Evaporation Rate of a Single Water Droplet on Hot Solid Surface*

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Synopsis

The evaporation rate of a single water droplet on the hot solid surface has been studied.

A theoretical rate equation is derived on the assumption that the evaporation rate at the bottom part of the droplet is controlled by the heat transfer through vapour film and the evaporation rate from the outer surface of the droplet by mass transfer.

According to this theory, the evaporation rate at the outer surface of the droplet is given as a function of its volume while that in the bottom part is proportional to the 0.7 power of its volume. It is found reasonable to neglect the inertial contribution to the vapour flux through narrow channel between the bottom and surface of the solid at least below 900°C of the surface temperature of the solid.

I. Introduction

As the basic information relating to the continuous casting and the spray-cooling of steel, the evaporation rate of a water droplet on hot solid surfaces is of much importance.

The life period of water droplets resting upon the heated steel surface was previously studied and the heat transfer rate was established to give a simple empirical equation in terms of the surface temperature and the spherical-equivalent radius of the droplets.3,4

As then referred to, however, much has been left in estimating the heat transfer rate in spray-cooling operations from the available data of the single droplet behaviour. In this work, these problems—the substantial mechanisms in the evaporation process of water droplets on hot solid surfaces are considered and the equations expressing the evaporation rate are derived.

A few publications2-4 are present, in which the radial flow of viscous vapour underneath droplets is considered and the simplified equation of motion are solved to give the vapour film thickness. In these papers only numerical methods were adopted because of the complexity of the analyses on the droplet shape and the over-simplification tended to result in their poor coincidences with the observations.

II. Evaporation Models of Water Droplet

1. Shape of the Droplet

An evaporation model has been proposed by Gottfried and his collaborators3 on the assumption of the spherical droplet. The shape of a water droplet smaller than 0.005 g may possibly be spherical,3,4 droplets of 0.015-0.032 g, however, were used by Gottfried, et al.3

Aylor and Bradfried4 employed the disk-type model of water drop, in which a water droplet was assumed to shrink in only the radial direction and retained the original depth during evaporation. This model leads to a simple rate expression such as being proportional to \( V^{2/4} \), but the disk-type model is appropriate to droplets larger than about 1 g.4

Wachters and his co-workers3 numerically solved the Laplace equation for the balancing between surfacial and volumetric forces and first discussed the exact shape of droplets in the film-boiling stage; their results well coincided with the above cited ones from photographic studies by Aylor, et al.4

Referring to the paper by Aylor, et al.4 the shape of the droplet in the film-boiling stage can be described as an ellipsoid around the shorter axis with the bottom being partly cutted-off. The radius of the cutted-off cross section, \( R_0 \), from the observation is related with the droplet volume \( V \) as follows:

\[
R_0 = V^N, \quad N = 0.45 \pm 0.01 \quad (1)
\]

In the present work Eq. (1) by Aylor, et al.4 is employed.

2. Evaporation Model

For film-boiling droplets of water, it is assumed in this paper, that the evaporation is controlled by the heat transfer process at the cutted-off surface in the bottom and by the mass transfer process at the outer surface of the droplet beside the cutted-off bottom.

The volume-decreasing rate of the droplet is, therefore, given by Eq. (2):

\[
- \frac{dV}{dt} = W_1 + W_2 \quad (2)
\]

1. Evaporation Rate at the Outer Surface of a Water Droplet, \( W_1 \)

As exact estimation of the upper surface area of the evaporating ellipsoid is not only tedious, but it is also inconvenient to use the estimated result owing to its extreme complexity. Herein, the outer surface area of the droplet is calculated as that of a spherical droplet cutted-off at the bottom part. Consequently, it is given by the following equation in terms of spherical-equivalent radius \( r_0 \) and radius of the cutted-off surface at the bottom \( R_0 \).

\[
S = 4\pi r_0^2 - \pi R_0^2 \quad (3)
\]

Substituting \( V \) of Eq. (1) into the above equation, it becomes as

\[
S = (36\pi)^{1/3} V^{2/3} - \pi V^{2N} \quad (4)
\]


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$W_1$ in Eq. (2) can be rewritten to the following equation by use of the assumption of mass transfer-controlling.

$$W_1 = k_v \cdot M \cdot S(e^8 - e) / \rho_i \quad \text{Eq. (5)}$$

See Nomenclatures for the symbols.

