Influence of Vertical Static Magnetic Field on Behavior of Rising Single Bubble in a Conductive Fluid

Xiao-Hong TIAN,1) Wan-Yuan SHI,1,2)* Tian TANG1) and Lin FENG1)

1) College of Power Engineering, Chongqing University, No.174, Sha-Zheng Street, Sha-Ping-Ba District, Chongqing, 400044 China. 2) Key Laboratory of Low-grade Energy Utilization Technologies and Systems, Ministry of Education, No.174, Sha-Zheng Street, Sha-Ping-Ba District, Chongqing, 400044 China.

(Received on August 23, 2015; accepted on November 5, 2015; J-STAGE Advance published date: December 25, 2015)

In order to investigate the influence of static magnetic field on behavior of rising single bubble in conductive fluid, a series of axisymmetric numerical simulations are carried out. A uniform vertical magnetic field with intensities ranging from 0 to 0.4 T (\(H_a=0−16.97\)) is superposed and the bubble radii range from 2 mm to 6 mm (\(R^* = 0.2−0.6\)). The rising velocity, instantaneous bubble shape and terminal height are discussed and whether the magnetic field restrains bubble rising simply or transits from positive effect to negative one with the increasing of magnetic field intensities is analyzed clearly. Besides, the discrepancy of bubble motion under magnetic field with weak or strong surface tension is compared. Numerical results show that vertical magnetic field elongates the bubble shape along vertical direction and a stronger magnetic field intensity contributes to a longer bubble shape. The imposed magnetic field has an inhibitory effect on the rising velocity for bubbles with strong surface tension as well as bubbles with weak surface tension but small sizes. However, for bubbles with weak surface tension but large sizes, the rising velocity is promoted by weak magnetic field intensities whereas inhibited by strong magnetic field intensities. The peak Hartmann number reaching the strongest positive effect and the critical Hartmann number turning from positive effect to negative effect are determined respectively.

KEY WORDS: rising single bubble; vertical static magnetic field; Lorentz force; numerical simulation.

1. Introduction

Bubble motion in liquid metal or magnetic fluid plays an important role in many industrial processes, for instance, the injection of argon gas into liquid metal to remove impurities during the continuous casting production process1) and the gas bubbles formation in foam metal production.2) In order to control the bubble flow, appending an external magnetic field is a contactless and effective way. Obviously, it’s significant to probe into the influence of magnetic field on bubble motion in conductive fluid.

Bubble motion in liquid has been studied both experimentally and numerically in many works.3−6) Among which, most of studies focused on the influence of buoyancy force, viscous force, surface tension force as well as bubble size on bubble movement. Recently, the influence of magnetic field on bubble motion has aroused a growing concern. Ishimoto et al.7) investigated the effect of non-uniform magnetic field on bubble rising through a transparent thin duct. Experimental results showed that bubble was elongated along the magnetic field direction and the bubble rising velocity declined in positive magnetic field gradient region while the bubble was accelerated in negative magnetic field gradient region. The same result was obtained in the experiments of single bubble rising in water based Fe3O4 magnetic fluid under a non-uniform magnetic field by Bi et al.8) and found that the bubble motion was stabilized in magnetic field. Zhang and coworkers9) examined the influence of a DC longitudinal magnetic field on single argon bubble rising in the eutectic alloy GaInSn. The significant modification of the bubble wake structures by magnetic field was observed and discussed. Bashkovoi et al.10) studied the chain flow of bubbles in magnetic fluid and observed the bubble chain separation in magnetic field. They found that the disintegration numbers of chains increased with the increasing of magnetic field intensity. Except for experiments, plenty of numerical simulations on magnetohydrodynamic (MHD) flow with bubbles were conducted in published papers. Merrouche et al.11) simulated the bubble rising in metallic liquid under vertical magnetic field and presented the profile of induced magnetic field. The two-dimensional axisymmetric bubble rising in a vertical magnetic field was calculated by Ueno and coworkers using Front Tracking method.12) They concluded that the shape of bubble was oblate in a weak magnetic field while it was prolate in a strong magnetic field. The influence of magnetic field on three-dimensional trajectory of an argon bubble in liquid metal was analyzed by Schwarz and Fröhlich.13) They pointed out that the effect on the trajectory
indeed resulted from the magnetic function on the bubble wake. Miao and coworkers\textsuperscript{14} simulated the effect of a static magnetic field on a bubble-driven liquid metal flow inside a cylinder using the Euler–Euler multiphase model and stated that the transverse magnetic field could cause distinct oscillations of the liquid velocity. Huang et al.\textsuperscript{15} investigated the terminal bubble shape of gas bubble with various initial shapes under magnetic field, finding that the initial shape had slightly influence on the terminal bubble shape when the magnetic intensity was strong. They also stated that the rising velocity declined in the additional magnetic field. Zhang and Ni\textsuperscript{16} found that if the original path of a rising bubble was non-linear without magnetic field, the rising velocity increased in weak magnetic field whereas it decreased in strong magnetic field. Magnetic field affected rising velocity by changing the ascending path. While for bubble which had a straight ascending path in non-magnetic field, the additional magnetic field slowed down rising velocity monotonously, which was consistent with the conclusion in Ref. 15). Nevertheless, Shibasaki et al.\textsuperscript{17} simulated bubble moving in a direct ascending path with or without magnetic field, finding that the weak magnetic field promoted bubble rising but the strong magnetic field presented an inhibiting effect, which was opposite to the conclusion in Ref. 16).

