Mathematical Model of Steady State Heat Transfer in Blast Furnace Hearth and Bottom*

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Synopsis

A method of determining the steady state temperature distribution in blast furnace hearth and bottom is given. Various cooling and lining systems were considered. A method of solutions coupling is applied. A fairly good agreement of computed and measured results is obtained.

I. Introduction

Cooling of blast furnace bottom and hearth is a problem of great practical importance. Some papers dealing with this problem have been published by Westbrook and Mac Pherson,^{1,2)} Paschkis and Mirsepassi^{3,4)} and some Russian researchers.^{5,6)} Most of these works based on the electric analog method. Etzel and coworkers⁷⁾ solved this problem numerically. In the cited papers the dependence of the thermal conductivities on temperature has been neglected, the temperature of the initial pig iron zone has been assumed constant. Malkiewicz⁸⁾ obtained the temperature distribution within the blast furnace hearth using finite differences and assuming temperature dependent heat conductivities. In this work the coal lining was assumed to be soluble in the liquid pig iron. Thus, the unknown location of the pig iron-coal lining interface has been defined by the pig iron melting temperature. However all the discussed above approaches were restricted to one particular

system of cooling. The aim of our work was to obtain a computer program capable of dealing with all existing types of both lining and cooling. The types of cooler considered in this paper and their heat connections are shown in Fig. 1.

In our model the non linear behaviour of the materials and their anisotropy have been taken into account. The problem of the location of the pig iron zone has been treated similarly as in Ref. 8. The erosion has been regarded as a process of solution of carbon blocks in the molten metal. It has been assumed that the solution of carbon blocks takes place until the temperature of the metal—lining interface reaches the pig iron melting point. Works of Ikeda *et al.*^{9,10)} offer a more realistic insight into the physical nature of the erosion. The program can also deal with cases when the carbon lining is covered by a protective material non soluble in the molten metal.

II. Solutions Coupling Method

This method is described in detail in Ref. 11). It is included here for completeness. The method consists mainly in splitting up the considered region into a number of subregions. On the new boundaries appearing, while dividing the region, an unknown

↔ indicates the heat is transmitted in one of these ways only.

1. Water spray cooler
2. Cooling plate
3. Cooling plate with uncooled part of blast furnace jacket
4. Radial underhearth cooler
5. Parallel flow cooler
6. Soil
7. Footing

Fig. 1.

Variations systems of cooling of a blast furnace hearth and bottom.

* Manuscript received on May 25, 1984; accepted in the final form on February 1, 1985. © 1985 ISIJ

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interface temperature is assumed. Further, the solutions of boundary value problems in every subregion are to be found. These solutions contain the unknown interface temperatures, which can be determined by substituting the subregion solutions into the boundary condition of the fourth kind.

Subregions, or more precisely the computer sub-routines corresponding to the subregions, can be coupled in arbitrary configuration. This is very convenient when considering various cooling systems of blast furnace.

The blast furnace bottom and hearth were selected to be the basic subregion, whereas the cooling elements were said to be the subsidiary subregions. To solve a problem, some of the subsidiary regions depending on the blast furnace design were coupled with the basic one. The boundary value problem in the basic subregion was solved numerically. In the subsidiary subregion, either analytical or numerical method was applied. Due to the discretization of both temperature and heat flux in the basic sub-region, also the temperature and heat flux at the interfaces of the subsidiary regions had to be discretized.

It was proved that in the subsidiary subregions the interface temperature and heat flux are interacting through equations

$$\dot{Q}^i = \tilde{G}^i T^i + \dot{Q}_0^i \dots\dots\dots(1)$$

It is to be noted, that $\tilde{G}^i$ and $\dot{Q}_0^i$ are constant when the boundary value problem in a subregion is linear, and depends on the temperature in case the problem is nonlinear. Equation (1) holds when applying either numerical or analytical method. This leads to considerable computational advantages. Equation (1) can be regarded as boundary conditions or constraints of the heat equation in the basic sub-region.

In the next chapters the assumptions under which the heat transfer equation in each subregion was solved, will be briefly discussed.