Within a smaller range of Reynolds number, the dimensionless Sherwood number is

$$Sh = 2k_v \rho_0 / D \approx 2$$

Thus,

$$k_v = D / \theta_0 = \left( \frac{4\pi}{3} \right)^{1/3} D \cdot V^{-1/3} \quad \text{Eq. (6)}$$

Substituting Eqs. (4) and (6) into Eq. (5) and rearranging leads to

$$W_1 = \beta \left[ 1 - e^8 \right] \left[ 1 - \left( \frac{\pi}{36} \right)^{1/3} V^{2/3} \right] V^{1/3} \quad \text{Eq. (7)}$$

where,

$$\beta = (48e^8)^{1/2} e^8 \cdot D \cdot M / \rho_i \quad \text{Eq. (8)}$$

From Eq. (7) it may be seen that $W_1$ is directly influenced by water vapour concentration in the atmosphere and that the evaporation stops in the saturation where $e = e^8$ and increases to a maximum in the dry atmosphere where $e = 0$.

$W_1$ is also dependent on the droplet volume $V$ and monotoneously decreases with decreasing $V$, as shown in Fig. 1.

**2. Evaporation Rate at the Bottom Surface of Water Drop, $W_2$**

If the film thickness of vapour at the bottom of a droplet and the effective thermal conductivity at the bottom are termed as $\xi_0$ and $k_v$, respectively, the evaporation rate of vapour per unit surface area at the bottom, $v_b$, is given by Eq. (9), like in Wachters, et al.'s situation.

$$v_b = \frac{1}{\rho_0(JH)} \left[ \frac{k_v JT}{\rho_0(JH)} \right] \left[ \frac{1}{\xi_0} \right] \quad \text{Eq. (9)}$$

So, $W_2$ becomes as

$$W_2 = \pi R_0 (\rho_0 / \rho_i) v_b = \left[ \frac{\pi k_v JT}{\rho_0(JH)} \right] \left[ \frac{V^{2/3}}{\xi_0} \right] \quad \text{Eq. (10)}$$

The term in the parentheses in the right hand side of Eq. (10) is a constant determined by both known experimental conditions and materials properties. The expression of $\xi_0$ in terms of $V$ is next desired.

![Fig. 1. Evaporation rate from the upper surface of water drop, calculated by Eq. (7)](image)

![Fig. 2. Vapour film produced between drop and hot surface)](image)

**III. Film Thickness of Vapour, $\xi_0$, during Evaporation**

So far, the fluid flow in vapour films in the film-boiling regime has been taken as a radially laminar flow with a negligibly small inertial contribution. Regardless of this simplification, either the complication in calculations or the poor correspondence with experimental results stems from the matter of shapes of the droplet. The problem whether the inertial contribution can be safely ignored also requires a further examination.

1. **The Case Taking Accounts of Both Inertial and Viscous Contributions**

Assuming that the axial component $v_a$ may be neglected in comparison with the radial one $v_r$ in the vapour flow, then

$$v_a = 0 \quad \text{Eq. (11)}$$

Mass and momentum balances at the quasi-stationary state within vapour film are given by Eqs. (12) - (14), according to a schematic model in Fig. 2.

