Behavior of Gas Jet and Plume in Liquid Metal

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Synopsis

Measurement of the distribution of gas holdup in nitrogen injection into mercury was made by means of the electroresistivity probe technique with the aid of high speed data processing system. On the basis of the measurement near an orifice or a nozzle, two regimes of gas flow are distinguished: bubbling and jetting. In the jetting regime, the gas jet issuing from the tuyere into mercury becomes very thin and rises rapidly. The radial distribution of gas holdup expands with increasing vertical distance due to entrainment of the surrounding liquid. The vertical distance at which transition of flow behavior from jet to plume occurs is about 30–40 mm above the tuyere under the present experimental conditions. The effect of bath depth on the gas holdup is discussed.

The upward flow of gas-liquid mixture in the plume zone is analyzed on the basis of macroscopic mass and momentum balances. The calculated results of plume radius and gas holdup are compared with the experimental results. It is presumed that the circulating flow in the bath affects the dispersion of nitrogen.

I. Introduction

Techniques of gas injection into molten iron have made great progress in the pretreatment of hot metal, bottom blowing oxygen steelmaking process (Q-BOP, AOD), combined blowing oxygen steelmaking processes, and ladle metallurgy. Consequently, the advantages of gas injection into molten metal have been well recognized such as the enhancement of stirring of metal bath and the increase in refining rates.

Many studies have been made on the dispersion of gas injected into liquids, upward flow of gas-liquid mixture and circulating flow in the bath. However, since the phenomena occurring in the processes are very complicated, many problems remain unsolved.

In the present paper, the dispersion behavior of nitrogen injected into mercury was studied by means of the electroresistivity probe technique. The distribution of gas holdup in the bath was measured to study the behavior of gas jet in the vicinity of the gas exit (bubbling and jetting), entrainment of the surrounding liquid into the upward flowing gas-liquid mixture, and possibility of the occurrence of channeling phenomena. The upward flow of gas-liquid mixture is analyzed on the basis of macroscopic mass and momentum balances. By comparing the calculated result with the experimental one, the entrainment phenomena are discussed.

II. Experimental

The experimental apparatus used is shown in Fig. 1. The vessel containing mercury was a stainless steel column of 15 cm inner diameter and 40 cm high. Nitrogen was injected through an orifice or a nozzle positioned at the center of the vessel bottom. A hole bored in the bottom plate was used as the orifice.

The nozzle tip was machined to be tapered so that the inner and the outer diameters at the gas exit were nearly equal. The tip of the nozzle was located at 10 mm above the vessel bottom. The diameters of the gas exit were 1 and 2 mm.

An electroresistivity probe technique was employed to detect the gas (or bubble) in the bath. The molybdenum probe was insulated electrically except for the tip. The probe tip was of 0.2 mm diameter. The opening and closing of the electric circuit caused by passing of bubbles over the probe tip gave square waves of voltage. The present electronic equipment was designed to enter the data at 7 points in the bath simultaneously into a microcomputer. The minimum sampling time was about 4 ms and the time of measurement was 10–33 s.

The bath depth was 20–140 mm. The gas flow rate \( V_g \) was 140–2 200 cm\(^3\)/s under the conditions at the gas exit. The nominal Mach number \( M' \) was 0.5–3.5. The gas holdup defined below was mea-

![Experimental apparatus](image)

1. Gas cylinder
2. Float meter
3. Pressure gauge
4. Mercury vessel
5. Gas reservoir
6. Heat exchanger
7. Cyclone
8. Gas reservoir
9. Mercury manometer
10. Mercury reservoir
11. Chain block
12. Electroresistivity probe
13. Regulated DC power supply
14. Input interface
15. Micro-computer

\[ M' = \frac{V_g}{A a} \]

where, \( A \): the cross sectional area of the gas exit
\( a \): the sonic velocity at room temperature.

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† \( M' \) is defined as \( M' = \frac{V_g}{A a} \) where, \( A \): the cross sectional area of the gas exit
\( a \): the sonic velocity at room temperature.
sured at vertical distances, $h$, of 5 to 120 mm. Seven probes out of the fifteen probes installed at intervals of 5 mm were chosen for the measurement.

