Acoustic Emission Investigation of Pop-in Crack Propagation*

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Synopsis

Ni-Cr-Mo steels of the secondary hardening type were heat treated to give pop-in crack propagations under bending load. The fatigue pre-cracked bending specimens were instrumented with an acoustic emission detecting system to obtain information on acoustic emission during pop-in crack propagation. A relationship was theoretically developed between the total count of acoustic emission and RMS (root mean square) voltmeter output for a given pop-in crack propagation and was shown to be in good agreement with experimental results. It was also presented that the observed relation between the total count of acoustic emission and the energy released by a pop-in crack propagation could be interpreted on the basis of a simplified model concerned with the acoustic emission signal for a single acoustic emission event.

I. Introduction

Since the pioneering work by Jones and Brown,1 many investigations on the combined use of the acoustic emission technique and fracture mechanics have been made. The important parameter in the fracture mechanics \( K \) (stress intensity factor) or COD (crack opening displacement) has been proposed to be directly related to the total count \( N \) of acoustic emission through a semi-empirical relation of the form

\[
N = A_1 K^{m_1} \quad N = A_2 (\text{COD})^{m_2} \quad \ldots \ldots \ldots (1)
\]

where, \( A_1, A_2, m_1 \) and \( m_2 \) are constant. The assumption leading to such relations is that the total count of acoustic emission is proportional to the volume2 or to the area3 of the plastically strained zone at the crack tip, the extent of which can be described in terms of \( K \) or COD. The theoretical basis for such proportional relations is, however, still to be clarified.

Several attempts have been made to relate the energy released by the propagation of a crack to the resultant acoustic emission.4-7) For developing a theoretical relation between the total count of acoustic emission and crack propagation, it is necessary, as the first step, to consider the relation under the condition of a single acoustic emission event. After the present work was well under way, the report by Gerberich, et al.8) came to our attention. They proposed a direct relation between load drop due to a pop-in and acoustic emission amplitude under the above mentioned condition. Consideration on such relations for a single acoustic emission event may be useful also for understanding the relationships for an accumulative phenomenon as presented by Eq. (1).

The purpose of the present work is to present results which will relate the acoustic emission activity to the energy released by a pop-in crack propagation. An attempt is made to explain the results using simplified models concerned with acoustic emission.

II. Test Materials and Experimental Procedures

The materials used were melted in a 20 kg vacuum induction furnace, solution-treated at 1 200°C for 8 hr in a hydrogen atmosphere, and forged and rolled at 1 200°C into bars with a section of 16 mm x 16 mm. The tensile and bending specimens shown in Fig. 1 were machined. The heat treatment of the specimens was as follows: austenitized at 950°C for 20 min, oil-quenched, and aged at 450°, 500° or 550°C for 100 min. The chemical compositions of the materials used are tabulated in Table 1.

In all bending specimens, a fatigue precrack was developed at room temperature by cyclic three-point bending at the base of the machined notch, using a Vibrophore fatigue machine of 10³ N capacity. The maximum cyclic load at the final stage of the fatigue precracking was chosen lower than one half of the first pop-in load in the subsequent static bending test. The length of the precrack (machined notch and fatigue crack) was changed from specimen to specimen, ranging from about 4.5 to 6.5 mm. The total number of the loading cycles was of the order

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of $10^8$ in order to obtain a desired precrack length.

After precracking, the specimens were tested at room temperature with an Instron type machine (10$^5$ N capacity), using a compression load cell to drive Y-axis of a multi-pen X-Y plotter and a multi-pen t-Y plotter. This t-Y plotter displayed the deflection of the specimen at the loading point in the loading direction. The COD at the notched surface of the specimen was measured with a standard clip gauge and registered simultaneously by the X-Y plotter.

The block diagram of the acoustic emission instrumentation is shown in Fig. 2. A differential type PZT transducer resonant at 140 kHz is banded and coupled by a viscous resin to the side of the bending anvil as shown in the figure. This permitted a constant coupling condition through the present experiment at some expense of sound coupling between the transducer and the testing part. The signals from the transducer were led to a preamplifier which gave a nominal electronic gain of 40 dB and then to a gain-variable amplifier set at 35 dB gain for the present experiment. The amplifier bandpass filter was set to pass frequencies between 100 and 300 kHz. The amplified and filtered signals were fed to the acoustic emission data presentation system. The acoustic emission data recorded with the t-Y plotter consisted of: count rate defined as the number per second of the amplified and filtered signals exceeding the threshold voltage, total count defined as the summation of such counts, and d-e output from an RMS voltmeter fed with the amplified and filtered signals. The last two were the primary interest for the present work.