It can be seen that the divergence remains in the studies about bubble motion under magnetic field. The problem of whether the additional magnetic field promoting bubble ascending or inhibiting it hasn’t been analyzed thoroughly. Besides, for the situation of magnetic field function reverses with Hartmann number increasing, the peak Hartmann number and the critical Hartmann number haven’t been exposed in the existing literature. Furthermore, bubble motion under magnetic field is actually quite complex. Other factors such as surface tension, viscosity ratio and density ratio may play a part in the process too. Among these factors, the surface tension is believed to have a significant effect on bubble shape that affects the bubble motion markedly. In other words, even the same magnetic field intensity may give different effect on bubbles if their surface tension coefficients are different. Thus the effect of surface tension is taken into consideration additionally.

In this paper, the investigation of rising single bubble under different magnetic field intensities is performed through a series of numerical simulations and the effect of magnetic field on the rising velocity, instantaneous bubble shape, the peak Hartmann number as well as the critical magnetic field are discussed carefully. The influence of surface tension on rising velocity and instantaneous bubble shape is taken into account. The mechanisms of the effect of magnetic field on them are illuminated clearly. For simplification, a 2D axisymmetric model is adopted for all the simulations. Stewart\textsuperscript{18} and Liu\textsuperscript{19} pointed out that in liquid with high viscosity a bubble passes through a nearly straight path. Here, our simulations also focus on fluids with high viscosity, thus the assumption of 2D axisymmetric model is acceptable.

### 2. Physical and Mathematical Models

A computational domain with the sizes of $4R \times 16R$ is shown in Fig. 1. Axisymmetric condition and wall boundary condition are applied on the symmetry axis and the other three boundaries respectively. All of the wall boundaries are assumed to be electrically insulating. Initially, the center of bubble is placed on the symmetry axis at $3R$ far from the bottom. Both bubble and liquid are set to be static in initial moment. Uniform magnetic field of $B$ is appended vertically in computational domain. Related computational parameters are listed in Table 1.

