The Solidification Time of Cylindrical Sand Mold Castings of Various Metals Affected by the Decrease in Surface Temperature*

By Eisuke NIYAMA**

Synopsis

Validity of the assumption of constant surface temperature in Chvorinov's and Adams' equations for the solidification time of castings was examined by numerical calculations. In the case of metals having high heat diffusivity like copper or aluminum, the decrease in surface temperature during solidification is negligible and Adams' equation can satisfactorily be used without correction, while Chvorinov's equation gives excessively large solidification time for cylinders. In the case of metals with low heat diffusivity like iron or lead, the decrease in surface temperature in cylinder is so large that corrections are necessary for Adams' equation. In such a case, Chvorinov's equation often gives a better approximation than Adams', if no corrections are made. A formula was proposed to predict the temperature drop in a given combination of metal and mold and to correct the figures to be used in Adams' equation. With these corrections, Adams' equation can consistently be a most reliable method to estimate the solidification time of a casting in sand mold.

I. Introduction

Chvorinov's equation¹⁾ has been widely used to estimate the solidification time of sand mold castings. Although the details of solidification profiles can be predicted with the aid of computer which is extensively used in the present, this equation can still be valuable for the daily casting designing.

In order to improve the precision of the equation, some attempts have been made from both experimental²⁾ and theoretical stand points.³⁾ The purpose of the present work is to further improve the precision by considering the change in the surface temperature of a casting during solidification, which has so far been assumed to be constant.

According to Chvorinov, the solidification time of a casting is

$$t_f = \frac{1}{q^2} (V/A)^2 \dots\dots\dots(1)$$
$$= K(V/A)^2 \dots\dots\dots(2)$$

The symbols used throughout this work are listed in Table 1. In the above equation, q is the solidification constant to give the thickness of the solidified layer X .

$$X = q \sqrt{t} \dots\dots\dots(3)$$

Assuming that the surface temperature of mold θ_i is constant, q for a plate casting is

$$q = 2b_1(\theta_i - \theta_1) / \sqrt{\pi \rho_2 L} \dots\dots\dots(4)$$

If there is superheat $\Delta\theta$, L may be replaced with $L + c_2\Delta\theta$. Chvorinov found experimentally on steel castings that a single value of q or K can commonly

Table 1. Symbols

θ	Temperature	°C
θ_f	Solidification temperature (mp)	°C
θ_c	Casting temperature	°C
θ_1	Initial temperature of mold	°C
θ_i	Interface temperature between metal and mold	°C
θ_e	θ_i when solidification ends at center	°C
θ_a	Average temperature of casting when solidification ends at center	°C
θ_m	Mean effective θ_i during solidification	°C
t	Time	sec
t_f	Solidification time	sec
L	Latent heat of solidification	cal/g
k_1, k_2	Thermal conductivity	cal/cm sec deg
ρ_1, ρ_2	Density	g/cm ³
c_1, c_2	Specific heat	cal/g deg
α_1, α_2	Temperature diffusivity ($k/\rho c$)	cm ² /sec
b_1, b_2	Heat diffusivity ($\sqrt{k\rho c}$)	cal/cm ² deg sec ^{1/2}
q	Solidification constant	cm/sec ^{1/2}
K	Chvorinov's constant	sec/cm ²
V	Casting volume	cm ³
A	Casting surface area	cm ²
X	Thickness of solidified layer	cm
P	Adams' correction factor	
T^*	Relative surface temperature $(\theta_e - \theta_1)/(\theta_f - \theta_1)$	
A^*	Relative temperature diffusivity α_1/α_2	
B^*	Relative heat diffusivity b_1/b_2	
L^*	Relative latent heat $L/c_2(\theta_f - \theta_1)$	
Suffix	1 for mold, 2 for metal	

be used for plates, cylinders and other shapes. Adams showed theoretically that q must be a function of the geometrical shape of the casting. For a plate, Eq. (4) can be used. For a cylinder, the modified q is

$$q = [2b_1(\theta_i - \theta_1) / \sqrt{\pi \rho_2 \{L + c_2(\theta_e - \theta_f)\}}] \left(\frac{1 + \sqrt{1 + P}}{2} \right) \dots\dots\dots(5)$$

where,

$$P = \pi \rho_2 \{L + c_2(\theta_e - \theta_f)\} / 4\rho_1 c_1(\theta_i - \theta_1) \dots\dots\dots(6)$$

The term of P is introduced to cover the 'corner' effect, that is, heat diffuses in increasingly wider space in the mold as the distance from the casting surface increases.

