Application of Heat Transfer Coefficient Estimation Using Data Assimilation and a 1-D Solidification Model to 3-D Solidification Simulation

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Solidification simulations are effective in designing a casting process to improve the quality of steel slabs and ingots. In this study, a new method was developed to efficiently estimate multiple heat transfer coefficients to improve the accuracy of three-dimensional (3-D) solidification simulation for a casting process. The heat transfer coefficients for the two heat transfer directions—side and bottom—of a prismatic mold were independently estimated by data assimilation using one-dimensional solidification simulations near the boundaries. The optimum values of the heat transfer coefficients at the side and bottom boundaries were elucidated by comparing the 3-D solidification simulation and experimental cooling curves. The maximum and average errors between the cooling curves of the 3-D solidification simulation with the optimum values and those of the experiment were less than 1.8% and 0.2%, respectively.

KEY WORDS: data assimilation; heat transfer coefficient; solidification; casting; simulation.

1. Introduction

An appropriate design of casting process is required to improve the quality of continuously-cast slabs and/or ingots of steels. Numerical simulations are methods of choice for effective designing of casting processes. There are various commercial software packages available to simulations of the solidification phenomenon during the casting process.1) Solidification simulation has become an important technique for predicting the temperature field to prevent delayed solidification in continuous casting and for designing the riser in ingot casting. Note that the accuracy of solidification simulations significantly depend on input parameters. Among them, the heat transfer coefficient between the cast and mold is one of the key parameters for accurate simulations. However, the accurate value of this parameter is generally not available because it depends on not only the types of cast and mold but also the type of process and the process conditions. Therefore, it is necessary to estimate the heat transfer coefficient for each process type and each process condition of interest in a trial-and-error way. The evaluation of the heat transfer coefficient corresponds to inverse analysis.2–11) The heat transfer coefficient is often assumed a constant value in the inverse analysis by focusing on processes for a short time period. To improve the accuracy of the solidification simulation, however, the time dependence needs to be considered by assuming its function form such as the exponential function. Unfortunately, it is difficult to express the heat transfer coefficient with a simple function because, for instance, the air gap formed at the mold–alloy boundary due to solidification shrinkage causes complicated time-dependence of the heat transfer coefficient in some cases. Moreover, the trial-and-error approach requires significant time to estimate an appropriate heat transfer coefficient. Therefore, there is an urgent need for reliable methods for estimating the heat transfer coefficient.

Various inverse analysis methods 12–22) to estimate the heat transfer coefficient have been recently proposed using a neural network,15,16) Bayesian approach,17) and genetic algorithm.18) However, the application of most inverse analysis methods are limited to special conditions such as steady state condition and no time dependency. Data assimilation is an interesting technique for overcoming these limitations, and can be used more generically than other methods for various casting conditions.19–22) Data assimilation methods, including Kalman,23) ensemble Kalman,24) and particle25) filters, and four-dimensional variational methods,26,27) are often used for parameter estimation. The particle filters are easy to
implement in simulation models. Recently, Oka et al. proposed a method for simultaneously estimating the heat transfer coefficient and thermal conductivity in unidirectional solidification using a particle filter. Based on this method, we developed a method for estimating the time-dependent heat transfer coefficient in unidirectional casting experiments and confirmed that the simulation with use of the heat transfer coefficient estimated by this method can accurately reproduce the cooling curves for the unidirectional casting of Al-1mass%Si alloy. We also observed that the position of the cooling curve(s) used in the estimation was the most important factor and that a cooling curve measured as close to the mold surface as possible should be employed for accurate estimation. Note that the heat transfer in region close to the mold surface is often one-dimensional (1-D) problem and therefore, it was possible to estimate an appropriate heat transfer coefficient using the local one-dimensional (L1D) heat transfer at the mold–alloy boundary.