The local gas holdup $\phi_{loc}$ at each location of the probe tip is given by

$$\phi_{loc} = \frac{\Sigma t_B}{t} \tag{1}$$

where, $\Sigma t_B$: the sum of the periods during which the bubble occupies the probe tip

$\:t$: the total time of measurement.

### III. Experimental Results

Figure 2 shows an example of the radial distributions of gas holdup at various vertical distances $h$. The orifice used was of 1 mm diameter. It is seen that $\phi_{loc}$ is the maximum on the center axis of the vessel, and gradually decreases with increasing radial distance from the axis. In the range of $h > 40$ mm, $\phi_{loc}$ decreases in the central part of the vessel and increases in the outer part with increasing $h$. Thus, the injected gas disperses radially during the time of rising in the bath.

Contour maps of gas holdup are plotted in Fig. 3. In Fig. 3(a), the gas holdups for 1 and 2 mm diameter orifices are compared to examine the effect of the gas velocity (or the inertial force). In the range of $h < 30$ to 40 mm, the contour map for $d_0 = 1$ mm is different from that for $d_0 = 2$ mm. The local gas holdup $\phi_{loc} = 0.8$ is found in the case of $d_0 = 2$ mm. In the range of $h > 30$ to 40 mm, the two contour maps are almost the same. In Fig. 3(b), a comparison is made between the gas holdup distributions for a nozzle and an orifice. Little difference is observed, except in the vicinity of the gas exit, where expansion of the gas jet issuing from the orifice is somewhat larger than that for the nozzle.

As shown in Fig. 3(a), the local gas holdup $\phi_{loc}$ near the orifice is dependent on the orifice diameter. To investigate the phenomena more closely, the gas holdup on the center axis $\phi_{r=0}$ is plotted against $h$ in Fig. 4. At a gas flow rate, $V_g$, of 550 cm$^3$/s, in the range of $h < 30$ mm $\phi_{r=0}$ for $d_0 = 2$ mm (①) tends to increase with decreasing $h$. But $\phi_{r=0}$ for $d_0 = 1$ mm (④) in the same range shows a reverse tendency. This tendency is observed in the gas injection under the sonic conditions. In the range of $h > 30$ mm, $\phi_{r=0}$ increases with increasing gas flow rate (④③>③).

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**Fig. 2.** Radial distribution of gas holdup at various vertical distances from orifice ($d_0 = 1$ mm, $V_g = 550$ cm$^3$/s).

**Fig. 3(a).** Contour map of gas holdup.

**Fig. 3(b).** Contour map of gas holdup.

**Fig. 4.** Relation between gas holdup on center axis and vertical distance from orifice.
Figure 5 shows the effect of bath depth \( H \) on the dispersion of injected gas. As \( H \) becomes smaller, the gas becomes to be dispersed in a smaller region of the bath and the gas holdup decreases as a whole. It is clear that the gas holdup for \( H = 20 \) mm is very small. Although the sound produced by the gas injection for \( H < 40 \) mm was quite different from that for \( H > 40 \) mm, no abrupt change of gas holdup was observed at the bath depth of 40 mm.

**IV. Discussion**

1. **Jet and Plume**

   The motions induced by injection of a fluid into the same or another fluid are roughly classified into two regimes: jet and plume.\(^{15}\) The jet is a fluid motion induced by the inertial force of the injected fluid. The plume is used to represent a fluid motion induced by the buoyancy force acting on the injected fluid. In the case of gas injection into a liquid, the buoyancy force always contributes to the motion of gas-liquid mixture. However, since the inertial force of the injected gas in the sonic region is predominant in the vicinity of the gas exit, for simplicity the motion in that region is called "jet".*

   As shown in Figs. 3 and 4, while the distribution of gas holdup for \( h < 30 \) mm is affected by the gas inertial force, that for \( h > 30 \) mm is dependent only on the gas flow rate. In view of the gas dispersion, transition of flow behavior from jet to plume occurs at \( h \approx 30 \) mm under the present injecting conditions.

2. **Bubbling and Jetting in the Jet Zone**

   Previously,\(^{16}\) the behavior of gas jet injected into mercury was observed through a transparent bottom plate at the center of which an orifice was installed. Two phenomena were distinguished on the basis of spreading of bubble base attached to the plate. One is the formation of bubbles of various base diameters. This is called "bubbling". Another phenomenon is that an apparent coincidence between the bubble base diameter and the orifice diameter occurs over various time ranges. This is called "jetting". The bubbling-jetting transition was found to occur in the sonic region. However, the observation through the bottom plate could not reveal the behavior of jet inside the bath.