### III. Experimental Results

The mechanical properties of the materials are listed in Table 2. The apparent fracture toughness $K_{IC}$ was calculated by the following equation:

$$K_{IC} = \frac{3P_{Q}S}{2B \sqrt{a_{2}}} \left[1.93(a/W)^{1/2} - 3.07(a/W)^{3/2} + 14.53(a/W)^{5/2} - 25.11(a/W)^{7/2} + 25.80(a/W)^{9/2}\right]$$

where, $P_{Q}$: first pop-in load detected with the clip gauge

$S$: span length ($= 40$ mm)

$B$: specimen thickness ($= 10$ mm)

$W$: specimen width ($= 12$ mm)

$a$: crack length.

The typical load-deflection and load-COD curves are reproduced in Fig. 3. The corresponding acoustic emission data vs. time curves are shown in Fig. 4. Multiple pop-ins could be observed before the final fracture for all specimens tested.

Figure 5 shows schematically simplified load-deflection curves corresponding to the specimens with different crack length of $a_1$ and $a_2$, where, $a_2 - a_1 = 2a > 0$. The potential energy release due to the increase in the crack length from $a_1$ to $a_2$ can be expressed as follows, assuming that the crack propagation occurs under a constant loading condition of $P_1$ in Fig. 5:

$$JU = P_1(D_2 - D_1) - \frac{1}{2} P_1(D_2 - D_1) = \frac{1}{2} P_1(D_2 - D_1) = \Delta OAD$$

### Table 2. Mechanical properties of the materials used

<table>
<thead>
<tr>
<th>Steel No.</th>
<th>7401</th>
<th>7402</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aged at, °C</td>
<td>450</td>
<td>500</td>
</tr>
<tr>
<td>$a_2$, Nmm$^{-2}$</td>
<td>1,300</td>
<td>1,283</td>
</tr>
<tr>
<td>$K_{IC}$, Nmm$^{-3/2}$</td>
<td>2,530</td>
<td>2,000</td>
</tr>
<tr>
<td>$Q^*$, mm</td>
<td>9.5</td>
<td>6.1</td>
</tr>
</tbody>
</table>

* $Q = 2.5 \left(\frac{K_{IC}}{a_2}\right)^2$

Fig. 3. Typical curves of the bending load vs. crack opening displacement, measured with a standard clip gauge and the deflection of a specimen measured at the loading point in the loading direction.
Fig. 4. Experimental results on the bending load, total acoustic emission counts and the RMS voltmeter output vs. time for the same specimen as that in Fig. 3. The total electronic gain of the amplifiers used is 75 dB.

Fig. 5. A schematic of load vs. deflection curves of bending specimens with different crack length of \( a_1 \) and \( a_2 \), where \( a_1 - a_2 = \Delta a \rightarrow 0 \)

Under the condition of a constant deflection \( D_1 \), on the other hand, the strain energy release by the propagation of the crack from \( a_1 \) to \( a_2 \) can be described as:

\[
\frac{1}{2} (P_1 - P_2) D_1 = \Delta OAB \quad (4)
\]

These two are identical with each other for an infinitesimally small increment of the crack length or \( \Delta a \rightarrow 0 \). The condition under which an actual pop-in crack propagation occurs may be considered to be intermediate between the above two conditions. Since as the increase in crack length occurs, load drop as well as the increase in the deflection or in the COD takes place as seen in Fig. 3. Furthermore, an actual pop-in crack propagation cannot be, in general, assumed as an infinitesimally small increase in crack length. The released energy due to a pop-in crack propagation should be regarded as the area of the triangle \( OAC \) in Fig. 5, where, \( C \) denotes the condition of load and deflection under which the crack arrests.

The released energy \( \frac{1}{2} \) was obtained by planimeteric integration of the recorded load-deflection curve on the assumption that the compliance of the specimen is elastic after a pop-in (between \( O \) and \( C \) in Fig. 5).