Generally, there are multiple heat-removal directions and boundaries during the casting process. For example, in the continuous casting of slabs, the heat removal conditions (heat flux) on the wide and narrow surfaces of the mold must differ. It is therefore desirable to use different heat transfer coefficients when performing a solidification simulation. Thus, to perform the accurate solidification simulation of cast products, multiple heat transfer coefficients are required. However, the proper evaluation of multiple heat transfer coefficients, particularly, multiple time-dependent heat transfer coefficients, is difficult. Based on our previous study, it is desirable to simultaneously estimate multiple time-dependent heat transfer coefficients by combining 3-D solidification simulation and data assimilation. However, because of the high calculation cost of estimation by 3-D simulation, it is necessary to devise a method with a low calculation cost. According to our previous study, estimation of the heat transfer coefficient in the L1D region at the mold–alloy boundary is effective in overcoming the problem of multiple heat transfer coefficient evaluations. Additionally, we confirmed that the time-dependent heat transfer coefficient can be appropriately estimated by a particle filter with 1-D simulations. Therefore, in this study, we investigated the applicability of the heat transfer coefficient estimated for the L1D region using a particle filter—L1D estimation—for 3-D solidification simulation. The outline of the L1D estimation is as follows: First, a casting experiment was performed to obtain several cooling curves in the alloy. Subsequently, the heat transfer coefficients on the sides and bottom of the mold were independently estimated in the L1D region using data assimilation. Next, 3-D solidification simulations were performed using the heat transfer coefficient, estimated independently, and compared with the measured 3-D temperature distribution (cooling curves at several locations). The L1D estimation was evaluated based on the extent to which the cooling curve could be reproduced using the heat transfer coefficient obtained by L1D estimation.

2. Experiment and Model

2.1. Experimental Procedure

The casting experiment was performed using Al–Si alloy, with the mold schematically illustrated in Fig. 1. The employed mold and cap were made of heat-resistant brick. The Al-1mass%Si alloy was prepared from 99.99% pure Al and 99.999% pure Si. The alloy was melted in an electric furnace and poured into the mold at a casting temperature of 700°C, after which the cap was immediately placed on the mold. To measure the temperature change in the solidifying ingot and mold, four thermocouples were set in the alloy (positions i1–i4) and two thermocouples were set in the mold brick (m1 and m2). The cooling curves were obtained at one-second intervals. The ingot size obtained by the casting was 50 mm × 50 mm × 80 mm, and i3 and i4 were located 40 and 70 mm from the bottom of the mold, respectively.

2.2. Solidification Simulation Model

To simulate the 3-D temperature distribution in the alloy, we need the heat transfer coefficients at side and bottom boundaries. Such multiple heat transfer coefficients are estimated based on data assimilation using L1D simulation with the cooling curve closest to each mold wall. The heat transfer coefficients at the bottom (h1) and side (h2) of the mold were independently estimated by reproducing the cooling curves at i1 and i2, respectively. One-dimensional solidification analysis was performed to simulate the cooling curves at these positions. The scale of L1D calculation (L) was defined as the distance from the inner surface of the mold. The boundary condition at the mold surface was given by the heat flux $h_1(T_1 - T_{so})$ for the bottom and $h_2(T_2 - T_{so})$ for the side. Here, $T_{so1}$ and $T_{so2}$ are the measured temperatures at positions m1 and m2, respectively. The zero-Neumann condition was set as the boundary condition inside the alloy. The enthalpy method was adapted to calculate the heat flow including the release of latent heat during solidification and was used to simulate the L1D temperature distribution in the alloy. The heat conduction in the mold was not calculated,
and the temperature changes at the bottom and sides of the mold followed the measured cooling curves of \( m_1 \) and \( m_2 \), respectively. The enthalpy method used in this study is explained in detail in our previous paper.21

In the governing equation, the variation in enthalpy within each time step was calculated using Eq. (1):

\[
\rho \frac{\partial H}{\partial t} = \nabla \cdot (\kappa \nabla T) \quad \text{.................................(1)}
\]

where \( H, T, t, \kappa, \rho \) are the enthalpy, temperature, time, thermal conductivity, and density, respectively.

As the initial condition, the casting temperature was set to the initial temperature of all the calculation meshes in the 1-D calculation scale. Hence, the amount of heat removed from the mold increases with increasing \( L \), and the estimated heat transfer coefficient depends on \( L \). Therefore, the optimum value of \( L \) that yields the heat transfer coefficient used for accurate 3-D simulation cannot be known in advance. In this study, \( h_1 \) and \( h_2 \) were obtained for \( L = 6–15 \) mm at 1 mm intervals, and \( L_1 \) and \( L_2 \) are hereafter used to denote \( L \) for estimation of \( h_1 \) and \( h_2 \), respectively. Therefore, \( h_1 \) for \( L_1 = 6–15 \) mm and \( h_2 \) for \( L_2 = 6–15 \) mm (10 cases each) were independently estimated based on data assimilation with LID simulation. Subsequently, 3-D solidification simulations were performed for all set of \( h_1 \) and \( h_2 \) to elucidate the optimal set that yields most accurate result of the cooling curves at the four points \((i_1–i_4)\) in the casting. The mesh size was 1 mm in both the 1-D solidification simulations for estimating \( h_1 \) and \( h_2 \), and the 3-D solidification simulation. The \( \kappa \) and \( \rho \) values of the Al–Si alloy used in the simulations were 95 W K\(^{-1}\) m\(^{-1}\) and \( 2.39 \times 10^3 \) kg m\(^{-3}\), respectively.29