   The present measurement of gas holdup shown in Figs. 3(a) and 4 indicates that the gas holdup near the orifice in the sonic region is different from that in the subsonic region. To examine these phenomena in detail, rectangular waves of voltage measured at \( h = 5 \) mm and \( r = 0, 5 \) mm are compared in Fig. 6. Large waves are seen in Fig. 6a for \( M' = 0.5 \). Each wave indicates the formation of one bubble. The frequency of bubble formation can be obtained from the measurement at \( r = 0 \) mm and also at 5 mm. In the case of \( M' = 2.0 \), different phenomena are found in Fig. 6(b): in some time range, many small waves are seen at \( r = 0 \) mm, and yet no wave is detected at \( r = 5 \) mm. However, in another time range a large wave is shown to form simultaneously at \( r = 0 \) and 5 mm. These phenomena may be related to the two regimes of gas flow: bubbling and jetting.

   In the jetting regime the gas jet issuing from the gas exit is considered to become very thin and rise rapidly. With thinning of the jet, the probe tip at \( r = 0 \) mm becomes frequently to touch with mercury and to produce very small waves. This offers an explanation of the smaller value of gas holdup near the orifice for \( M' = 2.0 \) than that for \( M' = 0.5 \) in Figs. 3(a) and 4.

   It is presumed that the time fraction for bubbling corresponds to that for the occurrence of large waves and the time fraction for jetting to that for the occurrence of small waves. Hence, the time fraction for jetting \( R_j \) is defined as follows:

\[
R_j = \frac{\text{Sum of periods for occurrence of small waves}}{\text{Total time of measurement}} \quad \ldots\ldots(2)
\]

* Strictly speaking, the term 'buoyant jet' should be used.
In Fig. 7, \( R_j \) at two vertical distances is plotted against \( M' \). It is seen that \( R_j \) tends to increase with increasing \( M' \) in the sonic region. The previous result obtained by observing through the transparent vessel bottom is plotted in Fig. 7. The result is considered to be obtained at \( h=0 \) mm. Thus, it is seen that \( R_j \) decreases with increasing \( h \) due to the radial expansion of the gas jet.

3. Dimension and Gas Holdup of the Plume Zone*

Generally, the plume zone should be defined under the consideration of liquid motion. However, since the measurement of liquid velocity was not made in this study, the plume dimension is determined on the basis of the gas holdup distributions. Here, the plume radius \( r_p \), is defined as the radius of circle the cross section through which 90 % of the injected gas passes.

\[
0.9 = \int_0^R \frac{2\pi r \phi_{loc}(r) dr}{\int_0^R 2\pi r \phi_{loc}(r) dr}
\]

where, \( \phi_{loc}(r) \): the local gas holdup at \( r \)

\( R \): the vessel diameter.

For the derivation of Eq. (3), the gas velocity in the plume is assumed to be uniform in the radial direction.

The average gas holdup \( \phi \) of the cross sectional area of the plume is given by

\[
\phi = \frac{\int_0^{r_p} 2\pi r \phi_{loc}(r) dr}{\pi r_p^2}
\]

Typical examples of the plume radius \( r_p \), calculated from Eq. (3) are given in Fig. 8. As shown in the figure, while \( r_p \) for \( h < \sim 20 \) mm hardly changes with \( h \), \( r_p \) for \( h > \sim 20 \) mm increases in proportion to \( h \). The relation between \( r_p \) and gas flow rate \( V_g \) is shown in Fig. 9. It is seen from the figure that the effect of \( V_g \) on \( r_p \) is not large, and that the expansion of plume zone is only slightly dependent on \( V_g \).

Figure 10 shows the relation between \( \phi \) and \( h \). In the case of \( V_g=550 \) cm³/s, \( \phi \) decreases with increasing \( h \). At the gas flow rate of 2 200 cm³/s, \( \phi \) is small in

* Since the transition of jet to plume occurs at relatively short vertical distance from the gas exit, the term "plume zone" is used to represent the gas–liquid mixture zone.