III. Blast Furnace Bottom and Hearth—Basic Subregion

The boundary value problem in this subregion was a complicated matter. The subregion consisted of two-dimensional concentric layers of lining. The heat conductivities in these layers were anisotropic and depended on temperature. The location of pig iron lining interface was unknown. As mentioned above a numerical method has been applied to solve the boundary value problem in that region. The method used was the elementary balance method. It was described elsewhere.^{12,13)} This kind of approach is in fact a kind of finite differences method. The main advantage of the elementary balance method is that it has simple physical interpretation, because the difference equations are derived explicitly from the energy conservation rule. This allows to make the net of mesh independent of the materials layers location (it is easy to handle the case when

the layers intersect—the mesh).
It was assumed that the temperature field is axis-symmetrical. The heat transfer within the sub-region is governed by equation

$$\frac{1}{r} \frac{\partial}{\partial r} \left(\lambda_r r \frac{\partial T}{\partial r} \right) + \frac{\partial}{\partial z} \left(\lambda_z \frac{\partial T}{\partial z} \right) = 0.$$

The convection effect in the molten pig iron has been taken into account by increasing its heat conductivity. This approach was firstly proposed by Wilms.¹⁴⁾

The considered subregion is a truncated cone or a cylinder. Figure 2 shows the prescribed boundary conditions. Due to the nonlinearity of the discretized equation system, an iterative method of solving it had to be applied. The method used here was the successive overrelaxation method. The procedure started from some initial guessed temperature distribution. Assuming the temperature in the whole subregion made calculation of the heat conductivity within the subregion as well as determination of the solid-liquid interface possible. It was assumed that the molten pig iron is contained in the zone defined by inequality

$$T > T_m,$$

when the solid is soluble in the liquid pig iron (e.g., carbon blocks). In case the solid is insoluble, the interface is the initial solid boundary. It is possible for the zone containing some “frozen” pig iron to exist. The interface was approximated by a set of linear functions. It was assumed within the soluble solid that the interface lies between two adjoining grid points if only the temperature in one of these points is higher and in the second one lower than T_m . The exact location of the interface was established by linear interpolation. All of those assumptions led to a set of equations whose solution was an improved value of temperature field. This procedure was repeated until convergence was obtained. The convergence was considered achieved when both the maximum correction of temperature and the total heat balance error were sufficiently small. The iterative process was always convergent.

IV. Water Spray Cooler

This system of cooling was described by assuming

Fig. 2. Assumed boundary conditions for the blast furnace hearth and bottom.

the IIIrd boundary condition on the blast furnace jacket to be satisfied.

V. Cooling Plate

The cooling plate is a cast iron plate with a steel coil inserted. Water is usually the cooling medium. The compensation material layer and the blast furnace bottom were considered to be a part of a cooling plate. The boundary conditions are shown in Fig. 3.

The problem was assumed to be linear and the cooling medium temperature to be constant. Because of complex geometry and multilayer structure,

Fig. 3. The repeatable sector of cooling plate.

Fig. 4. Prescribed boundary conditions for the uncooled part of blast furnace jacket.

a numerical method was applied. The net of mesh and the method used were described in detail in Ref. 15). Under these assumptions, the relation between temperature and heat flux is given by Eq. (1).

VI. Uncooled Part of Blast Furnace Jacket

In some cooling systems only a part of the jacket is being cooled by cooling plates. The remaining part of the jacket dissipates heat into the environment. It is assumed that the heat transfer coefficient is known.

This part of the blast furnace jacket could have been inserted in the basic subregion, but that would result in complicating computer programming as the geometry of the basic subregion would become rather complex. The geometry of that part of the jacket and the prescribed boundary conditions are shown in Fig. 4.

