(1-f)}, & t_{trn}/M_d \geq -f/(1-f)
\end{array}\right. $$  \hspace{1cm}  (28)

Equation (28) is equivalent to the accumulative RTD of a system that consists of a well-mixed volume in series with a plug flow volume, \(^3\) provided that:

$$f = \frac{V_{pl}}{V}$$  \hspace{1cm}  (29)

Where $V_{pl}/V$ is the volume fraction of the well-mixed region. It is noteworthy that:

$$f = \frac{V_{pl}}{V} = 1 - \frac{t_{trn}/M_d}{V}$$  \hspace{1cm}  (30)

Therefore, $-(1-f)/f$ is equal to the volume fraction of the plug flow region, $V_{pl}/V$.

3.3. Consequences for Tundish Optimization

In order to reduce losses due to mix steel, one usually designs flow modifiers for the tundish that eliminate any bypass flow and increase the plug flow volume fraction. Indeed, no mix steel would be generated by an ideal tundish that does not have any bypass flow and behaves fully as a plug flow.

Based on the analyses performed in sections 3.1 and 3.2, these goals can be simultaneously satisfied by means of the maximalization of the parameter $f$. Therefore, this should be set as the objective of the tundish optimization process if one wish to minimize the amount of mix steel formed during ladle changes.

4. Application of the Analytical Solutions for Optimization of the Ladle Transition Practice

The analytical solutions obtained in section 2 indicated that the steel mix that occurs in the tundish during ladle change is a function of a unique dimensionless variable, $z$. This property enables one to easily identify the optimum transition practice that minimizes the amount of mix steel.

In this section, this issue is exemplified for a simple transition procedure, \(^3\) which is described below. At steady-state, a tundish operates with an internal steel mass $M_{ds}=M_{ds,max}$ (kg) and a constant inlet flow rate, $\Gamma_{in}$ (kg/s), that is equal to the outlet flow rate, $\Gamma_{out}$ (kg/s).

During the ladle change, the outlet flow rate is maintained constant and equal to the steady-state value. On the other hand, the inlet flow rate varies in three stages. In the first stage, which starts after the old ladle is closed, the inlet flow rate is null. This condition continues until the steel mass within the tundish is reduced to a fraction $\alpha$ of the steady-state value ($0 < \alpha < 1$), $M_d = \alpha M_{ds,max}$.

The second stage starts when a new ladle is opened. In this stage, the tundish is refilled using an inlet flow rate $\Gamma_{in} = R_l \Gamma_{out}$, where $R_l$ is a positive constant ($R_l > 1$). Finally, the third stage starts when the steel mass within the tundish returns to the steady-state value, $M_d = M_{ds,max}$. In this stage, the inlet flow rate is reduced to the steady-state value, $\Gamma_{in} = \Gamma_{out}$.
Assume that the behavior of a tundish can be adequately described by the Cho and Kim model \(^1\) of parameter \(f\). In addition, consider that the steel grades involved in the ladle transition imposes that the steel mass of dimensionless concentration between \(F_1\) and \(F_2\), \(M_{12}\) (kg), need to be disqualified. This system is modeled below during the second and third stages so as to minimize the dimensionless ratio \(M_{12}/M_{\text{td,ss}}\).

The solution of Eq. (1) for this tundish is:

\[
M_d = \begin{cases} 
\frac{\alpha M_{\text{d,ss}} + (R_1-1)M_{\text{d,ss}}}{1 - \alpha} & M_{\text{d,ss}} \leq \frac{1 - \alpha}{R_1 - 1} \\
\frac{\alpha M_{\text{d,ss}} + (R_1-1)M_{\text{d,ss}}}{1 - \alpha} & M_{\text{d,ss}} > \frac{1 - \alpha}{R_1 - 1}
\end{cases}
\]  

(31)

The values of the dimensionless variable \(z\) that are associated with \(F_1\) and \(F_2\) through the analytical solutions presented in section 2 (\(z_1\) and \(z_2\), respectively) can be calculated as follows:

\[
z = \frac{\ln(1/f) - \ln(1 - F)}{1 - 1/f} \text{, } f \leq 1
\]

(32)

Where \(\max[a, b]\) expresses the maximum value between the arguments \(a\) and \(b\). This function is used to correctly calculate the variable \(z\) when \(f < 1\).

The transition between the second and third stages occurs when the dimensionless variable \(z\) is \(z_a\):

\[
z_a = \frac{R_1 - 1}{R_1} \ln \left(\frac{1}{1 - f}\right)
\]

(33)

Equations (33) follows from Eqs. (6) and (31).

