A Method for Coordinating Reloading Stress Relaxation Data into a Unique Curve*

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Synopsis

A method is proposed by which data from reloading stress relaxation tests performed under different conditions can be coordinated into a unique curve of the relative stress relief ratio at the final point of the i -th cycle of each reloading test, $\eta_{f,i}$, versus the cumulative plastic strain prior to the onset of the cycle, $\sum e_{p,i-1}$. This is possible if the value $K \equiv A_0 E \sigma_0^{n-1} \tau \cdot \exp(-Q/RT)$ for each measurement is constant, where A_0 , n , and Q are material constants, E Young's modulus, τ the reloading interval, σ_0 the initial stress, and RT has the usual meaning. Some experimental results for tests on a 12wt%Cr-steel justified this proposal. Applying the unique relationship between $\eta_{f,i}$ and $\sum e_{p,i-1}$, constructed only from short interval reloading data, it is easily possible to predict long interval reloading stress relaxation behavior.

I. Introduction

In connection with the retightening performance of bolts in, for example, high temperature power plant, it has been necessary to devise practical methods for predicting the long period reloading stress relaxation behavior of bolt materials. Draper¹⁾ pointed out long ago the suggestive fact that a unique relationship existed between the final stress remnant after each reloading cycle and the corresponding number of cycles, for two different types of data, one obtained in shorter period/higher temperature cycle tests and the other obtained in longer period/lower temperature cycle tests, no theoretical basis for which has yet been provided. Smith *et al.*²⁾ have found that a "deterioration" in stress relaxation strength, *i.e.*, an acceleration in stress relaxation rate, occurs in a specimen reloaded a number of times when the accumulated plastic strain attains a certain value. This deterioration phenomenon is serious for bolts because they are no longer serviceable after deterioration has occurred. Quaass³⁾ has proposed a graphical method for predicting reloading stress relaxation behavior, in which a diagram constructed using stress, total strain, and a temperature-time parameter was used as the basic data for prediction. Suzuki⁴⁾ has developed a mathematical approach to prediction based on the argument that the stress relaxation rate in the later stage of each cycle, named "Stage II" by him,^{4,5)} can be prescribed in terms of so-called "time hardening" theory. Both the method due to quaass and that due to Suzuki appear to have some faults, in that the accuracy of prediction may be low due to the usage of indirect basic data from types of tests other than reloading stress relaxation tests, or the many approximations in the computing methods.

In the present paper a modified method for predic-

tion is proposed, the basis of which has been reported elsewhere.⁶⁾ This method uses reloading stress relaxation data itself, but with a short reloading interval, to provide the basic data for prediction.

II. Derivation of a Temperature Compensated Time Law for Reloading Stress Relaxation Data

The stress relaxation rate during cycle i of the cyclic reloading on a specimen can be expressed, to a first approximation, by the following equation, based on "strain hardening" theory^{4,7)};

$$d\sigma/dt = -EA\sigma^n \varepsilon_p^m \dots\dots\dots(1)$$

where, E : Young's modulus

σ : the stress remnant at time t , which is counted from the onset of cycle i

$A(>0)$, $n(>1)$, $m(<-1)$: material constants which are here assumed to be approximately independent of the number of loading cycles.

A can be further expressed as $A=A_0 \exp(-Q/RT)$, where A_0 and Q are dimensional material constants and RT has the usual meaning. ε_p , the total plastic strain accumulated in the specimen at time t , is the only variable to change with the number of cycles, and is defined by

$$\varepsilon_p = \varepsilon_0 + \sum e_{p,i-1} \dots\dots\dots(2)$$

where, ε_0 : the initial strain

$\sum e_{p,i-1}$: the cumulative plastic strain prior to the start of cycle i .

Substituting Eq. (2) into Eq. (1), and then integrating Eq. (1) with respect to t and σ from the start to the end in cycle i , the following relationships are obtained;

$$\int_0^{\eta_{f,i}} (1-\eta)^{-n} [1 + \alpha q + q\eta]^p d\eta = K (\sum e_{p,i-1})^{-p} \dots\dots\dots(3)$$

using the following transformations, where τ_i , σ_0 and $\sigma_{f,i}$ are the period of cycle i , the initial stress, and the stress remnant at the end of cycle i , respectively;

$$\left. \begin{aligned} \eta &\equiv (\sigma_0 - \sigma)/\sigma_0 & (0 < \eta < 1) \\ p &\equiv -m & (m > 1) \\ q &\equiv \sigma_0/E(\sum e_{p,i-1}) & (0 < q < 1) \\ \alpha &\equiv E\varepsilon_0\sigma_0 - 1 & (\alpha \geq 0) \\ \eta_{f,i} &\equiv \eta \text{ at } t = \tau_i \equiv (\sigma_0 - \sigma_{f,i})/\sigma_0 \end{aligned} \right\} \dots\dots\dots(4)$$

and

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$$K \equiv AE\sigma_0^{n-1}\tau_i \equiv A_0 \exp(-Q/RT)E\sigma_0^{n-1}\tau_i \dots\dots(5)$$