The total count jump of acoustic emission at a pop-in crack propagation (see Fig. 4) is due to the energy released by the propagation. In Fig. 6 is demonstrated the experimental relation between the released energy \( \frac{1}{2} \) and the total count jump \( \Delta N \) at a pop-in crack propagation. It should be noted that the relationship is not influenced by the Kaiser effect, as the maximum fatigue load was chosen low enough in comparison with the pop-in load. In spite of the scatter of the results, it may be concluded that a large release of the energy corresponds to a larger total count jump of acoustic emission. The released energy is similarly plotted as a function of the maximum RMS voltmeter output \( V_{\text{max}} \) (see Fig. 4) at a pop-in crack propagation in Fig. 7. The general tendency is quite similar to that of Fig. 6, and the relationship between \( \ln \frac{1}{2} \) and \( \ln V_{\text{max}} \) tends to be linear with the slope of \( \frac{\ln \frac{1}{2}}{\ln V_{\text{max}}} = 2.0 \) in the range of larger \( \frac{1}{2} \) or \( V_{\text{max}} \). In the range of smaller \( \frac{1}{2} \) or \( V_{\text{max}} \), however, the slope deviates from the value of 2.0, showing an increasing tendency with the decrease of \( \frac{1}{2} \) or \( V_{\text{max}} \).


IV. Discussion

Harris, et al.\(^1\) have proposed, considering a given event of acoustic emission, that the voltage of the amplified signal from a transducer is a damped sinusoidal wave of the form

\[ V(t) = V_0 \exp(-\beta_1 t) \sin 2\pi ft \]

where, \( V_0 \): initial amplified and filtered voltage
\( \beta_1 \): damping constant
\( f \): linear frequency
\( t \): time.

By the definition of the total count of acoustic emission, the total count jump \( \Delta N \) due to a single event is given by

\[ \Delta N = f \cdot t_e \]

where, \( t_e \) is the time for the signal to be damped down to the threshold voltage \( V_t \) of the electronic counter. Therefore, from Eq. (5) it is found to be

\[ V_t = V(t_e) = V_0 \exp(-\beta_1 t_e) \]

Combining Eq. (6) with Eq. (7)

\[ V/t_0 = \exp(-\beta_1 t_e) = \exp(-\beta_1 \cdot f \cdot \Delta N) \]

Rewriting this equation yields

\[ \Delta N = \frac{1}{\beta_1} \ln \left( \frac{V}{V_t} \right) \]

By this equation the total count jump \( \Delta N \) due to a single acoustic emission event is related directly to the initial amplified and filtered voltage \( V_0 \).

The RMS voltmeter output is related, as pointed out by Hamstad and Mukherjee,\(^2\) both to the rate at which the acoustic emission events are occurring and to the amplitude of the stress waves generated by these events. But, considering that a pop-in corresponds to a single event of acoustic emission, the maximum RMS voltmeter output may be correlated to the amplitude of the signal.

As the amplified and filtered signal is not a continuous sinusoidal wave but a damped one as described by Eq. (5) for a single acoustic emission event, the RMS voltmeter output is difficult to define exactly. For the simplification of the matters, the RMS voltmeter output \( V_{\text{RMS}}(t) \) is supposed to be approximated for the damped signal by

\[ V_{\text{RMS}}(t) = \alpha \cdot V_0 \exp(-\beta_2 t) \]

where, \( \alpha \): a proportionality constant
\( \beta_2 \): damping constant of the system.

This \( V_{\text{RMS}}(t) \) is the input to the \( t-Y \) plotter and the recorded RMS voltmeter output \( V_{\text{ab}}(t) \) is a function of this RMS voltmeter output \( V_{\text{RMS}}(t) \). On the basis of the simplified assumption shown by Eq. (10), the maximum value of \( V_{\text{ab}}(t) \), termed by \( V_{\text{ab}}^{\max} \) and given by the following relation,

\[ V_{\text{ab}}^{\max} = V_{\text{ab}}(t = t_e) \]

may be shown to have a linear relationship with the initial amplified and filtered voltage \( V_0 \) for a pop-in crack propagation:

\[ V_{\text{ab}}^{\max} = C_0 V_0 \]

where, \( C_0 \) is constant. (see Appendix)

Substituting Eq. (12) in Eq. (9) yields the following relationship:

\[ \Delta N = \frac{f}{\beta_1} \ln \left( \frac{V_{\text{max}}}{C_0 \cdot V_t} \right) \]

As the threshold voltage of the electronic counter was set at 1 V for the present experiment, Eq. (13) can be rewritten as follows:

\[ \ln V_{\text{max}}^{\max} = \frac{\beta_1}{f} \Delta N + \ln C_0 \]

In Fig. 8 is plotted the relationship observed between the total count jump \( \Delta N \) and the maximum RMS voltmeter output \( V_{\text{ab}}^{\max} \) at a pop-in crack propagation, and it shows the linear relation as predicted by Eq. (14). This result is consistent with the assumption of Hamstad and Mukherjee\(^2\) on the RMS voltmeter output.

