The Mathematical Model of Hot Deformation Resistance with Reference to Microstructural Changes during Rolling in Plate Mill*

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Synopsis

Single-stage and multi-stage tension tests of Si-Mn and Nb steels have been conducted in the temperature range of 800 to 1 000 °C at strain rate of 1 to 10 sec\(^{-1}\) in order to evaluate the effect of metallurgical factors on hot deformation resistance. The flow stress for both steels has been found to be expressed by Misaka's type equation. But strain rate exponent, which is constant in Misaka's equation, is temperature dependent. The flow stress increases with decrease in austenite grain size and the addition of Nb. Nb in precipitation is found to cause the larger increase of the flow stress than Nb in solution. The flow stress in multistage tension test can be expressed by above-mentioned modified Misaka's equation if the strain term is replaced by the term considering strain accumulation. The strain accumulated during rolling can be expressed as a function of the observed deformation resistance and the calculated one for strain-free austenite. With this method on-line real time evaluation of static restoration process is practicable. On the basis of the experimental results, a mathematical model for prediction of roll force calculation in plate rolling has been developed. The roll force can be calculated with good accuracy by the model.

I. Introduction

Computer control of rolling process has been eagerly developed in modern rolling mills. There have been strong demands for increasing the efficiency in the process control and for improving the accuracy in the gage and shape control of rolled products. In order to satisfy the demands for high accuracy and high efficient rolling, it is necessary to establish favorable mathematical model of roll force calculation, which is one of the most important terms for process control. For this purpose it is necessary to predict the hot deformation resistance with good accuracy, since it is the predominant factor of the roll force model.

The controlled rolling technology, rapid development of which was motivated by the construction of large-diameter pipelines, has become one of the most important technologies in the thermomechanical treatment. The major purpose of controlled rolling is to control microstructural changes during rolling and to improve mechanical properties. The hot deformation resistance is very sensitive to microstructural change in high temperature range. In order to make proper control of microstructure, relation between microstructural change and deformation resistance must be obtained. Further it is necessary to make clear the effect of microstructure on the hot deformation resistance in order to predict roll force in controlled rolling process with good accuracy.

The effects of deformation conditions such as temperature, strain and strain rate on deformation resistance have been reported by many authors. The deformation resistance has been measured by hot torsion, compression and tension tests. Several mathematical models have been established on the basis of the experiments and applied to process control. However these models ignored the effects of microstructural changes taken place during multi-pass rolling. So the models can not be applicable to the computer control of controlled rolling in Nb steels, in which static restoration process is extremely retarded.

Strain accumulation during multi-stage deformation has been investigated by many authors. However, the effect of metallurgical factors on deformation resistance is not completely understood in the case of small reduction as in plate rolling. And only few papers have published on mathematical models which can be directly applicable to process computer control in plate manufacturing process. This requires further investigation into proper mathematical models.

The purpose of the present study is to make clear the effects of deformation conditions and microstructural changes on hot deformation resistance and thereby to establish mathematical models of deformation resistance applicable to computer control of controlled rolling in plate mill. Continuous and interrupted tension tests have been conducted in the temperature range form 800 to 1 050 °C at strain rates of 1 to 10 sec\(^{-1}\), using Si–Mn and Nb steels. The effects of deformation conditions such as deformation temperature, strain and strain rate are formulated into mathematical equations. The grain size, strain accumulation and precipitation of Nb are also taken into consideration. The mathematical models were successfully applied to process computer control in a production plate mill.

II. Method of Experiments

The tests were carried out on one Si–Mn and two Nb steels with different Nb content, and the chemical compositions of these steels are listed in Table 1. Specimens with a gage length of 50 mm and a diameter 10 mm were machined from continuous cast slabs.