* Received April 21, 1971.
** Hitachi Research Laboratory, Katsuta Works, Hitachi Ltd., Katsuta 312.

The measurement made by Adams on aluminum, copper, and tin cast in sand showed a close agreement with the theory. Here again θ_i is assumed to be constant and, in fact, it was found to be experimentally equal to the melting point of the metals within 10°C. According to Adams' equation, the solidification time of a cylinder or a sphere must be as much as 30% less than that of a plate having the same ratio of volume to surface area. However, the difference found by Chvorinov on steel castings was at most 10%. According to Pellini,²⁾ the solidification time of a steel bar, which can be regarded geometrically quite similar to a cylinder, is even larger than that of a plate by about 20%. It will be shown in the following discussion that all of these contradictions can be interpreted with Adams' equation by considering the decrease in surface temperature during solidification.

II. Numerical Calculations

Because of the difficulty associated with the exact analytical treatment, a numerical method was adopted to examine the solidification time and the change in the surface temperature of castings of various metals. The forward difference method, essentially that of Dusenberry,⁵⁾ was used. Cylindrical castings of 20 cm in diameter were concentrically sectioned in 5 to 10 sections. The molds were so thick as to be regarded as infinite and were sectioned in 20 to 25 sections. Perfect contact was assumed to be realized between casting and mold. The longitudinal heat flow was neglected. In this method, only pure metals were considered and the specific heat of the metal was raised in a narrow solidification range, for example 0.1°C, just below the melting point by an amount corresponding to the latent heat of solidification. Pouring temperature was assumed to be equal to the melting point, *i.e.* there is no superheat. It was also assumed that the initial temperature of the mold was 0°C. Thermal properties of the materials were as-

sumed to be constant and are listed in Table 2. These values were quoted from the book by Ruddle⁴⁾ and are only approximate, and thus the results obtained are not highly quantitative in nature. Besides the realistic values simulating real metals and mold materials, various combinations of hypothetical values were used in the calculation for the sake of comparison. Calculations were made on metallic molds as well as sand molds.

III. Numerical Results on Typical Castings

The calculated results for pure iron poured in cylindrical and plane sand molds are shown in Fig. 1. The surface temperature of the cylindrical casting in the moment of the end of solidification at the center is 1403°C. The decrease in surface temperature is 97°C and is by no means small. This fact indicates that before the end of solidification, the mold must have absorbed not only the latent heat of solidification but also the heat corresponding to this temperature drop. More precisely, the average temperature of the whole section of the casting at the end of solidifica-

Fig. 1. Results of numerical calculation on the temperatures at center and surface of iron plate and cylinder cast in sand molds. Thermal properties are listed in Table 2. Sand C.

Table 2. Thermal properties

	θ_f	L	c	ρ	k	α	b
Bi	270	11.2	0.03	9.8	0.02	0.068	0.0767
Pb	327	5.9	0.03	11.3	0.08	0.214	0.173
Sn	232	14.5	0.06	7.3	0.15	0.343	0.256
Fe	1 500*	65.0	0.20	7.5	0.10	0.067	0.388
Ni	1 455	71.7	0.15	8.9	0.15	0.112	0.448
Mg	650	88.8	0.30	1.7	0.30	0.588	0.391
Zn	420	27.0	0.10	7.1	0.25	0.352	0.421
Al	660	95.3	0.26	2.7	0.50	0.712	0.593
Cu	1 083	49.1	0.12	8.9	0.90	0.843	0.981
Hypothetical metals	1 000	0.02/100	0.1/0.2	0.1/30	10 ⁻⁵ /100	10 ⁻⁵ /100	10 ⁻⁴ /10
Sand mold** A	—	—	0.20	1.7	0.0015	0.0044	0.023
Sand mold B	—	—	0.20	1.7	0.0020	0.0059	0.026
Sand mold C	—	—	0.20	1.7	0.0030	0.0088	0.033
Sand mold D	—	—	0.15	3.0	0.0050	0.0111	0.047
Hypothetical molds	—	—	0.1/0.5	1/10	10 ⁻³ /0.2	0.01/0.1	0.01/0.1