The gas and liquid are assumed to be incompressible and immiscible. Joule heating and induced magnetic field are neglected. The continuity equation and momentum equation are expressed as follows:

$$\nabla \cdot u = 0 \quad \text{(1)}$$

$$\rho \left( \frac{\partial u}{\partial t} + u \cdot \nabla u \right) = -\nabla p + \nabla \cdot \left[ \eta \left( \nabla u + \nabla u^T \right) \right] + f_i + f_s + f_L \quad \text{(2)}$$

In the published papers, several numerical methods have been proposed and adopted for MHD two-phase flow simulations.\textsuperscript{20–23} Among which, the Volume of Fluid (VOF) method has been used most frequently to capture the interface since it can conserve the mass well in calculation.\textsuperscript{24–26} Hence, the VOF algorithm is adopted in this work. In the VOF method,\textsuperscript{27} the parameter of volume function $C$ is applied to index the interface of two phases, and the governing equation is employed as:

$$\frac{\partial C}{\partial t} + u \cdot \nabla C = 0 \quad \text{(3)}$$

In above equations, $u$ and $p$ represents velocity vector and pressure respectively. $\rho$ and $\eta$ are density and viscosity of

![Fig. 1. Schematic physical model of single bubble rising in conductive fluid under vertical static magnetic field. (Online version in color.)](image)

<table>
<thead>
<tr>
<th>Cases</th>
<th>$\rho/\rho_l$</th>
<th>$\eta/\eta_l$</th>
<th>$\sigma_\tau/\sigma_{\tau_l}$</th>
<th>$Re$</th>
<th>$Fr$</th>
<th>$We$</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>1 000</td>
<td>100</td>
<td>$3.6 \times 10^{10}$</td>
<td>6.45</td>
<td>0.102</td>
<td>7.59</td>
</tr>
<tr>
<td>B</td>
<td>1 000</td>
<td>100</td>
<td>$3.6 \times 10^{10}$</td>
<td>6.45</td>
<td>0.102</td>
<td>0.43</td>
</tr>
</tbody>
</table>
fluids respectively. The parameter \( C \) denotes the volume rate of fluid, so that the region \( C = 1 \) is gas phase, the region \( C = 0 \) is liquid phase and the area \( 0 < C < 1 \) represents the interface of gas and liquid. With the parameter \( C \), the density and viscosity are defined as:

\[
\rho = C \rho_s + (1 - C) \rho_l \quad \text{(4)}
\]

\[
\eta = C \eta_s + (1 - C) \eta_l \quad \text{(5)}
\]

The terms \( f_s, f_g, f_l \) in Eq. (2) are surface tension force, gravity force and Lorentz force respectively. In which, the gravity force is:

\[
f_g = \rho g \quad \text{(6)}
\]

The surface tension force is calculated by the CSF model:

\[
f_s = \sigma_s k \left( \frac{\rho \nabla C}{(\rho_s + \rho_l) / 2} \right) \quad \text{(7)}
\]

Where the subscripts \( l, g \) denote liquid and gas respectively. \( \sigma_s \) is surface tension coefficient, \( k \) is the curvature of interface which is estimated by \( k = -\nabla (\nabla C / |\nabla C|) \).

The Lorentz force is solved by the following equations:

\[
J_s = \sigma_s (-\nabla \phi + u \times B) \quad \text{(8)}
\]

\[
f_l = J \times B \quad \text{(9)}
\]

Where \( J, \phi, B, \sigma \) represent electric current density, electric potential, imposed magnetic field and electric conductivity. Here, \( \sigma = C \sigma_s + (1 - C) \sigma_l \). It should be noted that since the flow is axisymmetric, the electric potential would be zero all over the model.

In order to nondimensionalize the above governing equations, the following dimensionless variables and dimensionless numbers are introduced:

\[
(r^*, z^*) = \left( \frac{r}{R}, \frac{z}{L} \right), \quad R^* = \frac{R}{L}, \quad U^* = \frac{U}{L}, \quad p^* = \frac{p}{\rho U^2},
\]

\[
\rho^* = \frac{\rho}{\rho_l}, \quad \eta^* = \frac{\eta}{\eta_l}, \quad \sigma^* = \frac{\sigma}{\sigma_l}, \quad B^* = \frac{B}{B_l}, \quad J^* = \frac{J}{\sigma_l B_s U}, \quad Re = \frac{\rho U L}{\eta_l}, \quad Fr = \frac{U^2 g L}{B_s L},
\]

\[
We = \frac{\rho U^2 L}{\sigma_l \eta_l B_s L}, \quad Ha = \frac{\sigma_s B_s}{\eta_l}
\]