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* Based on the paper presented to the 101st ISIJ Meeting, April 1981, A45, at The University of Tokyo in Tokyo. Manuscript received on November 9, 1984; accepted in the final form on June 10, 1985. © 1985 ISIJ
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The tension test was conducted by a servo-controlled hydraulic hot tension machine named as Gleeble 1500. The specimens were preheated at the solution temperature of 1200 °C for 1 hr and quenched into water; Nb carbonitride is fully dissolved by this treatment, and Nb is again precipitated and dispersed in matrix of austenite at reduced reheating temperature below 1200 °C. Two series of experiments were performed in the present study: The first series is single stage tension tests. Specimens were reheated at 950 °C or 1150 °C for 1 min and deformed at temperatures from 800 to 1050 °C, and at strain rates of 1 to 10 sec⁻¹, and the effects of deformation temperature and strain rates on flow curve were studied. Further the effects of microstructural factors such as austenite grain size and Nb carbonitride on the flow stress were evaluated.

The second series is two-stage tension test. After reheating at 950 °C, the specimen was prestrained to 0.2, and subsequently deformed at 800 - 900 °C and strain rate of 10 sec⁻¹. Strain accumulated during the two-stage tension test was estimated as a function of holding temperature, holding time, prestrain and Nb content. The load, stroke displacement and deformation temperature were recorded in high speed visicorder. The minimum diameter of a specimen was measured after deformation, and flow stress, strain and average strain rate were calculated at minimum cross sectional area of the deformed specimen.

Typical flow curves of steels reheated at 1150 °C or 950 °C are shown in Fig. 1 and Fig. 2, respectively. At 1150 °C, Nb in 0.02 % Nb steel was completely dissolved, while at 950 °C Nb carbonitride particles dispersed in matrix in 0.035 % Nb steel. This enabled us to evaluate the both effects of Nb in solution and in precipitation on flow stress.

III. Experimental Result

1. Single-stage Experiment

1. Effect of Deformation Conditions

Figures 3 and 4 show relation between strain ε and stress σ for Si-Mn and Nb steels reheated at 1150 °C and 950 °C, respectively. From these results the following equation is obtained at constant deformation temperatures T and strain rate ε.

\[ \sigma = \sigma_1 \cdot \varepsilon^n \] ...........................(1)

where, \( \sigma_1, n \): constants.

Strain hardening exponent n's for both steels at both reheating temperatures are shown in Table 2. Nb increases the value \( \sigma_1 \), but has little effect on the value of n.

A temperature dependence of flow stress σ(ε=0.1) in steels reheated at 1150 °C is shown in Fig. 5: Arrhenius type of temperature dependence of flow stress is obtained as follows:

\[ \sigma = \sigma_2 \cdot \exp \left( \frac{A}{T} \right) \] ...........................(2)

where, \( \sigma_2, A \): constants.

The values of constant \( A \) are 4.10×10³ K, and 4.12×10³ K for Si-Mn steel and Nb steel, respectively, showing the addition of Nb has little influence on constant \( A \).

The effect of strain rate on flow stress σ(ε=0.1) for steels reheated at 1150 °C is shown in Fig. 6. The dependence of strain rate exponent m on deformation temperature T is shown in Fig. 7. The temperature dependence of the exponent m for Si-Mn and Nb steel is different, but the difference is very small in the temperature range from 800 to 1050 °C, which enables one to derive the following equation:

\[ \sigma = \sigma_3 \cdot \varepsilon^m = \sigma_4 \cdot \varepsilon^{m+\frac{A}{T}+\frac{A}{T}} \] ...........................(3)
On the basis of the above results, the flow stress $\sigma$ is expressed as a function of deformation temperature $T$, strain $\varepsilon$ and strain rate $\dot{\varepsilon}$:

$$\sigma = \sigma_0 \dot{\varepsilon}^{m'} \varepsilon^n \exp \left( \frac{A}{T} \right) \quad \text{...(4)}$$

where, $\sigma_0$, $\dot{\varepsilon}$, $m'$, $b$, $m''$: constants.

Equation (4) is Misaka's type equation. But strain rate exponent, which is constant in Misaka's equation, is temperature dependent. Values of $n$, $m''$ and $A$ are not influenced by deformation conditions, but by the microstructure of steel. The addition of Nb does not influence constants $A$ and $n$, but increases the constant $m''$, though it decreases the constant $m'''$.

### 2. Effect of Metallurgical Factors

The austenite grain size and strain-induced precipitates such as Nb (C, N) are major microstructural factors governing the hot deformation resistance.