The observed relation shown in Fig. 8 gives a rough estimate of the numerical value to the constants in Eq. (14):

\[ \beta_1 f \approx 1.15 \times 10^{-4} \]
\[ C_0 \approx 0.12 \]

The value of \( C_0 = 0.12 \) means that the maximum RMS voltmeter output \( V_{\text{ab}}^{\max} \) is reduced to 12% of the magnitude of the initial amplified and filtered voltage \( V_0 \) which could be reasonably expected.

The energy of acoustic emission \( E_{\text{AE}} \) can be esti-
estimated to be proportional to the $n$th power of the initial voltage $V_0$, as suggested by Harris, et al., as the following form:

$$ U_{AE} = \varphi V_0^n $$ \hspace{1cm} (16)

where, $\varphi$ is a proportionality constant, which is dependent on the total electronic gain of the amplifiers used. This acoustic emission energy should be a part of the released energy $JU$ due to a pop-in crack propagation. Therefore, assuming, as a generalized form, a high power proportional relation between these two, and using Eq. (12),

$$ JU = A \cdot U_{AE} = A(\varphi V_0^n)^m = C(V_0^{max})^m $$ \hspace{1cm} (17)

where, $m=nk$ and $C=A(\varphi C_0^n)^k$ are constant. Rearranging the equation yields

$$ \ln JU = m \ln V_0^{max} + \ln C $$ \hspace{1cm} (18)

Figure 7 shows this relationship and gives

$$ m \approx 2.0 $$
$$ C \approx 2.0 \times 10^3 $$ \hspace{1cm} (19)

in the range of larger $JU$ or $V_0^{max}$. The deviation of the observed value from $m=2.0$ in the range of smaller values of $JU$ may be explained as follows.

Because of the limited stiffness of the testing machine, load drop due to a small pop-in crack propagation is difficult to detect exactly and weakened as it would be. Moreover, a small pop-in crack propagation only at the middle section of the specimen, where the plane strain condition prevails more strictly, may lead to little deflection change of the specimen. These possibilities suggest that in the range of smaller values of $JU$ or $V_0^{max}$, the released energy $JU$ can be underestimated and the slope $d\ln JU/d\ln V_0^{max}$ may deviate from that in the range of larger values of $JU$ or $V_0^{max}$, showing a tendency to a steeper slope with decreasing in the value of $JU$ or $V_0^{max}$.

The observed value of $m=2.0$ is in agreement with that presumed by Gerberich and Hartford and Harris, et al.,

From Eqs. (14) and (18) the following relation can be obtained:

$$ \ln JU = m \frac{\varphi}{f} JN + \ln (C_0^n \cdot C) $$ \hspace{1cm} (20)

Substituting Eqs. (15) and (19) in Eq. (20), it is found for the present experiment that

$$ \ln JU = 2.0(1.15 \times 10^{-4}) JN + 3.30 $$ \hspace{1cm} (21)

This relationship is shown in Fig. 6, indicating a good agreement with the plotted points in the range of larger values of $JU$ or $JN$.

Equation (20) can be expressed, as a general form, by

$$ \ln JU = 2.0 \frac{\varphi}{f} JN + \ln (C_0^n \cdot C) $$ \hspace{1cm} (22)

where, the last term is dependent on the total electronic gain of the amplifiers and on the coupling condition of the transducer. The application of Eq. (22) is limited to the case of the threshold voltage $V_t=1$ through Eq. (13) but could be generalized to other cases.

It should be noted that the two different steels, each with three different aging treatments, showed pop-in crack propagations and that all the data on these steels could be presented on the same plots, as seen in Figs. 6 to 8. This implies that differences in material properties should not cause differences in the acoustic emission data such as presented in the present work.

V. Concluding Remarks

Ni–Cr–Mo steels of secondary hardening type were heat treated to give pop-in crack propagations under bending load. The fatigue pretrained bending specimens were instrumented with an acoustic emission detecting system to obtain information on acoustic emission during pop-in crack propagation. The experimental results could be discussed on the basis of reasonable conceptions.