Where \( Re, Fr, We, Ha \) are the Reynolds number, Froude number, Weber number and Hartmann number respectively. \( U \) and \( L \) represent the characteristic velocity and the characteristic length respectively. The characteristic velocity is defined as the terminal velocity’s order of magnitude, which is given to be 100 mm/s here. In the relevant literatures the bubble diameter is usually set as the characteristic length, but not in this work. The reason is that in our discussions the bubbles with different sizes will be simulated. If the bubble diameter is set as the characteristic length the Hartmann number would not be equal even under the same magnetic intensity of \( B \). To avoid misunderstanding to readers, a fixed value of 10 mm is chosen as the characteristic length. In fact, the order of magnitude of bubble diameters involved in the current paper is 10 mm.

Based on the dimensionless variables and dimensionless numbers above, the Eqs. (1)–(3) and the boundary conditions (not listed above) are non-dimensionalized and rewritten as follows:

\[
\nabla^* \cdot \mathbf{u}^* = 0 \quad \text{(10)}
\]

\[
\frac{\partial \phi^*}{\partial t} + \nabla^* \cdot \mathbf{u}^* = 0 \quad \text{(11)}
\]

\[
\frac{\partial \mathbf{u}^*}{\partial t} + \nabla^* \cdot \mathbf{u}^* \nabla^* \mathbf{u}^* = -\nabla^* p^* + \frac{1}{Re} \nabla^* \left[ \eta^* (\nabla^* \mathbf{u}^* + (\nabla^* \mathbf{u}^*)^T) \right] - \frac{1}{We} \nabla^* \left( \frac{\rho^* \nabla^* C}{|\nabla^* C|} \right) (\rho_s / \rho_l + 1) / 2 + \frac{\rho^* g}{Fr} \eta^* \frac{Ha^2}{Re} (\mathbf{u}^* \times \mathbf{B}^*) \quad \text{(12)}
\]

The boundary conditions are:

- At the axisymmetric line,

\[
\frac{\partial \mathbf{u}^*}{\partial r} = 0, \quad \frac{\partial \phi^*}{\partial r} = 0 \quad \text{............(13a,b)}
\]

- At the top and bottom walls,

\[
\mathbf{u}^* \cdot \mathbf{n} = \mathbf{u}^* \cdot \mathbf{\tau} = 0, \quad \frac{\partial \phi^*}{\partial z} = 0 \quad \text{............(14a,b,c)}
\]

- At the right wall,

\[
\mathbf{u}^* \cdot \mathbf{n} = \mathbf{u}^* \cdot \mathbf{\tau} = 0, \quad \frac{\partial \phi^*}{\partial r} = 0 \quad \text{............(15a,b,c)}
\]

Where \( \mathbf{n} \) denotes the normal unit vector and \( \mathbf{\tau} \) is the tangent unit vector of wall.

The partial differential Eqs. (10)–(12) and the boundary conditions (13)–(15) are discretized with a finite volume method. The PRESTO! (Pressure Staggering Option) scheme is adopted in pressure term discretization and the Geo-Reconstruct scheme which using a piecewise-linear approach is used to represent the interface between fluids. The coupled algorithm between pressure and velocity is the PISO (Pressure Implicit with Splitting of Operators) method. The commercial software of Fluent is utilized as a computational tool.

3. Numerical Validation and Mesh Independence

3.1. Numerical Validation

To confirm the validity of our calculation, a simulation is conducted to calculate single gas bubble rising in water using the current algorithm. The dimensionless radius of the bubble is \( R^* = 0.5 \). The density ratio and viscosity ratio between water and gas are 1000 and 100 respectively. The initial velocities of the bubble and the liquid are 0. Simulation result is compared with the calculation of Hua and Lou\(^{(29)}\) as shown in Fig. 2. During the rising process the bottom of bubble concaves inward and finally it turns into spherical-cap shape. The instantaneous bubble shapes at \( t^* = 2, t^* = 6 \) and \( t^* = 10 \) are in good agreement with those in Ref. 29. Thus the present method is reliable.