As shown in the above section the hot deformation resistance increases with the addition of Nb. This is considered to be due to the increase in constant $\sigma_0$ in Eq. (4). The values of deformation resistance of Nb
Steel with various levels of Nb contents were measured and compared with that of Si-Mn steels, of which the chemical composition is similar to Nb steels. The experimental conditions are the same as the single stage experiments as described in Chapter II.—The amount of precipitation of Nb carbonitride was measured by chemical analysis of residuals extracted by electrochemical dissolution method. In steels reheated at 950 °C, more than 95 % Nb is in precipitation. The ratio of deformation resistance of Nb steels to that of Si-Mn steel is not influenced by deformation temperature, strain and strain rate. Figure 8 shows the relation between the ratio of the deformation resistance and the amount of Nb in solution or Nb in precipitation. The deformation resistance increases proportionally with the amount of Nb in solution or in precipitation, and the increase due to the precipitation is larger than that of solution. 0.05 % Nb in precipitation increases the deformation resistance by about 20 %.

It was already reported that the refinement of austenite grain size increased the hot deformation resistance. In the present study the effect of austenite grain size was investigated in temperature range from 800 to 1 050 °C. The result is shown in Fig. 9, together with the result of Ouchi et al., which was obtained in the similar experimental condition. It is clear both experimental results are consistent. The flow stress increases about 10 % with decrease in austenite grain size from 150 to 25 μm.

2. Multi-stage Experiment

It is well known that hot deformation resistance significantly increases with the decrease in rolling temperature and also by the retarding effect of recrystallization of austenite due to Nb addition. Strain introduced in the previous pass of rolling is not completely recovered, thereby affecting the hot deformation resistance in subsequent passes. The hot deformation resistance under these conditions is dependent not only on alloying elements and deformation conditions of the concerning pass, but also on thermomechanical history.

In order to make clear the effect of deformation temperature and holding time on static restoration process, the interrupted tension tests were conducted under the following conditions: after reheating at 950 °C, the two-stage tensile testing with interrupted time from 1 to 60 sec was performed in the temperature range from 800 to 900 °C.

It was proposed that the effect of the retardation of static restoration process during interruption in the two-stage deformation test could be formulated in terms of strain accumulated: The flow stress  in the second deformation is expressed as follows:

\[ \sigma_{II} = \sigma_{I}(\varepsilon_{II} + \Delta \varepsilon)^n \] ..............................(5)

where, \( \sigma_{I} \) : constant
\( \varepsilon_{II}, \Delta \varepsilon \) : strain in the second deformation and strain accumulated by the first deformation, respectively.

The value of  \( \Delta \varepsilon \) can be expressed as a function of the first and the second flow stress as:

\[ \Delta \varepsilon = (\sigma_{II}/\sigma_{I})^{1/n} - \varepsilon_{II} \] ..............................(6)

The fraction of unrecovered strain  \( A \) is defined as the following equation.

\[ A = \Delta \varepsilon/\varepsilon_{I} \] ..............................(7)

where, \( \varepsilon_{I} \) : strain in the first stage deformation. The value of  \( A \) is the important parameter with which the kinetics of recovery process during rolling is estimated. Figures 10(a) and (b) show the variation in the fraction of unrecovered strain  \( A \) of Si-Mn and Nb steels deformed at 800 and 900 °C with interrupted time  \( \Delta t \). It is evident the addition of Nb remarkably retards static restoration progress. The present experiment adopted the very high reheating rate of 50 °C/sec as well as the very short holding time of 1 min at 950 °C, and these reheating conditions may tend to form the much finer size of Nb(C, N) precipitates compared with the case of reheating conditions adopted in the mill production. These fine Nb(C, N) precipitates formed at 950 °C in Nb steel may prevent the grain boundary migration, resulting in the marked retardation of static recovery and recrystallization of austenite.
Iv. Mathematical Model

The differential equation describing the restoration process is derived as follows. The variation in ε after deformation is given by

\[ \frac{d\varepsilon}{dt} = -f(\varepsilon) \cdot g(T) \] ........................(8)