VII. Soil

The following assumptions were made. Soil is a semi-infinite region. The temperature field has symmetry about the blast furnace axis. The thermal conductivity is uniform. The temperature on the soil surface is equal to the environment temperature except for a circle interior of radius R (the interface) where the temperature is approximated as

$$T^i(r, z=0) = T_e + \sum_{i=1}^m A_i \left[1 - \left(\frac{r}{R} \right)^2 \right]^{i+0.5}, \quad r \leq R \quad \dots\dots\dots(2)$$

This type of approximation was chosen because it enables the analytical solution of the heat conduction problem within the soil. The heat flux on the interface was determined as

$$\begin{aligned} \dot{q}^i(r, z=0) &= \lambda \frac{\partial T^i(r, z=0)}{\partial z} \\ &= \frac{\lambda}{R} \sum_{i=1}^N A_i \frac{\Gamma(1.5+i)\Gamma(1.5)}{\Gamma(i+1)} \\ &\quad \times F\left(1.5, i; 1, -\left(\frac{r}{R}\right)^2\right) \quad \dots\dots\dots(3) \end{aligned}$$

The circular net of mesh of the blast furnace bottom was projected onto the soil surface. This resulted in dividing the interface into $M \geq N$ coaxial rings, having internal radius r_i and external radius r_{i+1} ($i=1, 2, \dots, M, r_1=0, r_{M+1}=R$). In the center of those rings a nodal point R_i was chosen

$$R_i = (r_i + r_{i+1})/2.$$

Temperature and heat flux both discretized were defined as

$$T_i^i = T^i(R_i, z=0) \quad \dots\dots\dots(4)$$

$$\dot{Q}_i^i = \int_{r_i}^{r_{i+1}} \dot{q}^i(r, z=0) 2\pi r dr. \quad \dots\dots\dots(5)$$

The unknown coefficients A_i were removed demanding the polynomial (2) to be the least square approximation of temperature in nodal points R_i . Finally the relation between temperature and heat flux on the

interface took the form of Eq. (1).

VIII. Radial Underhearth Cooler

This type of cooler consists of two circular horizontal plates between which cooling air flows radially. The flow of the cooling air in this type of cooler is shown schematically in Fig. 5. The heat in this cooler was assumed to be transported by convection and radiation only. The heat conduction in the supporting radial beams was neglected. The problem was evaluated numerically using a net of mesh defined in the previous chapter. The discretized interface heat fluxes can be written as

$$\begin{aligned} \dot{Q}_{uj}^i(T_u^i, T_L^i) = \pi(r_{j+1} - r_j)^2 \left\{ C_o \varepsilon_u \varepsilon_L \sum_{l=1}^M \right. \\ \left. \times \varphi_{jl} \left[\left(\frac{T_{uj}^i}{100} \right)^4 - \left(\frac{T_{Ll}^i}{100} \right)^4 \right] + \alpha_j (T_{uj}^i - T_a) \right\} \\ (j = 1, 2, \dots, M) \dots\dots\dots(6) \end{aligned}$$

for the upper plate and

$$\begin{aligned} \dot{Q}_{Lj}^i(T_L^i, T_u^i) = \pi(r_{j+1} - r_j)^2 \left\{ C_o \varepsilon_L \varepsilon_u \sum_{l=1}^M \right. \\ \left. \times \varphi_{jl} \left[\left(\frac{T_{Lj}^i}{100} \right)^4 - \left(\frac{T_{ul}^i}{100} \right)^4 \right] + \alpha_j (T_{Lj}^i - T_a) \right\} \\ (j = 1, 2, \dots, M) \dots\dots\dots(7) \end{aligned}$$

for the lower one.

The set of the nonlinear equations was transformed into Eq. (1). The standard Newton-Raphson method was used to obtain the inverse relation.

IX. Footing

One of the blast furnace footings is a cylinder of radius R and height H over the soil surface. The assumptions made when solving the heat transfer equation in this subregion are as follows:

- the temperature field has symmetry about the furnace axis,
- the heat conductivity is uniform within the footing, and
- the heat loss through the bottom of the footing is negligible.

Because of the big radius, the temperature at the flank of the cylinder can be considered to be equal T_e . On the interface (*i.e.*, on the upper surface) the temperature can be approximated as

$$T^i(r, z=0) = T_e + \sum_{i=1}^{\infty} J_0 \left(\mu_i \frac{r}{R} \right) \dots\dots\dots(8)$$

The problem was solved analytically. After some simple algebra we arrived at the linear equation (1).