4. Analysis of Plume by the Macroscopic Balance Method

1. Macroscopic Mass and Energy Balances

Since the difference between the densities of gas
and liquid is very large, the buoyancy force acting on the gas in the liquid is larger than the inertial force of the injected gas. Here, in taking a macroscopic momentum balance for the upward flow of the plume zone shown in Fig. 11, the contribution of the inertial force is neglected. In addition, it is assumed that there are no radial distributions of gas, liquid velocities and gas holdup in the plume zone.

The macroscopic mass balances for the gas and liquid phases are given by the following equations:

\[ \frac{dm}{dh} = \frac{d}{dh} (\pi r_{p}^{2} (1 - \phi) \rho g \mu_{s}) = \frac{5}{3} \rho_{t} \{ V_{p} (\rho_{l} - \rho_{g}) g \}^{1/3} \rho_{l}^{2/3} h^{2/3} \] .................................(6)

where, \( \rho \): the density
\( u \): the velocity
\( m \): the mass flow rate in the plume
\( k_{e} \): the entrainment coefficient
\( g \): the acceleration of gravity

Suffixes \( g, l \): gas and liquid, respectively.

The macroscopic momentum balance for the plume is

\[ \frac{d}{dh} \left( \pi r_{p}^{2} (1 - \phi) \rho g \mu_{s} \right) = \pi r_{p}^{2} \rho_{t} g + \frac{d m}{dh} \] .................................(7)

where, \( v_{l} \): the velocity of the entrained liquid.

The first term of the right hand side of Eq. (7) is the buoyancy force.

The macroscopic mechanical energy balance for the plume is expressed as

\[ E_{w} = \frac{1}{2} (u_{l} - v_{l})^{2} \frac{dm}{dh} \] .................................(8)

where, \( E_{w} \): the energy dissipation due to the liquid entrainment.

By comparing Eq. (8) with Eq. (7), \( E_{w} \) is given by

\[ E_{w} = \frac{1}{2} (u_{l} - v_{l})^{2} \frac{dm}{dh} \] .................................(9)

The ratio of \( u_{l}/u_{s} \) in Eq. (8) is the fraction of stirring power of the injected gas effectively used for generating the upward flow.

There is an additional relation among \( u_{s}, u_{l} \) and \( \phi \).

\[ u_{l} - u_{s} = \frac{U_{r}}{\phi} - u_{s} = \frac{u_{m0}}{1 - \phi} \] .................................(10)

where, \( U_{r} \): the superficial gas velocity in the plume
\( u_{m0} \): the rising velocity of a single bubble.

The following equation is used for the calculation of \( u_{m0} \).

\[ u_{m0} = \sqrt{0.5 d_{p} \rho_{l} g} \] .................................(11)

where, \( d_{p} \): the bubble diameter, which is estimated from the correlation obtained previously.

\[ d_{p} = 0.091 \left( \frac{a}{\rho_{l}} \right)^{1/2} \frac{U_{r}^{0.44}}{g} \] .................................(12)

where, \( a \): the surface tension.

The variables in Eq. (12) are represented by C.G.S. units.

Equations (5) to (7) (or (8)) and (10) are solved numerically as simultaneous equations of \( r_{p}, \phi, u_{l} \) and \( u_{s} \). The unknown parameters, \( k_{e} \) and \( v_{l} \), can be determined from the comparison of the calculated results with the experimental ones of \( r_{p} (h) \) and \( \phi (h) \). In the calculation, the boundary conditions for \( u_{s} \) and \( u_{l} \) at \( h=0 \) mm are given as follows: The plume radius \( r_{p0} \) at \( h=0 \) is obtained by extrapolating the experimental value of \( r_{p} \). The liquid velocity \( u_{l0} \) at \( h=0 \) is determined so as to agree calculated plume radius near the gas exit with the experimental one.

2. Comparison of the Experimental Results and the Calculated Ones

The experimental results were compared with the calculated ones for the plume radius and are shown in Fig. 8. If the liquid velocity, \( v_{l} \), of the entrained liquid is assumed to be zero, the experimental plume radius agrees with the one calculated with the entrainment coefficient \( k_{e} \) of 0.25. Likewise, the following combinations of \( k_{e} \) and \( v_{l} \) are obtained: \( v_{l}=0.25 u_{l}, k_{e}=0.29; v_{l}=0.5 u_{l}, k_{e}=0.36; v_{l}=0.75 u_{l}, k_{e}=0.47. \)

Since the definite value of \( v_{l} \) is not known, one cannot determine the value of \( k_{e} \) by the comparison shown in Fig. 8.