If \( f(\varepsilon) \) is a continuous function at a closed interval \([0, \varepsilon]\), \( f(\varepsilon) \) can be uniformly approximated by polynomials of \( \varepsilon \) by Weierstrauss approximation theory.\(^{21}\)

Assuming Arrhenius type of temperature dependence, the variation in ε after deformation is formulated into the following differential equation:

\[ \frac{d\varepsilon}{dt} = -\exp \left( \frac{Q}{RT} \right) \sum_{i=1}^{N} C_i \varepsilon^i \] ........................(9)

where, \( N \): definite integer
\( Q \): apparent activation energy
\( C_i \): coefficient being dependent on the chemical composition.\(^{5}\)

Equation (9) is easily solved by direct integration method; we obtain hyperbolic equation for \( \varepsilon \). The hyperbolic equation is expressed by

\[ F(\varepsilon_t) - F(\varepsilon_0) = -\Delta t \exp \left( -\frac{Q}{RT} \right) \] ........................(10)

where, \( F(\varepsilon) \): indefinite integral of the reciprocals of right side of Eq. (9)
\( \varepsilon_t \): prestrain
\( \Delta t \): interrupted time.

Practically we can approximate restoration process in plate rolling with good accuracy if \( N=2 \). In this case \( A \) can be expressed by:

\[ A = \frac{C_1}{C_2} \exp \left( \frac{C_2 \Delta t}{C_1} \right) \exp \left( \frac{-Q}{RT} \right) - C_2 / C_1 \] ........................(11)

On assumption that \( C_2 \) is greater than \( C_1 \), Eq. (11) is linearized as:

\[ \ln C_1 + 0.5(1/ \varepsilon_1 + 1)(C_1/C_2) - Q/RT = \ln \left[ (1/ \varepsilon_1 - 1)/(\varepsilon_t, \Delta t) \right] \] ........................(12)

Coefficients \( C_1 \), \( C_2 \) and \( Q \) in Eq. (11) were determined by regression analysis method, using two-stage tensile test results with interruption time of 1 to 300 sec, prestrain of 0.05 to 0.3 and deformation temperature of 800 to 1 000 °C. Coefficients for Si–Mn and Nb steels are listed in Table 3.

In all cases \( C_3 \) is much greater than \( C_4 \). The retarding of restoration process in Nb steel is considered to be brought about by the increase in the values of apparent activation energy \( Q \).

Static softening process is accelerated with increase in prestrain because it is clear from Eq. (11) that the value of \( A \) decreases with \( \varepsilon_t \): \( \partial A/\partial \varepsilon_t < 0 \).

2. Prediction of Roll Force in Plate Rolling

With use of the mathematical models of deformation resistance and of temperature calculation, on-line real time roll force calculation is feasible. The method of roll force calculation is made in the following way.

The mathematical model consists of two equations: The first one is the fundamental equation which expresses deformation resistance for strain free austenite \( k_{\text{m}}^{\text{obs}} \) as a function of deformation temperature \( T \), strain \( \varepsilon \) and strain rate \( \dot{\varepsilon} \), and the other one is the differential equation describing static restoration process during pass interval. The equation is formally in accord with ones on the basis of the high temperature tensile test described in previous sections.

Systematic rolling experiments were performed in the plate mill for the purpose of developing the mathematical model. Roll force \( P \), entry and exit thickness \( H \) and \( h \), motor speed, plate width \( W \), interpass time \( t_{\text{om}} \) were measured. Mean deformation resistance \( K_{\text{m}}^{\text{obs}} \) was calculated by the following equation:

\[ K_{\text{m}}^{\text{obs}} = \frac{P}{Q^2 \sqrt{R^2 - (H-h)W}} \] ........................(13)

where, \( Q^2 \): radius of flattened roll
\( Q_P \): geometrical function of \( H \), \( h \) and \( R \).

The value of \( Q_P \) can be calculated by Sims’s formula, if \( (H+2h)/3 < \sqrt{R^2 - (H-h)} \). If \( (H+2h)/3 > \sqrt{R^2 - (H-h)} \)
peening effect must be considered: Slip line theory for indentation is utilized in this case.