The left hand side of Eq. (3) is uniquely dependent on $\sigma_{f,i}$, and the right hand side is a unique function of $\sum e_{p,i-1}$ if K can be put as a constant, regarding τ_i as being constant for a given combination of test variables σ_0 and T in Eq. (5). In other words, Eq (3) implies the existence of a “ temperature compensated time ” law in reloading data, similar to the Larson–Miller relationship in creep or creep rupture, and provides a theoretical basis for the experimental results of Draper mentioned above. Thus it is possible in principle to predict the value of $\sigma_{f,i}$ for any cycle of a reloading stress relaxation test, however long the period of the cycle is, based on a $\eta_{f,i}$ vs. $\sum e_{p,i-1}$ curve which can be constructed from short period reloading test data only. It should be noted here that we need only to know the stress remnant at the end of a cycle, since the tightening stress of a bolt needs only to be kept above a certain critical level during the bolt tightening.

III. Application and Discussion

Table 1 shows the results of reloading tests on two heats (A and B) of a 12wt%Cr heat resisting steel originally obtained for somewhat different purposes^{6,8)} than for checking the method proposed here. In this table $\sum \tau_i$ is the summation of the stress peroids before cycle i , from which it is possible to compute the period of each cycle. Figures 1(a), (b) and (c) show the variation of $\eta_{f,i}$ with $\sum \tau_i$, $\sum e_{p,i-1}$, and $\log \sum e_{p,i-1}$, respectively.

It can be seen that the curve for the 600°C/140 h cycle test and that for the 550°C/800 h cycle test are separate from each other in Fig. 1(a), while the data points for both the tests lie close to a single curve of $\eta_{f,i}$ vs. $\sum e_{p,i-1}$ (Curve 2) in Fig. 1(b), as well as on a single curve of $\eta_{f,i}$ vs. $\log \sum e_{p,i-1}$ in Fig. 1(c). Figure 2 shows the variation of $\eta_{f,i}$ with $\sum e_{p,i}$ (*N.B.* not $\sum e_{p,i-1}$), demonstrating the existence of a similar unique relation between these quantities to that in Fig. 1(b).

The good agreement found in the data for the 600°C/140 h and 550°C/800 h tests shown in Figs. 1 (b), 1(c) and 2 was mainly chance, at least in this case. However this agreement would seem to indicate some justification for the existence of the temperature compensated time law deduced in the preceding section, since the value of Q computed from Eq. (5), based on the 600°C/140 h and 550°C/800 h agreement, was reasonable, as shown below. The disagreement between the data for the 550°C/800 h and 550°C/140 h tests, or between the data for the 600°C/140 h and 550°C/140 h tests would seem also to support this argument, since the value of K for each test above was never identical, as can be seen from Eq. (5). In this point there is a problem to be considered. In order to deduce the relationship of Eq. (3), Eq. (1) was assumed, based on strain hardening theory. Therefore the value of $\eta_{f,i}$ estimated from Eq. (3), $\eta_{f,i}$ (cal), would be rather different from the experimental value, $\eta_{f,i}$ (obs), becoming more