The time instants \(t_1\) (s) and \(t_2\) (s), which correspond to \(z_1\) and \(z_2\), respectively, can be obtained from the solution of Eq. (6):

\[
\Gamma = \frac{\alpha - \frac{1}{R_1} \exp \left(\frac{1 - \alpha}{R_1 - 1}\right)}{1 - \alpha - \frac{1 - \alpha}{R_1 - 1} z_a}
\]

(34)

Finally, the relative mass of mix steel, \(M_{12}/M_{\text{td,ss}}\), is found:

\[
\frac{M_{12}}{M_{\text{td,ss}}} = \frac{\alpha M_{\text{d,ss}} + (R_1-1)M_{\text{d,ss}}}{1 - \alpha} \frac{1}{R_1 - 1} |z_a - z|, z < z_a
\]

(35)

**Figure 1** shows the variation in the relative mass of mix steel, \(M_{12}/M_{\text{td,ss}}\), with parameter \(R_{1}\) for the five hypothetical transition conditions described in Table 1 (cases A to E). This table also presents some relevant model results, which are discussed later. The case A is the reference condition for the analysis. In the other cases, one or more parameters of case A are modified in order to illustrate their relative effect on \(M_{12}/M_{\text{td,ss}}\).

All curves exhibit an abrupt change in behavior at a specific point. It is related to the completion of the tundish refilling stage before the chemical composition transition range is exceeded. Mathematically, this point occurs when \(z_a = z_2\).

Cases B and C show the effect of the parameters \(F_1\) and \(F_2\), which are determinate by the steel grades involved in the ladle change, on the relative mass of mix steel. In both cases, the difference \(F_2 - F_1\) was maintained the same as in case A. In spite of that restriction, the relative masses of mix steel of these cases are substantially different. In general, the set of equations predicts that the relative mass of mix steel increases dramatically when \(F_2\) approaches to unity.

Case D shows that increasing the parameter \(\alpha\) augments the relative mass of mix steel. On the other hand, according to Eqs. (33) to (35), the relative mass of mix steel tends to zero when the parameter \(\alpha\) tends to zero.

Finally, case E exemplifies the discussion performed in section 3. Increasing the parameter \(f\), the relative mass of mix steel is reduced.

In the cases C and E, the minimum point is related to a value of the parameter \(R_1\) close to unity for which the tundish refilling is negligible. The corresponding relative mass of mix steel is:

\[
\lim_{f \to 1} \frac{M_{12}}{M_{\text{td,ss}}} = \frac{\alpha}{1 - \alpha}
\]

(36)

In the cases A, B and D, the minimum occurs when \(z_a = z_2\) so that the parameter \(R_{1}\) is equal to the critical value \(R_{1,c}\):

\[
R_{1,c} = \frac{1}{\ln(\alpha)}
\]

(37)

When \(R_{1} = R_{1,c}\), the relative mass of mix steel is:

\[
\frac{M_{12}}{M_{\text{td,ss}}} = \frac{1}{R_{1,c}} - \frac{1}{R_{1,c}} \exp \left(\frac{1 - \alpha}{R_{1,c} - 1}\right)
\]

(38)

For the five cases considered, the maximum amount of mix steel occurs when the parameter \(R_{1}\) is indefinitely large so that:

\[
\lim_{k \to +\infty} \frac{M_{12}}{M_{\text{td,ss}}} = \frac{1}{\alpha}
\]

(39)

\[
\lim_{k \to +\infty} \frac{\Gamma_{\text{max}}}{M_{\text{td,ss}}} = \frac{1}{\alpha}, z < z_a
\]

(40)

However, if \(z_2 < \ln(1/\alpha)\), Eqs. (38) to (40) predict that the relative mass of mix steel tends to zero when \(R_{1}\) is indefinitely large.

5. Conclusions

This work provides general analytical solutions for the Cho and Kim model that facilitates model use. The analytical solutions also help researchers optimize ladle change practice as it was illustrated by an application example. In addition, the meaning of the model parameter \(f\) was investigated in order to gain to a better understanding of the model. If \(f \leq 1\), \((1 - f)\) is the fraction of bypass flow, otherwise \(- (1 - f)\) is the volume fraction of the plug flow region.

Acknowledgments

The authors also acknowledge the financial support of CAPES/PROEX to the graduate program.

The doctoral scholarship, No. 1487157, from CAPES to B. Braga is acknowledged.

REFERENCES