Variation in rolling temperature was calculated by thermal equation based on Fourier equation of heat conduction:

\[
\rho C_p \frac{\partial T}{\partial t} = \lambda \nabla^2 T + Q \tag{14}
\]

with boundary condition:

\[
-\frac{1}{\rho} \frac{\partial T}{\partial n} = q \tag{15}
\]

where, \( T, \rho, C_p \): temperature, density and specific heat of steel, respectively

\( Q, q, \lambda \): generation of heat, heat flux and thermal conductivity, respectively

\( n \): unit vector perpendicular to boundary surface.

The following sequential process of temperature change is considered; (1) air cooling, (2) water cooling and (3) rolling. Optimum values of heat flux were determined in order to minimize error in temperature calculation.

In the first step, deformation resistance for strain-free austenite \( K_m^{\infty} \) is determined by regression analysis. For this purpose the following criteria are adopted; rolling temperature is higher than the critical temperature \( T_{cr} \) and interpass time \( t_{om} \) is longer than the critical interpass time \( t_{cr} \). We assumed \( T_{cr} \) to be 950 °C and \( t_{cr} \) 60 sec for Nb steels and \( T_{cr} \) 900 °C and \( t_{cr} \) 30 sec for Si-Mn steels. If the rolling temperature is higher than \( T_{cr} \) and the interpass time is longer than \( t_{cr} \), the effect of strain accumulation on deformation resistance is negligible as shown in Fig. 10. Making regression analysis, we obtained dominant terms similar to Eq. (4), with which \( K_m^{\infty} \) is expressed as follows:

\[
\ln K_m^{\infty} = b_0 + b_1/T + b_2 \ln \varepsilon + b_3 T \ln \dot{\varepsilon} \tag{16}
\]

where, \( b_0 - b_3 \): coefficients, being dependent on chemical composition and austenite grain size.

Equation (16) describes deformation resistance for strain-free austenite.

The effect of strain accumulated, \( \Delta \varepsilon \), can be formulated in the following way; the difference between observed mean deformation resistance \( K_m^{\text{obs}} \) and \( K_m^{\infty} \) calculated by Eq. (16) is assumed to be attributable to \( \Delta \varepsilon \) during pass interval. By using total strain, defined as \( \varepsilon + \Delta \varepsilon \) instead of \( \varepsilon \) in Eq. (16), \( K_m^{\text{obs}} \) in low temperature austenite range is represented by the following equation:

\[
\ln K_m^{\text{obs}} = b_0 + b_1/T + b_2 \ln (\varepsilon + \Delta \varepsilon) + b_3 T \ln \dot{\varepsilon} \tag{17}
\]

Making use of Eqs. (16) and (17), \( \Delta \varepsilon \) is expressed by

\[
\Delta \varepsilon = [(K_m^{\text{obs}}/K_m^{\infty})^{1/b_3} - 1] \varepsilon \tag{18}
\]

For the purpose of predicting \( \Delta \varepsilon \), it is necessary to formulate the recovery process after hot deformation in mathematical equations. Let us consider the recovery process during pass interval between \((n-1)\)-th and \(n\)-th passes. Schematic illustration of recovery process is shown in Fig. 11. Fraction of unreleased strain during \((n-1)\)-th and \(n\)-th passes \( A_{n-1} \) is expressed by the following equation:

\[
A_{n-1} = \Delta \varepsilon_n/(\varepsilon_n + \Delta \varepsilon_n) \tag{19}
\]

where, \( \Delta \varepsilon_n \) and \( (\varepsilon_n + \Delta \varepsilon_n) \) are strain accumulated at \(n\)-th pass and total strain at \((n-1)\)-th pass, respectively. \( \Delta \varepsilon_n \) is represented as the sum of terms involving \( \varepsilon \) and \( \dot{\varepsilon} \):

\[
A_{n-1} = \sum_{i=1}^{n-1} \prod_{j=i}^{n-1} A_j \varepsilon_i \tag{20}
\]

Equation (20) shows that knowledge of \( A \) enables us to calculated \( A_{n-1} \). As shown in previous sections \( A_{n-1} \) can be expressed by

\[
A_{n-1} = C_1/(C_1(\varepsilon_n + \Delta \varepsilon_n) + C_1) \exp (C_1 t_{om}) \times \exp (-Q/T) - C_2(\varepsilon_n + \Delta \varepsilon_n) \tag{21}
\]

By using Eqs. (18) (19) and (21) \( A \) can be evaluated. Equation (21) is linearized in order to make regression analysis: Coefficients \( C_1, C_2 \) and \( Q \) are determined.