3.2. Mesh Independence

In order to choose a suitable mesh size, the mesh independence analysis is performed for a single bubble rising
using three kinds of structural grids with grid intervals of $D/20$ ($40 \times 160$), $D/50$ ($100 \times 400$) and $D/80$ ($160 \times 640$) (Where $D$ is the diameter of bubble) respectively. The terminal bubble shape and the instantaneous position of bubble’s apex under different meshes between $t^* = 2$ − $3$ are presented in Fig. 3. It can be seen that bubble shapes are nearly the same. However, the locations of bubble under grid intervals of $D/20$ have significant difference from those under $D/50$ and $D/80$. While the bubble locations under grid intervals of $D/50$ and $D/80$ show little discrepancy. The maximum relative error for the instantaneous position of bubble’s apex between $D/50$ and $D/80$ is $0.19\%$. It means that grid intervals of $D/50$ can ensure the calculating accuracy. Therefore, the following calculations are conducted with grid interval of $D/50$.

4. Results and Discussions

As gas is non-conductive fluid, magnetic field is unable to affect the motion of bubbles directly. However, the Lorentz force influences the flow of the conductive liquid around the bubble, which is affected by magnetic field indirectly. Besides, the Lorentz force imposed on the air-liquid interface impacts the bubble shape largely.

Let us first discuss the effect of uniform static vertical magnetic field on a single bubble with fixed surface tension. The bubbles with different sizes of $R^* = 0.3$, $0.4$ and $0.5$ are calculated under variable magnetic field intensities of $0$ T, $0.1$ T, $0.2$ T and $0.4$ T (the corresponding Hartman numbers are $Ha = 0$, $4.24$, $8.49$ and $16.97$ respectively). The calculation parameters are listed in Table 1 (case A). The locations and the shapes of the bubbles at $t^* = 0$, $1$, $2$, $3$ are shown in Fig. 4.

In the situation of $Ha = 0$ (see the left column in Fig. 4), the buoyancy force drives the bubble rising up. In the initial moment, the jet flow on the bottom of bubble concaves the bubble inwardly. The bubble deforms continuously along the ascending process until it reaches the final shape. For the
bubble of $R^* = 0.3$, the final shape is hemisphere, while it is spherical-cap for $R^* = 0.4$ and 0.5. And the bottom concavity in bubble of $R^* = 0.5$ is deeper than that in $R^* = 0.4$.

When the bubble is exposed in magnetic field, it is clear that the bubble shape is elongated along vertical direction by Lorentz force. Similar results were obtained in Refs. 7) and 12). With the increasing of magnetic field intensity, the Lorentz force is larger so that bubble shape is getting longer. When $Ha = 16.97$, final shape presents semielliptical-cap for $R^* = 0.3$ and deep semielliptical-cap for $R^* = 0.4$ and 0.5.

The aspect ratio $\lambda$, defined as the ratio between the vertical axis and the initial diameter, is introduced to measure the deformation of bubble. The evolutions of the $\lambda$ during the rising process under different magnetic field are illustrated in Fig. 5. At initial time, the aspect ratio is equal to 1 since the bubble is spherical. Then the aspect ratio $\lambda$ are modified with the changing of the bubble shape and the aspect ratio of final shape of bubble is defined as $\lambda_f$. In the condition without magnetic field, the aspect ratio $\lambda < 1$, which means the bubble is stretched horizontally. Under magnetic field, the aspect ratio $\lambda_f$ is larger than that without magnetic field and the bubble is elongated vertically, as seen from Fig. 4. The elongation effect of magnetic field becomes more remarkable for large bubbles. For bubbles with $R^* = 0.3$, the aspect ratio $\lambda_f = 0.81$ when $Ha = 0$, and it is 1.12 when $Ha = 16.97$, which is 1.38 times larger. For bubbles with $R^* = 0.4$, the aspect ratios $\lambda_f = 0.65(Ha = 0)$ and 1.19($Ha = 16.97$) respectively. The ratio of these two values is 1.83. Furthermore, for bubbles with $R^* = 0.5$ they are 0.56($Ha = 0$) and 1.22 ($Ha = 16.97$) respectively and the ratio rises to 2.18. The increasing of elongation effect of magnetic field with increment of bubble’s size can be explained by the Yang-Laplace equation. The equation indicates that the larger the bubble size is, the lower pressure difference between inside and outside of the bubble will be. Hence, the elongation effect of magnetic field becomes more significant for large bubble.
since it is easier to deform with lower pressure difference. 