$$\frac{1}{r} \frac{d}{dr} \left[ r d \tilde{v}_r \right] = \frac{v_0}{\xi_0}, \quad \tilde{v}_r = \frac{1}{\xi_0} \int_0^\xi \frac{v_i}{d \xi} \quad \text{Eq. (12)}$$

$$\rho_v \frac{d \tilde{v}_r}{d r} + \frac{d \tilde{P}}{d r} = \mu \frac{d^2 \tilde{v}_r}{d \xi^2} \quad \text{Eq. (13)}$$

$$\frac{d \tilde{P}}{d \xi} = \rho_v g \quad \text{Eq. (14)}$$

Here, it is considered that vapour is compressible and $\rho_v$ and $\mu$ are both constant at an average temperature $(T_s + 100) / 2$. Directly from Eq. (14), we have

$$\frac{d \tilde{P}(r, \xi)}{d r} = \tilde{P}(r, \xi) dr \quad \text{Eq. (15)}$$

Since nonlinearity of Eq. (13) presents a difficulty, the average radial velocity $\bar{v}_r$ is now substituted for $v_r$, involved in the differential term at the left hand side of Eq. (13).

Thus,

$$\rho_v \frac{d \bar{v}_r}{d r} + \frac{d \tilde{P}}{d r} = \mu \frac{d^2 \bar{v}_r}{d \xi^2} \quad \text{Eq. (16)}$$

Next, if Eq. (12) is integrated over $r = 0 \sim r$,

$$\bar{v}_r = \left( v_0 / 2 \xi_0 r \right) \quad \text{Eq. (17)}$$

The boundary condition is that

$$v_r(r, o) = v_r(r, \xi_0) = 0 \quad \text{Eq. (18)}$$

If $v_b$ in Eq. (16) is replaced by Eq. (17) and if the
The equation is solved under the above condition, \( v_r \) can be given as,
\[
v_r = \left( \frac{2\tilde{z}_0}{\rho_p\rho_0} \right) \left( \frac{dP}{dr} \right) \left[ \frac{\sinh (\lambda z_0) + \sinh (\lambda (\tilde{z}_0 - z))}{\sinh (\lambda \tilde{z}_0)} \right] - 1
\]
where \( \lambda \) is a characteristic root of Eq. (16) and given as,
\[
\lambda = \sqrt{\rho_p\rho_0/2\rho_0^2}
\]
Integrating Eq. (19) we have
\[
\tilde{v}_\theta = - \left( \frac{2}{\rho_p\rho_0} \right) \left( \frac{dP}{dr} \right) \left[ \frac{2}{\lambda} \left( \frac{1 - \cosh (\lambda \tilde{z}_0)}{\sinh (\lambda \tilde{z}_0)} \right) + \tilde{z}_0 \right] - 1
\]
Eliminating \( \tilde{v}_\theta \) from Eqs. (17) and (21), the integration under the following boundary condition,
\[
P(R_0, 0) = 0
\]
gives
\[
P(r, \tilde{z}_0) = \left( \frac{\rho_0}{8\tilde{z}_0} \right) (R_0^2 - r^2) \left[ \frac{2}{\lambda} \left( \frac{1 - \cosh (\lambda \tilde{z}_0)}{\sinh (\lambda \tilde{z}_0)} \right) + \tilde{z}_0 \right] - 1
\]
Since the droplet weight balances with the vapour pressure at the top of the film, it follows that
\[
V_{p,g} = \int_0^{R_0} 2\pi r P(r, \tilde{z}_0)dr
\]
When \( v_0 \) and \( R_0 \) in Eq. (24) are substituted from Eqs. (1) and (9), with rearranging, Eq. (25) can be derived.
\[
\tilde{z}_0 = \left( \frac{\pi(k, J^T)^2}{16 (J^H)^2 \rho_p \rho_0 g} \right)^{1/4} \cdot A \cdot V^{1/4}
\]
where,
\[
A = \left[ \frac{2}{\lambda} \left( \frac{1 - \cosh (\lambda \tilde{z}_0)}{\sinh (\lambda \tilde{z}_0)} \right) + 1 \right]^{1/4}
\]
\( \tilde{z}_0 \) in the preceding equation is a dimensionless group involving the known parameters alone and given as,
\[
\tilde{z}_0 = (k, J^T/2\rho_p (J^H))^1/2
\]
Thus the film thickness of water vapour \( \tilde{z}_0 \) can be estimated from Eq. (25) without any unknown parameters and it is proportional to \( V^{0.5} \) for \( N=0.45 \). \( \tilde{z}_0 \) is calculated to be some 10^-2 cm depending on the surface temperature of the solid (Fig. 3).**