* A low solidification temperature was used for iron to simulate steel.
** Sand A, B, C, and D simulate silica at low, medium, and high temperature and zircon sand, respectively.

tion is calculated to be 1 428°C. Hence, the average temperature drop is 1 500°–1 428°=72°C and the heat to be absorbed is $L + c_2\Delta\theta = 65 + 0.2 \times 72 = 79.4$ cal/g.

Rate of heat absorption by the mold, on the other hand, is governed by the surface temperature of the mold θ_i as already mentioned in Eq. (5). Since θ_i changes from 1 500° to 1 403°C during solidification, the effective mean surface temperature of the mold could not be 1 500°C but must lie between 1 500° and 1 403°C, the simplest estimate being the arithmetic mean of the two, 1 452°C.

Thus, two corrections are necessary, if the solidification time of a cylinder is estimated by Adams' equation; the correction for the heat absorbed before the end of solidification and the correction for the surface temperature of the mold. Both corrections will tend to increase the estimate of solidification time. The solidification times calculated from Chvorinov's and Adams' equations with and without these corrections are listed in Table 3. Necessary figures for the corrections were obtained from the numerical results.

Let us regard the result of numerical method which is considered most accurate as the standard value. The solidification time calculated from Chvorinov's equation is 88% of the standard value. The value obtained from Adams' equation without any corrections is, as expected, much smaller and is only 63% of the standard, which is improved up to 92% by the introduction of the correction for the heat absorbed and up to 97% by the corrections for both the heat absorbed and the mold temperature. From this fact, it is obvious that these corrections, particularly the correction for the heat absorbed are very important.

By comparing the numerical results, the solidification time for a cylinder is 98% of that for a plate. This means that the constant K of Eq. (2) for a plate differs by only 2% from that for a cylinder. It is found from the foregoing that such a precision is accidentally obtained due to the mutual cancelling of two inaccurate assumptions, namely, the neglect of corner effect and the assumption of constant surface temperature.

On a plate casting, Chvorinov's value is 86% of the standard. The surface temperature is constant during solidification but is lower than the melting point by 24°C. If two corrections are similiary made on Eq. (4), the value is improved up to 94%. Since the improvement is small for the plate casting, the correction may be omitted in most cases.

Figure 2 shows that the decrease in the surface temperature of copper cast in sand is as small as 7°C even in the case of a cylinder. It is seen from Table 3 that Adams' equation without correction gives a quite satisfactory value for copper, while Chvorinov's value is much larger than that.

Generalizing the above results, it may be said that Adams' equation, instead of Chvorinov's, should be applied to those cylindrical castings whose decrease in surface temperature during solidification is small. However, at least, correction for the heat absorbed

Table 3. Effect of various corrections on the calculated solidification times of iron and copper

	t_f by Eq. (4) (Chvorinov)	t_f by Eq. (5) (Adams) Correction of temp. drop			t_f by numerical solution
		No	Heat	Heat & mold temp.	
Iron cylinder	2 024 (88%)	1 450 (63%)	2 125 (92%)	2 247 (97%)	2 310 (100%)
Copper cylinder	4 692 (139%)	3 285 (97%)	3 341 (99%)	3 358 (99%)	3 380 (100%)
Iron plate	2 024 (86%)			2 211* (94%)	2 350 (100%)

* Correction on plate was made by Eq. (4).
* Cylinder diameter 20 cm and plate thickness 10 cm. Thermal properties from Table 2, Sand B for copper and sand C for iron.