In Fig. 9, the theoretical relation between the plume radius and the gas flow rate is shown by the solid line. As shown in the figure, the dependence of the plume radius on the gas flow rate is not large. From Figs. 8 and 9, it is clear that the expansion of the plume zone due to the entrainment of the surrounding liquid can be explained by the present mathematical model.

In Fig. 10, the calculated gas holdup \( \phi \) is plotted against the vertical distance \( h \). In the range of \( h>40-60 \) mm, the calculated results, for \( v_{l}=0.75 u_{l} \) and \( k_{e}=0.47 \) agree well with the experimental ones. Thus, it is presumed that the dispersion of gas in
mercury under the present injecting conditions is influenced by the circulating flow in the bath. The disagreement in the range of \( h < 40 \sim 60 \text{ mm} \) is due to the effects of bubbling–jetting phenomena and the inertial force of the injected gas on the gas dispersion in the initial jet zone.

The volumetric flow rate \( V_i \) of mercury in the plume is obtained with the calculated values of plume radius \( r_p \), gas holdup \( \phi \) and liquid velocity \( u_l \) from the following equation:

\[
V_i = \pi r_p^2 (1-\phi) u_l \quad \text{.................(13)}
\]

The liquid flow rate \( V_l \) calculated from Eq. (13) is shown in Fig. 12. Three sets of \( k_p \) and \( v_l \) are used in the calculation. It is to be noted that \( V_i \) is proportional to \( r_p \), roughly to the one-third power. This is in accord with the previous studies.\(^5,7\) The liquid flow rate \( V_l \) is strongly dependent on \( v_l \). This dependence should be studied in further detail in the near future.

Previously,\(^{10}\) a macroscopic balance of mechanical energy for the plume was taken on the assumption that the cross sectional area of the plume \( A_p \) does not change with \( h \). The result for \( V_i \) is given by

\[
V_i = 1.17 \left( \frac{V_{g,m} g H A_p^2}{\rho L} \right)^{0.33} \quad \text{.................(14)}
\]

where, \( V_{g,m} \): the gas flow rate at a logarithmic mean pressure \( H \): the bath depth.

The variables in Eq. (14) are expressed by the units of meter and second.

The liquid flow rate \( V_l \) is calculated from Eq. (14) and is plotted by a dotted line in Fig. 12. For the calculation of \( V_l \), the plume radius \( r_p \) at the surface obtained in the present study is used to estimate \( \phi \). The value of \( V_l \) calculated from Eq. (14) agrees with that for \( v_l = 0.5 u_l \). This is due to the fact that the mechanical energy balance of the previous study was made by assuming \( E_s = 0 \) and \( v_l = 0 \). This assumption corresponds to that of \( v_l = 0.5 u_l \) in the present study. It seems that the assumption of constant cross sectional area of the plume in the previous study does not give rise to a serious error in \( V_i \).

From the comparison of the calculated and experimental results shown in Fig. 10, the values of \( k_p = 0.47 \) and \( v_l = 0.75 u_l \) are obtained. The plume radius \( r_p \) and the gas holdup \( \phi \) are dependent on the definition of the plume zone. The present data of gas holdup are not sufficient enough to define the plume zone. Hence, it is thought that more detailed studies are necessary to make clear the velocities of the plume and the surrounding liquid. Moreover, the effect of liquid properties on the entrainment coefficient should also be investigated.

5. Effect of Bath Depth on the Distribution of Gas Holdup

Figure 5 shows that the gas holdup for \( H = 20 \text{ mm} \) is much smaller than those for \( H = 80 \) and \( 40 \text{ mm} \). This indicates that in the case of \( H = 20 \text{ mm} \) the injected gas rises very rapidly and channeling might have occurred. As mentioned earlier, however, the present measurement of gas holdup could not give the critical conditions of bath depth and gas flow rate for the occurrence of channeling.