Substituting Eq. (23) into Eq. (10) the evaporation rate at the bottom surface of the drop is given as,
\[
W_2 = \alpha(T_s) V^{5/4}
\]
where,
\[
* \text{The axial position corresponding to the maximum flow rate is given by}
\]
\[
\tau_{max} = \frac{1}{2\lambda} \ln \left( \frac{1 - \varepsilon_{z_0}}{1 - \varepsilon_{z_0}} \right)
\]
\[
** \text{Note that the calculation in Fig. 3 is done for cases where } V = 1 \text{ cm}^3
\]
Table 1. Constants and properties involved in evaporation rate expression \( \left( T = \left( T_s + 100 \right)/2 \right) \)

<table>
<thead>
<tr>
<th>( M )</th>
<th>18 g/mol,</th>
</tr>
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<tbody>
<tr>
<td>( \epsilon^* )</td>
<td>3.268 ( \times 10^{-2} ) mol/cm(^2),</td>
</tr>
<tr>
<td>( D )</td>
<td>0.380 cm(^2)/sec,</td>
</tr>
<tr>
<td>( k_s )</td>
<td>2.07 ( \times 10^{-7} T + 3.40 \times 10^{-4} ) ( \text{cal/cm} \cdot \text{sec} \cdot \text{deg} ) ( \text{C} ) **</td>
</tr>
<tr>
<td>( \mu )</td>
<td>2.25 ( \times 10^{-9} (273 + T)^{0.5} ) ( \text{g/cm} \cdot \text{sec} ) ( \text{m} )</td>
</tr>
<tr>
<td>( \rho_0 )</td>
<td>0.2194 ( \text{g/cm}^3 )</td>
</tr>
</tbody>
</table>

* Values at the boiling point of water
** Radiation is not considered

In order to compare the above-obtained results to each other, calculations are made. Numerical data of the constants and properties are listed in Table 1. The calculated results are given in Fig. 3, which illustrates that \( \xi_b \)'s from Eq. (23) substantially coincide with those from Eq. (32) and the both are three to five times larger than those from Eq. (36).

The inertia contribution can thus be neglected at lower temperatures than about 800°C so far as the viscous terms are taken account into calculations, which supports the assumption hitherto made by other investigators.

Plots of both \( \xi_b \) and \( A \) in Eq. (25) vs. the surface temperature of the solid are shown in Fig. 4. As a common property of gases, the viscosity of water vapour increases with increasing temperature, but the effect of the increase of the evaporation rate exceeds that of the viscosity. Consequently, \( \xi_b \) increases with temperatures and thus \( A \) decreases until the limiting value 1 when \( \xi_b \to \infty \).

A result such that the term in parentheses of Eq. (25) is the same as that of Eq. (36) suggests the progressive contributions of the inertia term to the boiling process with further increasing temperature. Therefore, it may be interpreted that Fig. 3 is a part of the schematic tendencies shown in Fig. 5.

Here, the assumption of much smaller \( \xi_b \) relative to \( R_0 \) which has been presented at the beginning of Section III. \( J \) is re-examined on the basis of the preceding results.

The ratio of \( \xi_b \) to \( R_0 \) at the droplet volume is given in Fig. 6, in which the ratio is smaller than 0.02 below 760°C. Thus, the assumption is verified.