Block diagram for data analysis is shown in Fig. 12.

The mathematical models were applied to on-line real time computer control of plate mill rolling.

Let us consider the case that after \((n-1)\)-th pass prediction of roll force at \(n\)-th pass is performed. With the help of on-line data processing system, plate temperature \( T \), strain \( \varepsilon \), strain rate \( \dot{\varepsilon} \) and observed mean
deformation resistance $K_m^{(n-1)}$ at $(n-1)$-th pass are calculated, with which mean deformation resistance for strain-free austenite $K_m^{(0)}$ is computed. Further strain accumulated at $(n-1)$-th pass, $\Delta e_{n-1}$ is estimated by Eq. (18). According to draft schedule determined in advance by process computer, $T$ at $n$-th pass is predicted and strain accumulated at $n$-th pass $\Delta e_n$ is evaluated by Eqs. (20) and (21). Making use of Eq. (17), mean deformation resistance $K_m$ is predicted. Finally roll force is calculated. Simplified formulae for radius of flattened roll $R'$ and geometrical factor $Q_p$ are utilized for saving computing time.\(^{16}\)

Applying the mathematical models to computer control system in a production plate mill, automatic control of entire controlled rolling practice has been performed. Adoption of the models results in improvement of accuracy in roll force prediction remarkably. Distribution in force correction factor, defined as the ratio of observed roll force to calculated one, in controlled rolling is shown in Fig. 13. Errors of roll force prediction are less than 5% of the calculated values.

**V. Discussion**

1. **The Form of the Mathematical Model**

Softening process after deformation has been extensively studied by many investigators. Static restoration process can be estimated by the techniques of interrupted deformation test.\(^{5-11}\) In this type of work fractional softening $X$ that takes place during a holding time is evaluated by the following equation\(^{81}\);

$$X = (\sigma_m - \sigma_2) / (\sigma_m - \sigma_1)$$  \hspace{1cm}  (22)

where, $\sigma_m$: flow stress in the first stage of straining
$\sigma_1$, $\sigma_2$: yield stress in the first and second stage of straining, respectively.

But for the purpose of predicting the hot deformation resistance, the variation in flow stress rather than one in yield stress must be taken into consideration because it is very difficult to measure yield stress during rolling, thereby to evaluate $X$ by Eq. (22) from rolling data analysis. Further, it was reported that the recovery in flow stress in the two-stage deformation test was smaller than one in yield stress.\(^{80}\)

The present treatment of stress-strain curve in multi-stage deformation given by Eq. (5) is based on the assumption that the flow curve on restraining in the interrupted deformation test closely approaches the corresponding uninterrupted flow curve if the origin of strain axis is properly shifted. In order to ascertain the above assumption the following analysis was performed. As shown in Fig. 14, stress-strain curve in two stage tension tests with interruption times of 1 sec and 10 sec closely approaches to that of continuous deformation, if the origins of strain axis are shifted by 0.18 and 0.10, respectively. The amount of shifting in strain axis is considered to be $\Delta e$.

If relation between dislocation density $\rho$ and strain $\varepsilon$ obeys the following equation:

$$\rho = h(t) \cdot \varepsilon$$  \hspace{1cm}  (23)

the first and second order terms in the right hand side of the differential equation (9) are considered to represent decrease density by absorption of dislocation at boundaries and pairwise annihilation dislocations in grain interior, respectively, at the initial stage of static restoration. Recrystallization is dominant softening mechanism in the next stage. In this stage the differential equation may represent decrease in dislocation density due to the interaction between migrating boundaries and dislocations in grain interior. But the detailed analysis is the subject of future study.