Table 1. Summary of results of reloading stress relaxation tests on TAF steel.*

Tests	Remarks	η_f^{**}	$\sum e_o(\%)$	$\sum \tau(h)$
550°C/ 140 h cycle test	Material B, Initial stress : 30 kgf/mm ²	1	0.015	0
		0.433	0.103	142.8
		0.347	0.174	281.1
		0.293	0.234	429.9
		0.287	0.292	580.0
		0.250	0.343	719.6
600°C/ 140 h cycle test	Material B, Initial stress : 25 kgf/mm ²	0.207	0.385	854.6
		1	0.015	0
		0.548	0.132	142.2
		0.440	0.224	279.9
		0.414	0.304	430.6
		0.404	0.382	581.1
550°C/ 800 h cycle test	Material A, Initial stress : 30 kgf/mm ² (Step down method)	0.376	0.460	721.1
		0.368	0.532	883.1
		1	0.015	~0
		0.550	0.128	600
		0.453	0.221	1 430
550°C/ 250 h cycle test	Material A, Initial stress : 30 kgf/mm ²	0.400	0.303	2 280
		0.387	0.381	3 450
		1	0.015	~0
575°C/ 175~300 h cycle test	Ditto	0.345	0.105	216
		0.315	0.169	479
		1	0.030	~0
600°C/ 150~300 h cycle test	Ditto	0.478	0.190	175
		0.465	0.295	496
		1	0.060	~0
		0.538	0.280	149
		0.538	0.407	600

* TAF steel is composed of 0.2C–11Cr–Mo–1/5W–1/5Nb–0.02B in wt%, and heat treated as 1 150°C, O.Q. → 700°C, A.C.

** η_f is the relative stress relief: $(\sigma_0 - \sigma_f)/\sigma_0$, where σ_0 : initial stress, σ_f : final residual stress, of the cycle.

different the greater the number of reloading cycles, because the stress relaxation rate during a reloading cycle estimated from Eq. (1) is not always in good agreement with experimental values.^{4,6,8)} The reason why the temperature compensated time law predicted from Eq. (3) nevertheless holds, as above, is possibly because the relative error in $(\eta_{f,i}(\text{obs}) - \eta_{f,i}(\text{cal}))/\eta_{f,i}(\text{obs})$ was almost constant during each reloading cycle test having the same value of K . Putting $\eta_{f,i}(\text{cal}) = F(K, \sum e_{p,i-1})$, it follows that $\eta_{f,i}(\text{obs}) \cong F(K, \sum e_{p,i-1})/(1 - \text{const.})$, so that a temperature compensated time law could hold even for the $\eta_{f,i}(\text{obs})$ vs. $\sum e_{p,i-1}$ data.

In Fig. 1(b) the data for the 550°C/250 h cycle test, with σ_0 equal to 30 kgf/mm² (~300 MPa), is located a little apart from and to the left of Curve 1, which is for the 550°C/140 h cycle test, using the same initial stress. This seems somewhat anomolous in view of Eq. (3), from which it is to be expected that the 550°C/250 h data will be located to the right of Curve 1, since the value of K is greater than that in Curve

Fig. 1. Relationships between the relative stress relief η_f , the cumulative time Σt , and the cumulative plastic strain $\Sigma \epsilon_p$, for reloading stress relaxation tests on TAF steel.

Fig. 2. Relationship between $\eta_{f,i}$ and $\Sigma \epsilon_{p,i}$ (N.B. not $\eta_{f,i}$ vs. $\Sigma \epsilon_{p,i-1}$ in Fig. 1(b)), where $\Sigma \epsilon_{p,i}$ is the cumulative strain up to the end of the i -th cycle.

1. This anomaly may result from the scatter in stress relaxation strength from specimen to specimen, which is characteristic of this type of steel.⁹⁾
The data for the 600°C/150~500 h and 575°C/175~300 h tests, marked ▲ and ■ respectively in Fig. 1, cannot be directly compared with Curves 1 and 2 in (b) of this figure, because the values of τ_i , and thus of K , are not constant in the former but are constant in the latter. However, the fact that the data for the 575°C/175~300 h test intersects Curve 2 in Fig. 1 (b) suggests that the period τ_i of a constant K test at 575°C with a K identical with that in

Table 2. Values of the constant Q used in the temperature-compensated time relationship.

Materials	Comparative testing conditions	Q (kcal/mol)
TAF steel	550°C/800 h/ $\sigma_0=30$ kgf/mm ² :	98.2 ($n=10$)
	600°C/140 h/ $\sigma_0=25$ kgf/mm ²	78.0 ($n=6$)
Cr-Mo-V steel*	625°C/546 h/ $\epsilon_0=0.15\%$	78.5**
	650°C/157 h/ $\epsilon_0=0.15\%$	
12%Cr steel*	650°C/178 h/ $\epsilon_0=0.15\%$	94.0**
	625°C/1 095 h/ $\epsilon_0=0.15\%$	

* from Draper (See Ref. 1)).
** These values were calculated by regarding σ_0 as a constant.