The temperature–enthalpy relationship for Al-1 mass%Si alloy was obtained using Pandat software with the PanAl database. The liquidus and solidus temperatures were 654.6 \( \degree \)C and 607.0 \( \degree \)C, respectively. The enthalpy method used in this study is effective for estimating the time-dependent heat transfer coefficient, which is briefly described below. The method used in this study is the same as that used in a previous study.21

2.3. Estimation of Time-dependent Heat Transfer Coefficient Using Particle Filter

Data assimilation is employed to perform highly accurate estimation by combining observational data and a simulation model. Although it was initially developed for numerical weather prediction, it is also an effective and practical technique for estimating parameters or material properties that are difficult to measure experimentally. The particle filter is a data assimilation method that uses a simple algorithm and is easy to implement. Thus, we adopted a particle filter to estimate the time-dependent heat transfer coefficient, which is described below. The method used in this study is the same as that used in a previous study.21

Figure 2 is a schematic diagram explaining the estimation process of the time-dependent heat transfer coefficient \( h \) using a particle filter. In the initial state, arbitrary values of \( h \) were randomly assigned to each simulation called particle. In the present case, the particle corresponds to each LID simulation. The heat flow was calculated using the assigned value of \( h \) for each particle. The value of likelihood for each particle was then calculated, followed by re-sampling. This operation is called “filtering.” The likelihood was determined by the difference between the measured and calculated cooling curves. The likelihood of particle \( i \) at \( t \) (\( \lambda_i^t \)) was given by the following equation:

\[
\lambda_i^t = \frac{1}{\sqrt{2\pi}\Sigma} \exp \left[ -\frac{1}{2} \left( T_{\text{meas}} - T_{\text{cal}} \right)^T \Sigma^{-1} \left( T_{\text{meas}} - T_{\text{cal}} \right) \right] \quad \text{.................................(2)}
\]

where \( \Sigma \) is a variance-covariance \((m \times m)\) matrix, \( m \) indicates the number of different positions of the thermocouples used for temperature measurement, and \( T_{\text{meas}} \) and \( T_{\text{cal}} \) are vector notations of the measured and calculated temperatures at the different thermocouple positions, respectively. This work assumed \( \Sigma = \sigma_i^2 \mathbf{I} \), where the mean is zero, \( \sigma_i \) is the standard deviation of error in the temperature measurement, and \( \mathbf{I} \) is the \( m \times m \) unit matrix. For resampling after calculating the likelihood, the normalized likelihood of the \( j \)th particle \((\beta_j^t)\) was calculated using Eq. (3):

\[
\beta_j^t = \frac{\lambda_j^t}{\sum_{j=1}^{N} \lambda_j^t} \quad \text{.................................(3)}
\]

where \( N \) is the total number of particles.

In the subsequent resampling, particles with lower normalized likelihoods (smaller circles in Fig. 2), were randomly selected and removed, whereas those with higher normalized likelihoods (larger circles in Fig. 2) were duplicated. Note that the produced particles presented the same heat transfer coefficient as the original parent particle. Thus, system noise \((\nu)\) was added to all the particles, \( i.e., h + \nu \), after updating. The values of \( h \) for each particle after resampling were used for the heat flow calculations in the next step. The system noise was taken as the product of \( h \) for the particles with the highest likelihood and a random number given by the Gaussian distribution, with the mean and a standard deviation are 0 and \( \sigma_n \), respectively. The temperature profile calculated for the particles with the highest likelihood was updated as the initial temperature profile for the next step. In this study, \( h \) estimated at \( t \) was determined to be the one with the highest likelihood. By iterating these steps, we could estimate the time-dependent heat transfer coefficient. The values of \( \sigma_1 \) and \( \sigma_n \) used in this study were 2.0 K and 0.2, respectively, while the number of particles \( N \) was 2 048.
3. Results and Discussion