**IV. Comparisons of the Experimental Results with Calculations**

When the atmosphere is saturated with water vapour the following expression is obtained, as mentioned above.

\[
\frac{dV}{dt} = W_2 = a(T_s) V^{0.70}, \quad \epsilon = \epsilon^* \quad \text{(38)}
\]

From this, the life period of a water droplet is given as,

\[
t_d = V_0^{0.30}/0.30 a(T_s) \quad \text{............(39)}
\]

In Fig. 7, the calculated results from Eq. (38) are compared with the experimental results in a saturated air from the literature.1 The evaporation rates calculated by using \( a(T) \) or \( a(T_s) \) of Eq. (29) or (33), respectively, are in good agreement with the observed results, while the calculated rates in terms of \( a_0(T_s) \) of Eq. (37) are some five times larger than those observed. Therefore, the viscosity of water vapour through the thin layer at the bottom part of a drop should not be eliminated.

With dry atmospheres, i.e., when \( \epsilon = 0 \) in Eq. (7), the evaporation rate from the upper surface of the drop \( W_1 \) (Fig. 1) is to be included.

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The calculated results and the data by both Wachters, et al.\textsuperscript{29} and Aylor, et al.\textsuperscript{29} under the dry condition are compared in Fig. 8. The latters are between three- and one-fifth of the former and rather they are similar to the evaporation rate in a saturated air.

The previous experimental result reported by the present author\textsuperscript{19} was of the case in which any controlling of the atmosphere was not done. The observed results on the life period of water droplets upon the hot steel surface are shown in Fig. 9, together with the theoretical curves. The experimental data lie between the both curves when \( \epsilon=0 \) and \( \epsilon=\epsilon^* \), and this is not necessarily unsuitable to the present theories. Further investigations are to be done on the evaporation rate of a drop, especially in dry and/or unsaturated atmospheres.

\textbf{V. Conclusions}

The evaporation rates in the film-boiling regime of a water droplet resting on hot solid surfaces are theoretically derived. The evaporation rate at the outer surface of the drop is given as a function of its volume while that from bottom part is proportional to 0.7 power of its volume.

The vapour flow through a vapour film underlying the droplet is considered. At least when the surface temperature of the solid is lower than 800°C, viscosity of the vapour cannot be ignored and rather the inertial contribution to the vapour flow can be ignored.

The theoretical and observed data, under the saturated atmosphere, are in good agreement.

\textbf{Nomenclatures}

\( \epsilon, \epsilon^* \): concentration and saturated concentration of water vapour in the atmosphere (mol/cm\(^3\))
\( D \): diffusivity of water vapour (cm\(^2\)/sec)
\( g \): gravity constant (cm/sec\(^2\))
\( \Delta H \): heat of vaporization of water (cal/g)
\( k_e \): effective thermal conductivity of vapour film (cal/cm sec °C)
\( k_o \): mass transfer coefficient (cm/sec)
\( M \): molecular weight of water (g/mol)
\( P \): pressure difference between pressure in vapour film and that in the atmosphere (g/cm sec\(^2\))
\( R_0 \): radius of vapour film beneath drop (cm)
\( r_0 \): spherical-equivalent radius of drop (cm)
\( S \): upper surface area of drop (cm\(^2\))
\( T_s \): surface temperature of solid (°C)
\( t_s \): life period of drop (sec)
\( V, V_0 \): volume and initial volume of water droplet (cm\(^3\))
\( v_0 \): vapour flux at the bottom surface of drop (cm\(^3\) (vap)/cm\(^2\)-sec)
\( v_r, v_z \): velocity components in directions of \( r \) and \( z \) (cm/sec)
\( W_1, W_2 \): evaporation rates at the upper and the bottom surface of drop (cm\(^3\) (liq.)/sec)
\( \delta_o \): film thickness of vapour (cm)
\( \rho_w, \rho_r \): densities of water vapour and water (g/cm\(^3\))
\( \mu \): viscosity of vapour (g/cm·sec)
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REFERENCES