2. **Relation between Microstructure and Deformation Resistance during Rolling**

The precipitation behavior of Nb(C, N) during rolling was investigated by a rolling of composite slab with a thickness of 240 mm, which is produced by welding of an Nb steel slab section and a Si-Mn steel section placed in rolling direction.\(^{20}\) Chemical compositions of Nb and Si-Mn steels used in composite slab rolling are same with those of steels used in hot deformation test. After reheating at 1250 °C, the composite slab was rolled to 25 mm plate. It is so difficult to simulate the thermomechanical process in production mill rolling by a hot deformation test that a mathematical model is necessary to estimate the kinetics of Nb(C, N) precipitation during rolling. The outline of the mathematical model is shown in Fig. 13.
Appendix. Figure 15 shows changes in the amount of Nb(C,N) precipitates predicted by computer simulation during rolling together with the variations of austenite grain size, the surface temperature and the deformation resistance ratio of the Nb steel and Si-Mn steel. In the deformation in the high-temperature austenite region, Nb (C,N) scarcely precipitates, and precipitation proceeds abruptly during the interruption for waiting for a temperature decrease. Also the deformation resistance ratio of Nb and Si-Mn steels, which is evaluated with correction of the effect of austenite grain size difference between Nb and Si-Mn steels, is almost constant at about 1.03 in the high-temperature austenite region. In the pass after the interruption, however, this ratio increases to 1.06, showing the effect of Nb(C,N) precipitation during the interruption. This reveals that an increase of the amount of precipitates leads to an increase of deformation resistance. Almost the same result as the case of hot deformation test was also obtained in this composite slab rolling experiment, and it was confirmed that precipitates of Nb(C,N) increases the deformation resistance in a larger degree compared with solid solution effect of Nb atom.

The refinement in austenite grain size during rolling has an effect on the deformation resistance. Sel-lars and Whiteman\(^1\) proposed an empirical formula for estimating the austenite grain size after static recrystallization, and showed that the recrystallized austenite grain size, \(d_r\), can be approximated by the following equation:

\[
d_r = A'' \cdot \varepsilon^{-1} \cdot d_0^2 \left( \frac{1}{B} \ln \left( \frac{Z}{A} \right) \right)^{2/3}
\]

where, \(d_0\) : initial austenite grain size

\(Z\) : Zener–Hollomon parameter

\(\varepsilon\) : strain

\(A, A'', B\) : constants.

\(Z\) is a function of temperature and strain rate, and if the grain size upon reheating is denoted by \(d_0\), all the parameters in Eq. (25) take known values and it is possible to calculate the \(d_r\), in the first pass rolling. Therefore, the grain size in multi-pass rolling can be estimated by repeating the procedure in which \(d_0\) in the next pass is calculated in consideration of grain growth after recrystallization in preceding pass rolling and Eq. (25) is used again.\(^1\)

To quantitatively explain the relationship between the grain refinement and deformation resistance during rolling based on the present experiment result, a parameter called here as a standard deformation resistance, \(K_m^*\), is used. The value \(K_m^*\) is a calculated deformation resistance in the case of the austenite grain size of 150 \(\mu\)m. This is the deformation resistance obtained under the same deformation conditions as the rolling conditions in question by using a mathematical model on the basis of the hot deformation experiment, and the value is corrected with respect to the through-thickness temperature distribution. The effect of the grain size is reflected in the ratio of the mean deformation resistance, \(K_m\), calculated from the rolling load to \(K_m^*\), namely, \(K_m/K_m^*\). Attention is paid to changes in this value.

Variations in austenite grain size in Si-Mn steel reheated at 1 150 °C and 1 250 °C were simulated by using Eq. (15). Simulation results, along with variations in the above-mentioned \(K_m/K_m^*\), are shown in Fig. 16. Recrystallization results in grain refinement and the value of \(K_m/K_m^*\) increases. The austenite grain size of 400 \(\mu\)m, which is obtained at the reheating temperature of 1 250 °C, is refined and becomes finally about 65 \(\mu\)m, reaching the same grain size obtained in the case of the reheating temperature of 1 150 °C. Also the value of \(K_m/K_m^*\) is as low as 0.95 in the first pass, but it is 1.04 in the sixth pass, the same level as the case of reheating temperature of 1 150 °C.