X. Parallel Flow Cooler

This type of cooler also consists of two circular horizontal plates but the cooling air flows parallel between supporting beams not radially as in the previous chapter. The flow of cooling medium in this type of cooler is shown schematically in Fig. 6. Conduction in the beams was taken into account by treating the beams as fins (*i.e.*, only the vertical tem-

Fig. 5. The flow of the cooling air in the radial underhearth cooler.

Fig. 6. The flow of the cooling medium in the parallel flow cooler.

perature gradient was assumed to be non-zero). All the surfaces were assumed to exchange heat by radiation and convection. These assumptions led to a non-linear integro-differential equation. This equation was solved numerically. Both interface heat flux and temperature were related by the nonlinear equation (1).

XI. Numerical Example

The mathematical model of heat transfer in blast furnace bottom and hearth was a basis of a computer program. This program was applied to establish temperature and heat flux distributions which are of interest when designing both blast furnace cooling and lining systems. The program consists of subroutines corresponding to all the subregions described above. New systems of cooling can be easily introduced by writing adequate subroutines. There is no need to make any changes in subroutines already written, when introducing a new cooling system. The results obtained with the aid of the program are shown in Figs. 7 and 8.

XII. Comparison of Numerical and Experimental Data

For the second example the temperature was measured in few points of an operating blast furnace. The temperatures calculated and observed are given in Table 1.

The cooling air temperature rise was calculated and measured. The results in both cases appeared to be 7 K.

WPW-65: coal lining (carbon blocks)
MWW-194: coal compensation lining
Fig. 7. Temperature distribution in the blast furnace hearth and bottom—the first example.

WPW-65: coal lining
WPG-20: semigraphite lining
WGH-67: raphite lining
MWW-192: coal compensation material
Measured temperatures: 1 1 058 K
2 401 K
3 350 K
Fig. 8. Temperature distribution in the blast furnace hearth and bottom—the second example.

Table 1. Calculated and observed temperatures in blast furnace bottom.

No.	Coordinates (m)		Temperature (K)	
	<i>r</i>	<i>Z</i>	Observed	Calculated
1	4.5	2 830	1 058	1 008
2	0.0	4 665	401	396
3	0.0	5 155	350	356

XIII. Conclusions

A mathematical model of heat transfer in blast furnace hearth and bottom and some blast furnace

cooling systems was developed. The model was based on an original method called solutions coupling method. A computer program was developed, capable to deal practically with any existing blast furnace cooling system. A new system can be included in this program without any significant additional programming effort. A fairly good agreement of computed and measured results was obtained.

Nomenclature

- A_i : unknown constants
- C_o : Stefan radiation constant
- $F(\alpha, \beta; \gamma, x)$: hypergeometrical function
- G : matrix linking temperature and heat flux at the interface (Eq. (1))
- H : footing height
- $J_0(r)$: Bessel function
- M : number of nodal points at the interface
- N : number of unknown coefficients A_i (Eq. (2))
- R : external radius of blast furnace bottom
- r : radial coordinate
- R_i : nodal points of a cylindrical net of mesh
- r_i : cylindrical net of mesh
- $\dot{Q}^i$: discretized interface heat flux
- $\dot{Q}_0^i$: heat flux at the interface when this surface is subject to zero temperature
- $\dot{q}$: heat flux per unit area
- T : temperature
- T^i : discretized interface temperature
- T_a : cooling air temperature
- T_e : environment temperature
- T_j : nodal value of temperature
- T_m : pig iron melting temperature
- T_{Lj}, T_{uj} : temperature in j -th sector of upper and lower plate of a radial underhearth cooler, respectively
- z : distance normal to interface
- α : convective heat transfer coefficient
- ε : emissivity
- φ_{ji} : view factor
- $\Gamma(x)$: gamma function
- λ : heat conductivity
- λ_r, λ_z : heat conductivity in r and z directions, respectively
- μ_i : eigenvalues defined as the positive roots of $J_0(\mu_i)=0$

Superscripts

- i : refers to the interface

Subscripts

- a : air
- e : environment
- i, j, l : summation or nodal values
- L, u : lower and upper plate of the radial underhearth cooler, respectively

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