**Figure 6** shows the rising velocity of bubbles under different magnetic intensities. The bubble is assumed to be static at initial time and then it accelerates because of buoyancy force. The rising velocity finally sustains a relative stable value, which is defined as the terminal velocity. Discrepancies of terminal velocity appear between different magnetic field intensities as well as different bubble sizes. For $R^* = 0.3$, the rising velocity at $Ha = 4.24$ is close to that at $Ha = 0$. For bubbles with $R^* = 0.4$ and $0.5$, the ascending velocity is faster in such a weak magnetic intensity than that without additional field. While at $Ha = 8.49$, the rising velocity is reduced and it is lower than that without magnetic field except for the bubble with $R^* = 0.5$. Further increase of Hartman numbers, the rising velocity falls down sharply for all three sizes of bubbles. 

In order to deeply understand the effect of magnetic field on bubble shape and rising velocity, the instantaneous velocity vectors of $R^* = 0.5$ at $t^* = 1$ under different Hartman numbers are illustrated in **Fig. 7**. It is clear that in the condition of $Ha = 0$, the velocity above the bubble is much smaller than that below the bubble, which compresses the bubble to be spherical-cap shape. There is a vortex forms on each side of bubble owing to the return flow. When the
The magnetic field is appended, the flow perpendicular to the magnetic field direction is suppressed due to braking effect. Thus the horizontal velocity component is restrained and the vortexes disappear gradually, as seen in Figs. 7(b), 7(c) and 7(d). The bottom jet flow is also weakened by braking effect with the increase of \( Ha \). It is noted that the horizontal velocity components distribute radially and outwardly along the bubble’s upper surface. The Lorentz force then acts on the upside around bubble radially inward and the bubble contracts radially, which results in a vertical elongation. In brief, the vertical magnetic field weakens the bottom jet flow and elongates the bubble simultaneously.

As mentioned above, the influence of Lorentz force on bubble rising velocity is not simply positive or negative. In order to further investigate the effect of magnetic field on behavior of bubbles with different radii, single bubble under different magnetic intensities are calculated. Another two sizes of bubbles with \( R^* = 0.2 \) and \( R^* = 0.6 \) are computed as supplementary. Since it is difficult to make an accurate comparison among velocity values for they vary and fluctuate with time, the heights of bubble’s apex at a terminal time are considered to measure the whole ascending process. The heights of bubble’s apex at \( t^* = 3 \) are shown in Fig. 8. They show that for bubbles with \( R^* = 0.2 \) and \( R^* = 0.3 \), the terminal position of bubble’s apex at \( Ha = 0 \) is higher than those with magnetic field. The height declines continuously with the increase of magnetic field intensities. It means that the magnetic field inhibits the rising velocity of small bubbles with \( R^* \leq 0.3 \). Nevertheless for large bubbles with \( R^* \geq 0.4 \), the terminal position is higher under weak magnetic field than that without magnetic field, i.e. the positive effect, while an opposite situation happens in strong magnetic intensity, i.e. the negative effect. For \( R^* = 0.4 \), the peak Harmann number which reaches the strongest positive effect and the critical Harmann number turning from positive effect to negative effect are \( Ha_p = 4.24 \) and \( Ha_c = 8.06 \), respectively. Therefore a magnetic field with \( 0 < Ha < 8.06 \) gives a positive effect on the rising velocity of bubbles with \( R^* = 0.4 \) and the rising velocity reaches the maximum value when \( Ha = 4.24 \). Similarity, for \( R^* = 0.5 \) the peak Harmann number and the critical Harmann number are \( Ha_p = 6.36 \) and \( Ha_c = 11.46 \) while for \( R^* = 0.6 \) they are \( Ha_p = 6.78 \) and \( Ha_c = 14.42 \), respectively. In general, the larger the bubble is, the larger the peak Harmann number and the critical Harmann number will be.