Curve 2 lies within the range 170 to 300 h.
To confirm this it is necessary to know the values of Q , n , and E associated with Eq. (5). Assuming A_0 to be a constant, Q can be calculated from Eq. (5) as

$$Q = 2.303R[\log(\tau_{i1}E_1/\tau_{i2}E_2) + (n-1) \cdot \log(\sigma_{01}/\sigma_{02})]/(1/T_1 - 1/T_2) \dots\dots(6)$$

For Curve 2 in Fig. 1(b), putting $\tau_{i1} \approx 800$ h, $T_1 = 823$ K, $\sigma_{01} = 30$ kgf/mm² (300 MPa), and $\tau_{i2} \approx 140$ h, $T_2 = 893$ K, $\sigma_{02} = 25$ kgf/mm² (250 MPa), with $E_1 = E_2$, the value for Q can be computed to be ~ 78.0 kcal/mol (326.4 kJ/mol) if $n=6$ or ~ 98.2 kcal/mol (410.9 kJ/mol) if $n=10$. These magnitudes of Q appear to be reasonable compared with magnitude of the apparent activation energy for creep in heat resisting steels.⁹⁾ τ_i for a constant K reloading cycle test at 575°C can then be estimated to be ~ 190 h using the relationship $\tau_i \approx \exp(-Q/TR_1) \cdot \tau_{i1} / \exp(-Q/R \times 848)$, derived from Eq. (4), together with the values $Q \approx 80$ kcal/mol (334.7 kJ/mol), $T_1 = 823$ K and $\tau_{i1} \approx 800$ h. This magnitude of 190 h is within the range of 170 to 300 h estimated above, providing a further justification of the present theoretical deduction. Table 2 summarises the values of Q obtained from reloading tests, including those of Draper.¹⁾ It can be seen that the values of Q computed from Draper's results, with the assumption $E_1 \approx E_2$ and $\sigma_{01} \approx \sigma_{02}$, are comparable to the value of Q obtained above.

A point to be noted with respect to Draper's results is that the unique relation between data from longer period/lower temperature tests and that from short period/higher temperature tests held even in the range of cycles where the specimen deterioration was appreciable. This suggests the possibility that deterioration may initiate at a constant value of $\Sigma \epsilon_p$, even in reloading stress relaxation tests under different conditions, so long as the values of K are kept constant.

IV. Conclusions

(1) Reloading stress relaxation measured under various conditions with a constant K (which is defined by the equation

$$K \equiv A_0 E \sigma^{n-1} \tau \exp(-Q/RT)$$

where, A_0 , n , Q : material constants

E : Young's modulus

τ : the period of a cycle

σ_0 : the initial stress

RT : the usual meaning

can be coordinated into a unique curve of the relative stress relief at the final point of the i -th cycle in each test, $\eta_{f,i}$, vs. the cumulative plastic strain prior to the onset of the i -th cycle, $\sum e_{p,i-1}$.

(2) This was experimentally justified by results of reloading stress relaxation tests on a 12wt%Cr type steel. Applying the unique curve of $\eta_{f,i}$ vs. $\sum e_{p,i-1}$, constructed only from short period reloading data, it is possible to predict long period reloading stress relaxation behavior.

(3) The values of Q computed from the experimental results were 75~95 kcal/mol (314~397 kJ/mol), being comparable to the apparent activation energy usually observed during creep in heat resisting steels.

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REFERENCES

- 1) J.H.M. Draper: *Iron Steel*, **34**, (1961), 622.
- 2) A. Smith, D.J. Armstrong, M. F. Day and L.M.T. Hopkin: Joint Inter. Conf. on Creep, Sess. 6, London, (1963).
- 3) S. T. Quass: *J. Inst. Metals*, **96** (1968), 129.
- 4) F. Suzuki: High Temperature Strength in Metallic Materials, ed. by S. Taira, Yokendo Book Co., Tokyo, (1968).
- 5) S. Taira and F. Suzuki: *J. Soc. Mater. Sci., Japan*, **11** (1962), 169.
- 6) T. Shinoda: Doctoral Thesis to Tokyo Institute of Technology, (1973).
- 7) Yu. N. Rabotnov: Creep Problems in Structural Members, English translation ed. by F. A. Leckie, North-Holland Pub. Co., Amsterdam-London, (1969), 210.
- 8) T. Shinoda and S. Sasaki: *J. Soc. Mater. Sci., Japan*, **16** (1967), 150.
- 9) A. Hede and B. Aronsson: *JISI*, **207** (1969), 1241.