Fig. 2. Result of numerical calculation on the temperatures at center and surface of copper cylinder cast in sand mold. Thermal properties are listed in Table 2. Sand B.

is necessary when Adams' equation is applied to those cylindrical castings whose decrease in surface temperature is large. In the case of sand cast steel, two neglected factors in a cylinder fortunately cancel out each other to give a value of constant K nearly equal to that of a plate.

IV. Analytical Considerations

Recognizing the importance of the decrease in surface temperature during solidification, it would be worth-while to find the factors governing that amount. Although the exact solution can hardly be obtained, some analytical consideration may be necessary as a guide for the later numerical approach.

Assuming the metal and mold as semi-infinitely thick plates and neglecting the heat of solidification, the interface temperature is constant, independent of the time, and is given by the following formula.⁶⁾

$$\theta_i = \frac{b_1\theta_1 + b_2\theta_c}{b_1 + b_2} \dots\dots\dots(7)$$

Using the above equation and the values listed in Table 2, θ_i for iron and sand C becomes 1 386°C. If the latent heat of solidification of the metal is considered, b_1 in Eq. (7) must be replaced with b' defined as⁷⁾

$$b' = b_1 \operatorname{erf}(q/\sqrt{\alpha_2}) \dots\dots\dots(8)$$

where, q is the solidification constant used in Eq. (3) and is obtained by solving Schwarz's equation, not shown here, with successive approximation. Since this procedure is rather tedious, a simplified form is proposed here. By assuming that the surface temperature is constant for a plate casting and by considering the heat transfer within the casting alone, q is approximately given by Stefan's equation.⁷⁾

$$q \doteq \sqrt{2k_2(\theta_f - \theta_i)/\rho_2 L} \dots\dots\dots(9)$$

On the other hand, assuming also a constant surface temperature and considering the heat transfer within the mold alone, q is given by Eq. (4). The value of θ_i is obtained by combining the two together. When $\theta_f b_1^2/k_2 \rho_2 L$ is sufficiently small, the approximate solution can be written as

$$\theta_i \doteq \theta_1 + (\theta_f - \theta_1) \left\{ \frac{2b_1^2(\theta_f - \theta_1)}{\pi k_2 \rho_2 L} \right\} \dots\dots\dots(10)$$

This may be regarded as an approximation of Schwarz's method. For iron poured in sand C mentioned in Table 2, the approximate θ_i by Eq. (10) is 1 469°C and the exact θ_i by Eqs. (7) and (8) is 1 472°C. The numerical result on a plate shown in Fig. 1 also indicates a constant surface temperature, despite of the finite thickness. This can be expected only when superheat is zero degrees. The value of 1 476°C is also nearly equal to the exact solution for a semi-infinite plate.

The surface temperature of a cylinder as calculated numerically (Fig. 1) starts from the same value as the above mentioned and decreases gradually. This is reasonable, since the cylindrical interface may be regarded as a plane in the initial stage of cooling and the effect of geometry, or 'corner' effect, becomes gradually apparent as the rate of heat absorption by the mold relative to the rate of heat generation from the casting increases with time. The surface temperature at the end of solidification is 1 403°C and much lower than the starting temperature. It is rather near the value given by Eq. (7).

Since two factors, namely, the latent heat and the cylindrical shape, are neglected in Eq. (7), there is no direct reason why the surface temperature at the end of solidification of a cylinder must be near that value. Yet, it may serve as a guide in seeking a simplified expression for the temperature drop, when there is no exact solution available for this purpose.