Katoh et al. gave the critical bath depth \( H^* \) (m) for channeling,\(^{22}\)

\[
H^* = 0.18 \left( \frac{\rho_g Q^2}{\rho_l d_n^3} \right)^{1/3} \quad \text{.................(15)}
\]

where, \( \rho_g, \rho_l \): the densities of gas and liquid (kg/m\(^3\)) \( Q \): the gas flow rate (Nm\(^3\)/min) \( d_n \): the nozzle diameter (m).

The critical bath depth for nitrogen injection into mercury at \( V_g = 960 \text{ cm}^3/\text{s} \) and \( d_n = 1 \text{ mm} \) calculated from Eq. (15) is about 120 mm. As shown in Fig. 5, channeling did not occur at shallower bath depths. Thus, Eq. (15) cannot explain the present experimental results.

V. Conclusion

Nitrogen was injected into mercury through an orifice or a nozzle positioned at the center of vessel bottom. Gas holdup in the jet and plume zones was measured by means of the electroresistivity probe technique. The results were compared with the calculated ones on the basis of macroscopic mass and momentum balances. The conclusions obtained are summarized as follows:

1. The local gas holdup shows the maximum on the center axis of the vessel and decreases with increasing radial distance.

2. In gas injection under the subsonic conditions, the gas holdup \( \phi_{r<0} \) on the center axis decreases monotonically with the increase of vertical distance \( h \) from the gas exit.

3. In the sonic region, the gas jet issuing from the gas exit is observed to become very thin and jetting occurs. It is found that \( \phi_{r<0} \) shows the maximum at a distance of \(~30 \sim 40 \text{ mm} \) from the exit and then decreases with increasing \( h \) in the range of \( h > ~30 \sim 40 \text{ mm} \).

4. The distribution of gas holdup is dependent only on the gas flow rate in the range of \( h > ~30 \sim 40 \text{ mm} \), below which it is dependent also on the diameter of the gas exit. It is presumed that the transition of flow behavior from jet to plume occurs.
The plume expands with increasing $h$ by entraining the surrounding liquid. The dimension of the plume zone is influenced only a little by the gas flow rate.

(6) Dispersion of gas injected into liquid and upward flow of gas-liquid mixture are described quantitatively on the basis of macroscopic mass and momentum balances.

(7) In the case of a shallower bath depth ($h \approx 20$ mm), gas disperses in a smaller region of the bath and the gas holdup decreases as a whole. It is thought that in this case channeling phenomena occurs.

REFERENCES

Appendix
Comparison of the Plume Zones

Several investigators directed their attention to the size of the plume zone. In the previous studies, however, no general definition of plume radius has been given. The definitions used by the present and previous studies are compared as follows:

The radial distribution of gas holdup obeys the Gaussian distribution:

$$\phi_{\text{gas}} = \phi_{\text{gas}}^0 \exp \left( -\frac{r^2}{r_{1/2}^2/\ln 2} \right) \quad \text{(A-1)}$$

where, $r_{1/2}$: the plume half radius, namely the radial distance at which $\phi_{\text{gas}}$ becomes $\phi_{\text{gas}}^0/2$.

Substituting Eq. (A-1) into Eq. (3), it gives

$$r_p = 1.82r_{1/2} \quad \text{(A-2)}$$

which is the definition of plume radius of the present study.

Kawakami et al.\(^2\) showed that the radial distribution of bubble frequency follows the Gaussian distribution and defined the plume radius $r_{p,f}$ by

$$r_{p,f} = 2.55r_{1/2,f} \quad \text{(A-3)}$$

where, $r_{1/2,f}$: the plume half radius defined on the basis of bubble frequency.

Ebneh and Pluschke\(^2\) measured the velocity distribution in the plume, and expressed the plume radius $r_{p,v}$ as

$$r_{p,v} = 1.20r_{1/2,v} \quad \text{(A-4)}$$

where, $r_{1/2,v}$: the plume half radius defined by the velocity distribution.

According to Takelli and Maxwell\(^2\) the relation between $r_{1/2}$ and $r_{1/2,v}$ is

$$r_{1/2,v} = 0.7r_{1/2,v} \quad \text{(A-5)}$$

Combining Eqs. (A-4) and (A-5) gives

$$r_{p,v} = 1.72r_{1/2} \quad \text{(A-6)}$$

The present definition of plume radius agreed approximately with that of Ebneh et al. If $r_{1/2} \approx r_{1/2,v}$, the definition of Kawakami et al. gives much larger plume radius than the present one.