The austenite grain size can be estimated from...
variations in deformation resistance if the roll force and temperature can be accurately measured.

It is well known that static restoration process is affected by microstructure. It is necessary to obtain the relationship between microstructure and the fraction of unrecovered strain: coefficients in Eq. (11) should be described as a function of metallurgical factors. The coefficient \(Q\) in Eq. (11) is dependent on the amount of solid solution and/or precipitate of carbonitride forming element. The austenite grain size has an influence on the other coefficients in Eq. (11), \(C_1\) and \(C_2\). The further investigation is necessary to formulate the effect of the microstructure on static restoration.

Finally, the hot deformation resistance with reference to metallurgical factors is expressed as the following equation.

\[
\ln K_w = b_0 + b_1 T + b_2 \ln (1 + \dot{\varepsilon}) + b_3 T \ln \dot{\varepsilon} + b_4 \ln \dot{\varepsilon} + b_5 T \ln \dot{\varepsilon} + \ldots \quad (27)
\]

**VI. Summary**

The effect of microstructural changes during rolling on deformation resistance of Si-Mn and Nb steels was investigated by high speed hot tension tests. On the basis of experimental results mathematical models were formulated. The results are summarized as follows:

1. The flow stress is expressed by the Misaka's type equation. But strain rate exponent, which is constant in Misaka's equation, is temperature dependent. The flow stress increases with decrease in austenite grain size and with the addition of Nb. The Nb in precipitation give rise to the larger increase of the flow stress than Nb in solution.

2. The flow stress in multi-stage tension test can be expressed by the modified Misaka's equation, if the strain term is replaced by the terms considering strain accumulation. The softening curve after deformation is expressed in terms of temperature, interruption time and prestrain.

3. The strain accumulated during rolling can be expressed as a function of the observed deformation resistance and calculated one for strain-free austenite. With this method on-line real time evaluation of static restoration process is feasible.

4. The mathematical models of deformation resistance were determined by regression analysis. The model consists of two main equations: the fundamental equation and the restoration equation. The fundamental equation for deformation resistance of strain free austenite is described by the function of deformation temperature, strain, and strain rate. The restoration model expresses softening process in low temperature austenite during pass interval by simple equation.

5. The mathematical model was applied to automatic control of the plate mill rolling. The roll force can be calculated with good accuracy with use of the model.

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**Appendix**

The precipitation kinetics of Nb(C, N) can be calculated by classical nucleation and growth theory.\(^{22}\) The rate of nucleation per unit volume at a reaction time \(t\), \(J(t)\), is:

\[
J(t) = N(t) \cdot \beta^* \cdot Z \cdot \exp \left(-\frac{\Delta G^*}{kT}\right) \exp \left(-\frac{\tau}{t}\right)
\]

where, \(N(t)\): number of nucleation site per unit volume
\(\beta^*\): number of atoms reaching the critical nucleus surface per unit time
\(\Delta G^*\): Gibbs free energy upon forming the critical nucleus
\(\tau\): incubation time
\(k\): Bolzman constant.

The parameters contained in Eq. (A-1), \(\beta^*\), \(Z\), \(\Delta G^*\) and \(\tau\), are functions of interfacial energy of precipitate, \(\gamma\), the diffusion coefficient of Nb in austenite, \(D_{Nb}\), and shape factors, \(K\) and \(L\). The amount of precipitation \(X_p(t)\) is calculated by Avrami equation\(^{23}\):

\[
X_p(t) = 1 - \exp \left[-\int_0^t J(t') \cdot V(t, t') dt'\right]
\]

where, \(V(t, t')\): the volume that a precipitate nucleated at moment \(t'\) has attained by moment \(t\).

The growth rate of precipitate is estimated by volume diffusion mechanism.\(^{24}\)

In the present model, parameters \(N(t), \tau, K\) and \(L\) are unknown factors. The values of these parameters were determined so that best agreement with the reported experimental result\(^{25}\) could be obtained. The precipitation kinetics of Nb(C, N) in the single stage experiments were simulated by the present model. The calculated amounts of Nb(C, N) were in good agreement with measured ones by chemical analysis.