3.1. Experimental Cooling Curves

Figure 3 shows the cooling curve measured at each position in the casting experiment. Figure 3(a) shows the cooling curves at i₁, i₃, and i₄ in the alloy, corresponding to the temperature distribution in the centerline of the z-axis direction, while Fig. 3(b) shows the cooling curves at i₂ and i₅ in the alloy, corresponding to the temperature distribution in the centerline of the x-axis (or y-axis) direction. Figure 3(c) shows the cooling curves at m₁ and m₂ in the mold. From Figs. 3(a) and 3(b), the temperature gradient between i₁ and i₃ indicates that heat is removed from the bottom of the mold. In Fig. 3(b), it is understood that there is a temperature gradient between i₂ and i₅, the heat is removed from the side surface of the mold as shown in Fig. 3(c), the temperature at the bottom of the mold gradually increased to ~400°C, while it increased sharply to 500°C at its side. Therefore, we confirmed that the heat was removed from multiple (side and bottom) directions of the mold during solidification of the alloy.

3.2. Estimation of Heat Transfer Coefficient Using 1D Solidification Simulation

As described in Section 1, the time-dependent heat transfer coefficients of the bottom and side of the mold were estimated from the particle filter with the 1D solidification simulation. Before showing the estimated heat transfer coefficients, we start the discussion from the comparison between the measured and simulated cooling curves. Figures 4(a) and 4(b) show the cooling curves at i₁ and i₂ obtained by 1D estimation for L₁ = 10 mm and L₂ = 10 mm, respectively. The measured cooling curves are shown in Fig. 4 for comparison. The simulated cooling curves at i₁ and i₂ well agreed with the measured curves. Such agreement was observed for all values of L₁ and L₂ tested in this study. The relative error (ΔEᵣ) was calculated at each time using Eq. (4), and the average of ΔEᵣ (ΔEᵣave) was calculated using Eq. (5).

\[ \Delta Eᵣ[\%] = 100 \left( \frac{T_{\text{meas, } t} - T_{\text{sim, } t}}{T_{\text{meas, } t}} \right) \]  \hspace{1cm} \text{Eq. (4)}

\[ \Delta Eᵣave[\%] = \frac{1}{M} \sum_{i=1}^{M} \Delta Eᵣ \]  \hspace{1cm} \text{Eq. (5)}

where \( T_{\text{meas, } t} \) and \( T_{\text{sim, } t} \) are the experimental and simulation temperatures at time \( t \), respectively, and \( M \) is the number of time steps and \( M \) was set to 1 000. Table 1 lists the maximum ΔEᵣ (ΔEᵣmax) and ΔEᵣave for all values of L₁ and L₂. In the \( h₁ \) estimation, ΔEᵣmax and ΔEᵣave were less than 0.210% and 0.005%, respectively, for all L₁ values, while in \( h₂ \) estimation, they were less than 0.536% and 0.008%, respectively, for all L₂ values. The errors in the measured and simulated cooling curves were very small under all conditions, indicat-
ing that the cooling curve can be accurately simulated from the time-dependent heat transfer coefficient estimated using data assimilation. Note that the temperature distribution due to undercooling before nucleation of the primary \(\alpha\)-Al phase was also reproduced as seen in Fig. 4. The maximum error \(\Delta E_{\text{max}}\) is actually associated with the undercooled region for all conditions. The enthalpy method employed in this study uses the relationship between the temperature and enthalpy in the equilibrium state. Thus, the temperature variation in the undercooling and recalescence due to the nucleation of the primary \(\alpha\)-Al, in principle, cannot be reproduced in the present solidification model. This drawback of the simulation model is cured by employing the time-dependent heat transfer coefficient estimated by the particle filter. This is one of the advantageous features of particle filter.

Figures 5(a)–5(c) show the relationship between the time and \(h_1\) obtained by L1D estimation for \(L_1 = 7, 11,\) and 15 mm, while Figs. 6(a)–6(c) show the relationship between time and \(h_2\) obtained by L1D estimation for \(L_2 = 7, 11,\) and 15 mm. For all values of \(L_1\) and \(L_2\), the heat transfer coefficient increased sharply during undercooling and decreased during recalescence. Particularly, the heat transfer coefficient \(h_2\), which comprised a high degree of undercooling, increases to more than 1000 W m\(^{-2}\) K\(^{-1}\). When the temperature increased due to recalescence, \(h_2\) became negative. The negative value of \(h\) means that the heat transfer occurs from the mold to the alloy. However, this unphysical process does not occur in the present experiment. As mentioned above, the L1D simulation cannot reproduce the undercooling and thereby the subsequent recalescence, which usually causes the difference between the experimental and simulation results. However, the particle filter enables the accurate reproduction of the experimental results including the undercooling and recalescence even with the L1D estimation. Hence, the heat transfer coefficient exhibits the negative value during recalescence.