V. Factors Governing the Decrease in Surface Temperature

The physical situation suggests that the major factors involved in the present problem are k_1 , k_2 , c_1 , c_2 , ρ_1 , ρ_2 , L , $\theta_f - \theta_1$ and $\theta_e - \theta_1$. Introducing the π theorem,⁸⁾ four dimension-less products T^* , A^* , B^* and L^* were formulated. As explained in Table 1, T^* is the relative surface temperature at the end of solidification, A^* the relative temperature diffusivity, B^* the relative heat diffusivity, and L^* the relative latent heat of solidification. Now the aim is to find the actual form of the following relationship.

$$T^* = F(A^*, B^*, L^*) \dots\dots\dots(11)$$

Using these dimensionless variables and θ_i for θ_e , Eq. (7) can be rewritten as

$$T^* = 1/(1 + B^*) \dots\dots\dots(12)$$

and Eq. (10) as

$$T^* = 1 - (2B^{*2}/\pi L^*) \dots\dots\dots(13)$$

About one hundred numerical "experiments" were made with the aid of a computer and the relative surface temperature T^* thus obtained were plotted against various combinations of A^* , B^* , and L^* . Figure 3 is the plot suggested from Eq. (12). The agreement is fair, considering the wide range of B^* and T^* . This means that the relative error of θ_e by Eq. (12) is small as compared with the numerical solution. However, when the decrease in surface temperature $\theta_f - \theta_e$, instead of temperature θ_e is taken into account, the relative error is still considerable as indicated in Table 4. It was noticed that the error was particularly large in the case of metals whose A^*/L^* was large. Thus, another expression was tried and is plotted in Fig. 4.

$$T^* = \frac{1}{1 + B^{*4} \sqrt{A^*/L^*}} \dots\dots\dots(14)$$

The reduction in scatter of plotted values in Fig. 4 as compared with Fig. 3 is not so great. But, if this application is limited to real metals cast in sand, the error of $\theta_f - \theta_e$ is sufficiently small, as shown in Table 4. Trials based on Eq. (13) were not successful.

It is concluded that the decrease in surface temperature is mainly governed by the relative heat diffusivity B^* and Eq. (7) or (12) may be used for a rough estimate. In order to obtain a greater precision, the relative temperature diffusivity A^* and the relative latent heat of solidification L^* must be taken into consideration. Equation (14) is an empirical and

Table 4. Estimation of the decrease in surface temperature

Metal	Mold	A^*	B^*	L^*	$\theta_f - \theta_e$		
					From Eq. (12)	From Eq. (14)	Numerical solution
Bi	Sand A	0.0650	0.295	1.383	62	33	40
Pb	Sand A	0.0206	0.130	0.547	38	18	21
Fe	Sand D	0.169	0.122	0.217	163	155	150
Sn	Sand A	0.0129	0.088	1.042	19	7	7
Fe	Sand C	0.132	0.083	0.217	116	102	97
Ni	Sand C	0.0785	0.071	0.329	97	69	60
Mg	Sand A	0.0075	0.058	0.455	36	13	13
Zn	Sand A	0.0125	0.054	0.643	21	8	6
Al	Sand A	0.0062	0.038	0.555	23	8	8
Cu	Sand B	0.0070	0.027	0.378	23	11	7
Pb	Iron mold	0.308	2.24	0.547	226	216	235
Fe	Iron mold	1.0	1.0	0.217	750	891	861
Al	Iron mold	0.0927	0.655	0.555	261	195	250
Cu	Iron mold	0.0784	0.395	0.378	307	228	293

Fig. 3. The relation between the relative surface temperature $T_s^*=(\theta_e-\theta_1)/(\theta_f-\theta_1)$ and the relative heat diffusivity $B^*=b_1/b_2$ obtained from numerical calculations

Fig. 4. The relation between the relative surface temperature $T_s^*=(\theta_e-\theta_1)/(\theta_f-\theta_1)$, the relative heat diffusivity $B^*=b_1/b_2$, the relative temperature diffusivity $A^*=\alpha_1/\alpha_2$, and the relative latent heat $L^*=L/c_2$ ($\theta_f-\theta_1$) obtained from numerical calculations

tentative solution for this purpose and may be used for common metals cast in sand mold.