The relationship obtained between the released energy $JU$ and the total count jump of acoustic emission $JN$ during a pop-in crack propagation was as following:

$$ \ln JU = 2.0(1.15 \times 10^{-4}) JN + 3.30 $$

for the present experiment. This relation can be rewritten as

$$ \ln JU = 2.0 \frac{\varphi}{f} JN + \ln (C_0^n \cdot C) $$

where, the last term of the equation is constant, de-
pending on the total electronic gain of the amplifiers and on the coupling condition of the transducer used, and \( \beta_1 \) and \( f \) denote damping constant and linear frequency of acoustic emission signal, respectively. This relationship is not influenced by differences in the material properties in the range of the experiment.

Nomenclature

- \( a \): Crack length
- \( B \): Specimen thickness
- \( f \): Linear frequency
- \( K \): Stress intensity factor
- \( K_{eq} \): Apparent fracture toughness
- \( L \): Dead time
- \( N \): Total count of acoustic emission
- \( P_{eq} \): First pop-in load
- \( S \): Span length
- \( s \): Laplace transformation operator
- \( T \): Time constant
- \( t \): Time
- \( JU \): Released Energy by crack propagation
- \( V_{o}(t) \): Initial amplified and filtered voltage
- \( V_{oas}(t) \): Value of \( V_{RMS}(t) \) as recorded by plotter
- \( V_{RMS}(t) \): Maximum value of \( V_{o}(t) \)
- \( V_{RMS} \): RMS (root mean square) voltmeter output
- \( V_s \): Threshold voltage
- \( W \): Specimen width
- \( \beta_{1}, \beta_{2} \): Damping constants

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REFERENCES


Appendix

The following discussion is based, as the first step, on the simplified assumption of Eq. (10), which should be considered further. The RMS voltmeter output given by Eq. (10) is the input to the \( t-Y \) plotter, whose transfer function \( G(s) \) can be described by a generalized equation of the form

\[
G(s) = \frac{1}{1 + T s} \exp(-L s) \quad ......(A-1)
\]

where, \( s \): Laplace transformation* operator

\( L \): dead time

\( T \): time constant of the plotter.

The recorded RMS voltmeter output \( V_{oas}(t) \) is given by combining Eq. (10) with Eq. (A-1)

\[
V_{oas}(t) = \int_{-\infty}^{t} G(s) \cdot \int_{-\infty}^{s} V_{RMS}(t) ds \quad ......(A-2)
\]

Thus, from Eqs. (A-2) and (A-3), the following relation can be obtained,

\[
V_{oas}(t) = \int_{-\infty}^{t} \left[ \frac{1}{1 + T s} \exp(-L s) \right] \frac{a V_0}{s + \beta_2} \quad ......(A-3)
\]

\[
\int_{-\infty}^{t} \left[ \exp\left(-\frac{t-L}{T}\right) - \exp\left(-\beta_2(t-L)\right) \right] \quad ......(A-4)
\]

where, \( u(t-L) \) denotes the unit step function defined

\[
u(t-L) = 0 \quad \text{for } t<L \quad \text{and } u(t-L) = 1 \quad \text{for } t \geq L.
\]

The maximum RMS voltmeter output recorded \( V_{oas} \) is given at \( t_a \) when \( t_a \) is equal to \( t \) that satisfies the following condition \( t_a \geq L \)

\[
dV_{oas}(t) = 0 \quad ......(A-5)
\]
Solving the equation for $t$

$$t_m = \frac{\ln \beta_2 T}{\beta_2 - 1} + L = \text{constant} \quad \cdots \cdots \quad (A-6)$$

Thus, Eqs. (A-4) and (A-6) yield

$$V_{\text{max}}^{\text{ob}} = V_{\text{ob}}(t = t_m) \quad \cdots \cdots \quad (A-7)$$

This means

$$V_{\text{max}}^{\text{ob}} = C_0 V_0 \quad \cdots \cdots \quad (A-8)$$

where, $C_0$ is constant and given by

$$C_0 = \frac{n}{T} \left[ \exp \left( -\frac{t_m - L}{T} \right) - \exp \left( -\frac{\beta_2(t_m - L)}{T} \right) \right] \quad \cdots \cdots \quad (A-9)$$

for $t_m \geq L$. 