Excluding the time in the undercooled region, the estimated heat transfer coefficient was relatively close to a constant value for both \(h_1\) and \(h_2\). Table 1 lists the average values of the heat transfer coefficients for 100–1000 s, which increased with increasing calculation area \((L_1\) and \(L_2)\). This is because the longer is the 1-D area, the larger is the initial heat in the alloy. To remove a large amount of heat to the outside of the alloy while reproducing the measured cooling curve, the heat transfer coefficient must be large. Therefore, 10 different time-dependent heat transfer coefficients could be obtained for both \(h_1\) and \(h_2\), and we proceeded to determine the optimal set of \(h_1\) and \(h_2\) that yields the most accurate 3-D simulation result of temperature distribution in the alloy.

### 3.3. Applications to 3-D Simulation

To efficiently elucidate the optimum values of \(h_1\) and \(h_2\), we proceeded to perform a three-step evaluation: First, 3-D solidification simulations were performed using the

<table>
<thead>
<tr>
<th>Scales for L1D calculation, (L_1, L_2) [mm]</th>
<th>(\Delta E_{\text{max}}) [%]</th>
<th>(\Delta E_{\text{ave}}) [%]</th>
<th>(h_1)ave [W m(^{-2}) K(^{-1})]</th>
<th>(\Delta E_{\text{max}}) [%]</th>
<th>(\Delta E_{\text{ave}}) [%]</th>
<th>(h_2)ave [W m(^{-2}) K(^{-1})]</th>
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<tr>
<td>6</td>
<td>0.210</td>
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<td>22.70</td>
<td>0.409</td>
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<td>53.61</td>
<td>0.195</td>
<td>0.003</td>
<td>91.71</td>
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</table>
heat transfer coefficients estimated for $L_1 = L_2$. Next, we compared the simulated and measured cooling curves at $i_1$ and $i_2$ and evaluated the degree of agreement using the solidification time. Finally, the optimum values of $h_1$ and $h_2$ were thoroughly investigated.

**Figure 7** shows the cooling curves at $i_1$ and $i_2$ obtained by 3-D solidification simulation using the heat transfer coefficients estimated at $L_1 = L_2 = 10$, 11, and 12 mm. The accuracy of the simulation was evaluated by comparing the experimental and simulation solidification times. The solidification time is defined as the time when the temperature at the measurement position reaches the solidus temperature ($607^\circ$C) of the Al-1mass%Si alloy. The simulated cooling curves at $L_1 = L_2 = 11$ mm were in relatively good agreement with the measured cooling curve at both $i_1$ and $i_2$. On the other hand, the solidification time of the simulation with $L_1 = L_2 = 10$ mm was longer than that of the experiment, while the one with $L_1 = L_2 = 12$ mm was shorter. Therefore, based on the conditions $L_1 = L_2 = 11$ mm, the optimal set was thoroughly investigated using $h_1$ and $h_2$ estimated for $L_1 \neq L_2$ in the range $11 \pm 2$ mm.

**Figure 8** shows the cooling curves at $i_1$ and $i_2$ obtained by 3-D solidification simulation using $h_1$ ($L_1 = 9$–11 mm) and $h_2$ ($L_2 = 11$ mm). Although the solidification time of the simulation with $h_1$ for $L_1 = 9$ mm was slightly longer than that of the experiment, the simulated and measured cooling curves were in good agreement. The agreement in the case of $L_1 = 10$ mm is better than the cases of $L_1 = 9$ and 11 mm. **Figure 9** shows the cooling curves at $i_1$ and $i_2$ obtained...
by 3-D solidification simulation using $h_1$ ($L_1 = 11$ mm) and $h_2$ ($L_2 = 10–12$ mm). The solidification time in case of $L_2 = 10$ mm was longer than that of the experiment, while that of $L_2 = 12$ mm was shorter. Compared with the cooling curve of $L_2 = 11$ mm, these simulated cooling curves deviated significantly from the measured cooling curves. Table 2 shows the difference between the experimental and simulation solidification times at each position in the alloy.