The mechanism behind such phenomenon can be attributed to two factors as mentioned above, i.e., the elongation effect on bubble shape and braking effect on fluid flow by magnetic field. On one hand, since the bubble is elongated along the vertical direction in the magnetic field, it suf-
fers lower flow resistance which gives a positive effect on ascending velocity. On the other hand, the jet flow under the bubble is suppressed by magnetic field, which leads to a negative effect. Whether the magnetic field shows positive effect or negative one depends on which factor plays the dominant role. As shown above (see Figs. 4 and 5), elonga-

Fig. 8. The terminal ($t^* = 3$) position of bubble of case A. (a) $R^* = 0.2$, (b) $R^* = 0.3$, (c) $R^* = 0.4$, (d) $R^* = 0.5$, (e) $R^* = 0.6$.

Fig. 9. The evolution of bubble shapes of $R^* = 0.5$ under different magnetic field intensities of cases A and B. (a) case A, (b) case B. (Online version in color.)
tion effect on small bubble is weak so that the inhibition effect plays the dominant role all along. While for large bubble, the elongation takes the major role in weak magnetic field intensity so that the rising process is accelerated. However, when the magnetic field is strong enough, the braking effect plays a vital role gradually thus bubble goes up slowly. Furthermore, for the elongating function plays a more critical role for bubble with larger size, the larger the bubble size is, the greater the peak promotion point and the critical Hartman number are.

Now, let us consider another influencing factor, i.e., surface tension, $\sigma$. According to the Yang-Laplace equation, the deformation of bubble should be affected by surface tension due to the pressure difference between inside and outside of bubble, which is proportional to $\sigma$. A single bubble of $R^*=0.5$ in the parameters of case B is calculated in such case. All the parameters are the same as those in case A except for the Weber number. $We$ number in case B is smaller than that of case A. It means that the surface tension force plays more remarkable role in case B than that in case A. Comparisons of the locations and shapes at $t^*=0, 1, 2, 3$ between cases A and B are shown in Fig. 9. In case B, it can be seen that the vertical elongation of bubbles are much slighter than those in case A, especially under stronger magnetic field. The final aspect ratio in case B is 0.88 ($Ha=0$) and it is 1.01 ($Ha=16.97$) as seen in Fig. 10. The ratio of these two values is 1.14 while it is 2.18 in case A. This difference states that the strong surface tension force makes it more difficult to elongate the bubble by Lorenz force.

The rising velocity and the terminal position in case B are shown in Figs. 11 and 12, respectively. In the case A, both the rising velocity and the terminal position with $R^*=0.5$ first increase and then decrease with the increasing of Hartmann numbers. But in case B, the rising velocity and the terminal position decline continuously with the increase of magnetic field intensity. That is to say, the magnetic field always gives an inhibition effect in case B. The reason is that the elongation effect is inhibited by strong surface tension force and therefore the braking effect plays the dominant role throughout.

5. Conclusions

The effect of static uniform vertical magnetic field on behavior of a single rising bubble with both weak and strong surface tension were investigated through a series of VOF simulations. The bubble radii range from 2 mm to 6 mm ($R^*=0.2$ to 0.6) while the range of Hartman numbers is $Ha=0−16.97$. Numerical results indicate that bubble is elongated by Lorentz force and the stronger the magnetic field is, the more serious the elongation is. There are two influencing factors that attribute to the rising velocity of bubble in magnetic field, namely the elongation effect and braking effect respectively.