From Eq. (12), it is predicted that a large temperature drop is expected either when the heat diffusivity of the cast metal is small, *e.g.* iron, or when the heat diffusivity of the mold is large, *e.g.* metallic mold. In fact, if a metal is poured into a mold made of the same metal, θ_e will be half of θ_f , because B^* is unity. However, neither Eq. (12) nor (14) gives θ_e with a high accuracy for metal mold castings as indicated in Table 4. In any case, the present method can not be applied to metal mold castings since air gap formation is neglected.

It is noted that in any of the above equations the decrease in surface temperature is independent of the casting size. This is because the temperature profiles in geometrically similar castings are similar to each other, though the time factors are much dependent on the size. Although there is an experimental report⁹⁾ that the decrease in surface temperature during solidification of castings is dependent on the casting

Fig. 5. Experimental arrangement for temperature measurement

Fig. 6. An example of the temperature measurement on a lead casting poured in sand mold

size, it is suggested from the foregoing discussion that there could be some errors in the measurement.

VI. Examination of Experimental Data

Cylindrical castings of lead, aluminum, and iron, one for each, were poured in sand molds, and the temperatures of the metal at the center and near the surface were recorded. The detail of the experimental procedures is shown in Fig. 5 and Table 5. An example of the measurement is shown in Fig. 6, and the results are summarized in Table 6.

As a result of the use of inaccurate thermal properties, the agreement between the experiment and the calculation is rather poor, particularly with respect to the solidification time. However, the order of magnitude of the measured temperature drop agrees well with the prediction. There is a reason to suspect that the measured temperature drop in iron was excessively large due to the influence of the temperature of the mold side of the interface.

For further confirmation, Pellini's data^{10,11)} were examined and the results are summarized in Table 7. Here again the comparison is not highly quantitative, since, besides the same reason mentioned above, the castings were square in section and alloys, instead of pure metals, were used in the experiment. Yet, the agreement with the prediction is fairly satisfactory.

Kondic's data,⁹⁾ though on spheres, are also qualitatively in accordance with the present theory in that the temperature drop for iron is much larger than those for aluminum and copper.

Table 5. Experimental procedures

Cast metal		Pb	Al	Fe
Purity		99.99%	99.9%	Electrolytic
Casting	Diameter	6 cm	6 cm	5 cm
	Height	30 cm	30 cm	25 cm
Riser	Diameter	None	6 cm	5.5 cm
	Height		6 cm	5.5 cm
	Treatment		Pre-heat	Pre-heat, exothermic top cover
Mold		Silica sand (Nikko No. 5, 5) bonded with CO ₂ and dried at 150°C		
Pouring		Poured with no super-heat		
Thermo-couple	Type	CA	CA	Pt/Pt-13Rh
	Diameter	0.03 cm	0.03 cm	0.05 cm
	Lead insulation	Asbesto	Asbesto	Alumina powder in silica tube
	Hot junction insulation	Thin coating with silica and zircon powder		

Table 6. Results of temperature measurements

	Experiment			Numerical calculation			Mold
	θ_f	$\theta_f - \theta_e$	t_f	θ_f	$\theta_f - \theta_e$	t_f	
Pb	324	18	180	327	21	152	A
Al	659	1	225	660	8	394	A
Fe	1 537	138	90	1 500	97	144	C
	/1 530			1 500	150	91	D

Table 7. Comparison with the data by Pellini^{10,11)}

Base metal	Alloy	Experiment		Estimation	
		Liquidus temp. (θ_L)	$\theta_L - \theta_e$	$\theta_f - \theta_e$	Mold
Pb	Pb	324	30	18	A
Fe	0.5%C steel	1 477	50	100	C
	12%Cr steel	1 488	50	101	C
	18-8 steel	1 449	55	99	C
Ni	Monel	1 335	40	64	C
Al	Al-8.5%Mg	602	0	7	A
	Al-4.5%Cu	643	0	8	A
	Al-7%Si	610	5	7	A
Cu	88-10-2 bronze	1 004	3	10	B
	60-40 brass	896	15	9	B

* Estimation was made from Eq. (14) by using A^* , B^* , and L^* of the base metal and measured liquidus θ_L as θ_f .