Table 3 lists the average ($\Delta E_{\text{ave}}$) and maximum ($\Delta E_{\text{max}}$) errors between the cooling curves of the experiment and 3-D solidification simulation using the set of $h_1$ and $h_2$ at $L_1 = 10$ mm and $L_2 = 11$ mm. The cooling behavior at each position was investigated in detail. As shown in Fig. 8(b), the simulated cooling curve at $i_2$ well agreed with the measured cooling curve, including the undercooled region. This cooling curve was accurately reproduced because the cooling curve at this position was used to estimate $h_2$. However, in the cooling curve at $i_1$ used in the estimation of $h_1$, the simulated temperature was slightly lower than the measured temperature at 100–350 s (Fig. 8(a)). Also, $h_1$ was slightly convex at 100–350 s (Fig. 5). In the LID estimation, $h_1$ was estimated by removing all heat solely by 1-D heat transfer to the bottom of the mold, which might be overestimated as the heat transfer coefficient for 3-D simulation. In 3-D simulation, we believe that the heat removed from the bottom of the mold slightly decreased because the heat were removed

### Table 2. Difference between the experimental and simulation solidification times at each position in the alloy.

<table>
<thead>
<tr>
<th>$L_1$ [mm]</th>
<th>$L_2$ [mm]</th>
<th>Difference in solidification time</th>
<th>$i_1$</th>
<th>$i_2$</th>
<th>$i_3$</th>
<th>$i_4$</th>
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<tr>
<td>10</td>
<td>10</td>
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<td>11</td>
<td>15.8</td>
<td>12.8</td>
<td>12.8</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>116.2</td>
<td>112.4</td>
<td>112.6</td>
<td>101.4</td>
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</tr>
<tr>
<td>9</td>
<td>11</td>
<td>23.4</td>
<td>26.0</td>
<td>25.8</td>
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</tr>
<tr>
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<td>11</td>
<td>0.8</td>
<td>2.6</td>
<td>2.8</td>
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<tr>
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<td>12</td>
<td>101.8</td>
<td>99.8</td>
<td>100.0</td>
<td>89.0</td>
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</table>

- Positive and negative values indicate respective longer and shorter solidification times than those observed in the experiments.

Figure 10 shows the cooling curves at $i_3$ and $i_4$ obtained by 3-D solidification simulation using the optimal set of $h_1$ and $h_2$.
from the side of the mold. Therefore, this excessive heat transfer to the bottom of the mold by the higher $h_1$ value estimated from 1-D condition was simulated as a temperature drop at 100–350 s. The same temperature drop seen at $t_1$ was also observed in the simulated cooling curve for $h_2$, which is located far from the bottom of the mold. The Al–Si alloy exhibits a high thermal conductivity and the size of the mold used in this study is relatively small. This temperature drop may have therefore been affected by heat removal from the bottom of the mold. Because steels with lower thermal conductivities than those of Al alloys tend to have a local 1-D temperature gradient near the mold wall, the heat transfer coefficient might be accurately obtained by L1D estimation. In addition, the calculation time required for L1D estimation approximated several tens of minutes, using a laptop computer for each $L$ condition. Even if the optimal set of heat transfer coefficients is searched by the 3-D solidification simulation, the calculation cost of the present method is still lower than that of the simultaneous estimation of multiple heat transfer coefficients by the 3-D model.

4. Conclusions

In this study, we investigated whether a 3-D solidification simulation can be performed accurately using the heat transfer coefficient estimated in a L1D region. To estimate the heat transfer coefficient in this region, we used the particle filter with the L1D solidification simulation developed in our previous study.21) The following conclusions were drawn from this study:

(1) Estimation of the heat transfer coefficient by the particle filter using L1D simulation enabled accurate simulation of the measured cooling curve (maximum error < 1%).

(2) Data assimilation enables estimation of the time-dependent heat transfer coefficient that can accurately simulate the cooling curve including the undercooling and recalescence that the model does not explicitly consider.

(3) By estimating the heat transfer coefficient under various L1D conditions and selecting the optimum condition from the estimated heat transfer coefficients, the 3-D solidification simulation that requires multiple heat transfer conditions could be performed with high accuracy.

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REFERENCES