When the surface tension is weak, for small bubbles ($R^* \leq 0.3$), the braking effect is dominate influencing factor in the ascending process and the increasing of magnetic field intensity will decrease the rising velocity since the jet flow under bubble is suppressed. For large bubbles ($R^* \geq 0.4$), the rising velocity of bubbles is increased by elongation effect in a weak magnetic field since the flow resistance of bubbles is reduced. But the further increase of magnetic field will lead a notable decrease of rising velocity because of the braking effect. Besides, the peak Hartmann number $Ha_p$ which reaches the strongest positive effect and the critical Hartmann number $Ha_c$ turning from positive effect to negative effect are increased for a larger bubble size. The increase of surface tension of bubbles will weaken the elongation effect since the bubble is difficult to deform and
the braking effect will result in a continuous decrease of rising velocity of bubbles.

Acknowledgements
This work was supported by National Natural Science Foundation of China (grant No. 51176210).

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B$</td>
<td>magnetic field, T</td>
</tr>
<tr>
<td>$B_0$</td>
<td>characteristic magnetic field, $B_0=</td>
</tr>
<tr>
<td>$C$</td>
<td>volume rate of fluid</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter, m</td>
</tr>
<tr>
<td>$f_g$</td>
<td>gravity force, N/m$^3$</td>
</tr>
<tr>
<td>$f_s$</td>
<td>surface tension force, N/m$^3$</td>
</tr>
<tr>
<td>$Fr$</td>
<td>Froude number, $Fr=U^2/gL$</td>
</tr>
<tr>
<td>$g$</td>
<td>gravity acceleration, m/s$^2$</td>
</tr>
<tr>
<td>$Ha$</td>
<td>Hartmann number, $Ha=(\sigma_u/\eta)^{1/2}B_0L$</td>
</tr>
<tr>
<td>$J$</td>
<td>electric current density, A/m$^2$</td>
</tr>
<tr>
<td>$k$</td>
<td>curvature, m$^{-1}$</td>
</tr>
<tr>
<td>$L$</td>
<td>characteristic length, m</td>
</tr>
<tr>
<td>$n$</td>
<td>normal unit vector</td>
</tr>
<tr>
<td>$\tau$</td>
<td>tangential unit vector</td>
</tr>
<tr>
<td>$p$</td>
<td>pressure, Pa</td>
</tr>
<tr>
<td>$r$</td>
<td>radial coordinate, m</td>
</tr>
<tr>
<td>$R$</td>
<td>radius, m</td>
</tr>
<tr>
<td>$Re$</td>
<td>Reynolds number, $Re=\rho UL/\eta$</td>
</tr>
<tr>
<td>$t$</td>
<td>time, s</td>
</tr>
<tr>
<td>$u$</td>
<td>velocity vector, m/s</td>
</tr>
<tr>
<td>$U$</td>
<td>characteristic velocity, m/s</td>
</tr>
<tr>
<td>$v$</td>
<td>rising velocity of bubble, m/s</td>
</tr>
<tr>
<td>$z$</td>
<td>axial coordinate, m</td>
</tr>
</tbody>
</table>

Greek symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>electric potential, V</td>
</tr>
<tr>
<td>$\eta$</td>
<td>viscosity, kg/(m·s)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>aspect ratio</td>
</tr>
<tr>
<td>$\rho$</td>
<td>density, kg/m$^3$</td>
</tr>
<tr>
<td>$\sigma_e$</td>
<td>electric conductivity, S/m</td>
</tr>
<tr>
<td>$\sigma_s$</td>
<td>surface tension coefficient, N/m</td>
</tr>
</tbody>
</table>

Superscript

* dimensionless variable

Subscripts

* critical
| $f$ | final |
| $g$ | gas |
| $l$ | liquid |
| $p$ | peak |

References