VII. Estimate of Solidification Time

So far the surface temperature of cylindrical castings at the end of solidification has been discussed. To estimate the solidification time of the casting using

Table 8. Effect of surface temperature corrections on the calculated solidification time

Metal	Mold	B^*	Solidification time, t_f			
			Eq. (4) Chvorinov	Eq. (5) Adams	Cor- rected*	Numer- ical solution
Bi	Sand A	0.295	6 349 (100%)	4 437 (70%)	6 317 (100%)	6 340 (100%)
Pb	Sand A	0.130	1 597 (95%)	1 305 (77%)	1 575 (94%)	1 684 (100%)
Fe	Sand D	0.122	922 (63%)	683 (47%)	1 308 (90%)	1 460 (100%)
Sn	Sand A	0.088	8 000 (127%)	5 405 (86%)	5 759 (91%)	6 295 (100%)
Fe	Sand C	0.0825	2 024 (88%)	1 450 (63%)	2 308 (100%)	2 310 (100%)
Ni	Sand C	0.0715	3 687 (119%)	2 521 (81%)	3 155 (102%)	3 100 (100%)
Mg	Sand A	0.058	2 071 (114%)	1 652 (91%)	1 785 (99%)	1 810 (100%)
Zn	Sand A	0.054	8 003 (146%)	5 408 (98%)	5 706 (104%)	5 495 (100%)
Al	Sand A	0.038	5 841 (133%)	4 130 (94%)	4 281 (98%)	4 380 (100%)
Cu	Sand B	0.027	4 692 (139%)	3 285 (97%)	3 424 (101%)	3 380 (100%)

* Correction was made by Eqs. (14), (15), (16), and (5).

Adams' equation, one must first know the average temperature over the entire section of the casting at the end of solidification and the effective mean surface temperature of the mold from the start to the end of solidification. Since the latter is of less importance, a satisfactory result may be obtained with a simple formula

$$\theta_m = 0.5 \times (\theta_f + \theta_e) \dots \dots \dots (15)$$

For the former, the numerical results were examined and the following approximate relationship was established.

$$\theta_a = 0.2 \times \theta_f + 0.8 \times \theta_e \dots \dots \dots (16)$$

This becomes as the exact solution, when the temperature drop from the melting point at any location is proportional to the square root of the distance from the center.

Now the procedure to obtain the solidification time is as follows. The value of θ_e is obtained from Eq. (12) or, preferably by Eq. (14). Then, θ_m and θ_a are obtained by putting θ_e into Eqs. (15) and (16). The value of q is obtained by replacing θ_i in Eqs. (6) and (5) by θ_m and θ_f by θ_a . Finally t_f is obtained from Eq. (1).

Some examples are shown in Table 8. It is demonstrated that for metals with low heat diffusivity, Chvorinov's equation gives a better approximation than Adams', if no corrections are made. For metals with high heat diffusivity, Adams' equation is better than Chvorinov's. If the corrections with respect to the decrease in surface temperature are made to Adams' equation, consistently reliable results are obtained.

VIII. Significance of Surface Temperature Corrections on Riser Designing

The most important factor in determining the size of riser is the solidification time relative to that of the casting. It has been shown in the foregoing that the solidification time can be calculated from Eq. (2), provided that K is considered as a function not only of the thermal properties but also of the geometry of the casting. If a cylindrical riser is attached to a plate casting, K for a cylinder and K for a plate, or at least their ratio, must be known. Using the thermal properties listed in Table 2, the values of K were calculated for a few examples and are shown in Table 9.

For copper, the value of K for a cylinder is much smaller than that for a plate. This is resulted from the corner effect. For iron poured in normal sand, the value of K for a cylinder is nearly equal to that for a plate. For iron poured in high diffusivity mold, the value of K for a cylinder is large than that for a plate. This happens because the decrease in surface temperature during solidification of the cylinder is so large as to overcome the corner effect of the mold. This seems to be a more reasonable interpretation of Pellini's data²⁾ than the earlier one by Ruddie.⁴⁾

It is sufficient in the present paper to point out the dependence of K on the casting geometry even in iron or steel, depending on the mold thermal properties. The accurate values of K are best to be obtained from direct measurement of the solidification time, because the figures in Table 9 are based only on the approximate values of the thermal properties and, in fact, in most cases the precise properties are not known. Once K is obtained for a given shape and a combination of metal and mold, it can be used for any size.

IX. Conclusions

In an attempt to improve the precision of Chvorinov's equation, validity of the assumption of constant surface temperature was examined by means of numerical calculations with a few additional experiments and the following conclusions were obtained.

(1) The surface temperature of a cylindrical casting immediately after pouring is equal to that of a plate casting, which is somewhat lower than the melting point. The temperature decreases gradually, unlike a plate, and, hence, the surface temperature at the end of solidification is more or less lower than the starting temperature.

(2) The decrease in temperature defined as the difference between the melting point and the temperature at the end of solidification, depends mainly on the relative heat diffusivity, the ratio of heat diffusivity of the mold to that of the cast metal. It also depends on the relative temperature diffusivity and the relative latent heat. A formula was proposed to estimate the decrease in temperature in a given metal and mold.

(3) In the case of metals, like copper or aluminum, whose heat diffusivity is large, the decrease in surface

Table 9. Some examples of Chvorinov's constant K as calculated numerically for plates and cylinders

Metal**	Mold**	Shape	K^*	Temp. drop $\theta_f - \theta_e$
Copper	B	Plate	187	0
Copper	B	Cylinder	148	7
Iron	C	Plate	94	24
Iron	C	Cylinder	92	97
Iron	D	Plate	47	54
Iron	D	Cylindr	58	150

* K in Eq. (2), sec/cm².
** Thermal properties are listed in Table 2.

temperature is so small that it can be neglected in calculating the solidification time. Adams' equation, instead of Chvorinov's, should be used for cylinders of such metals.

(4) In the case of metals like iron, steel, or lead, whose heat diffusivity is small, the decrease in surface temperature is large and Adams' equation gives a satisfactory value only if necessary corrections are made to the temperature terms. Chvorinov's equation can be a good approximation for cylindrical steel castings poured in normal sand, due to a fortuitous mutual cancelling of two inaccurate assumptions, namely, the constant surface temperature and the neglect of 'corner' effect.

(5) In steel casting poured in high diffusivity mold, it is possible that the effect of the decrease in surface temperature overcomes the corner effect with the result that the solidification time of a cylinder is longer than that of a plate having the same ratio of volume to surface area.

(6) It is desirable to establish experimentally the constant K for various casting shapes and combinations of metals and molds. This will be of particular value for steel castings in various types of molding sands.

REFERENCES

1) N. Chvorinov: *Giesserei*, **27** (1940), 10, 177, 201.
2) W. S. Pellini: *Trans. AFS*, **61** (1953), 603.
3) C. M. Adams and H. F. Taylor: *Trans. AFS*, **65** (1957), 170.
4) R. W. Ruddie: *The Solidification of Castings*, (1957), Institute of Metals.
5) G. M. Dusenberre: *Heat-Transfer Calculations by Finite Differences*, (1961), International Textbook Co.
6) L. R. Ingersoll, O. J. Zobel, and A. C. Ingersoll: *Heat Conduction*, (1954), 91, Univ. Wisconsin Press.
7) H. S. Carslaw and J. C. Jaeger: *Conduction of Heat in Solids*, (1959), 285, Oxford.
8) W. H. McAdams: *Heat Transmission*, (1954), 192, McGraw-Hill.
9) J. Berry, V. Kondic, and G. Martin: *Trans. AFS*, **67** (1959), 449.
10) H. F. Bishop, F. A. Brandt, and W. S. Pellini: *Trans. AFS*, **59** (1951), 435.
11) F. A. Brandt, H. F. Bishop, and W. S. Pellini: *Trans. AFS*, **62** (1954), 646.