Theoretical Analysis of 3-roll Rolling Process by the Energy Method

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Synopsis
The 3-roll rolling process is analyzed by the energy method. A simplified admissible velocity field defined by three variable parameters is introduced. The solutions are derived by minimizing the consumed energy rate. Numerical calculations are done for hexagon-flat pass and the results are compared with the experiments. Both results on the roll torque and reduction in cross-sectional area agree satisfactorily.

Key words: rolling; bar and rod; simulation; 3-roll; energy method; roll torque; reduction in area.

I. Introduction
The 3-roll rolling process, which is well known as Kocks or Properzi process, is one of the useful processes for producing wire, rod, and bar. This process has an advantage of capability of high reduction per pass, low rolling pressure, high dimensional accuracy of products, and lower consumption for a given production rate. However, the deformation of material in 3-roll bar rolling is complicated in three dimensions. Therefore, the method of theoretical analysis is not well developed practically, comparing with the flat rolling.

This paper presents a simple method for theoretical analysis of the deformation in 3-roll rolling on the basis of the energy method. The analysis is applied to hexagon-flat pass, which is a typical rolling pass of 3-roll rolling pass schedule. The theoretical results are compared with the experimental results.

II. Theoretical Method
As the basic formulas of the energy method, the upper bound theorem in the 3-roll rolling is expressed by Eqs. (1) and (2), where is the energy consumption rate in a velocity field.

\[ 3 \cdot T \cdot \omega \leq \Phi \] .............................................(1)

\[ \Phi \equiv \int \sqrt{3} k_{eq} dV + \int k \cdot dV \cdot d^2 + \int \tau \cdot dV \cdot dS + T_b \cdot V_b + T_f \cdot V_f \] .............................................(2)

where, 
- \( k \): yield stress in pure shear
- \( S_c \): contact area of roll and material
- \( T \): roll torque for one of the rolls
- \( T_b, T_f \): back and front tension
- \( V \): volume of deformation zone
- \( V_b, V_f \): speed of material at the entry and exit of roll bite
- \( \Delta V \): velocity difference between roll and material on the contact surface
- \( \Delta V_1 \): velocity discontinuity
- \( l' \): surface of velocity discontinuity
- \( \varepsilon_{eq} \): equivalent strain rate
- \( \tau_f \): frictional shearing stress
- \( \omega \): roll angular velocity.

In the energy method, an admissible velocity field is specified with some parameters. An upper bound solution is derived for these parameters which minimize \( \Phi \).

For theoretical analysis, the axes of co-ordinate are taken with the origin at the geometric center of cross section of material entering the roll bite, with X-axis in the rolling direction, Y-axis parallel and Z-axis normal to the roll surface (Fig. 1).

1. Velocity Field
To assume the velocity field, the following assumptions are applied.
(1) The cross section perpendicular to the rolling direction (X-axis) remains as a plane.
(2) The contour of free surface at the gap between two rolls is similar to that before rolling.
(3) Elastic deformation of the roll and material is ignored.
(4) Rolled material is rigid, perfectly plastic, and notations for \( V_X \).

Fig. 1. Definition of the co-ordinate for theoretical analysis.
obeys Mises’ criterion.

5) The frictional shearing stress on the contact surface of roll and material is \( f = m \cdot k \); the shear factor \( m \) is constant.

In hexagon-flat pass, the assumption (2) means that the contour of free surface in any \( X \)-cross section remain straight and inclining to \( Y \)-axis in angle of \(-60^\circ\).

On these assumptions, the admissible velocity components (\( V_x \), \( V_y \), and \( V_z \)) are considered in sequence. Because of the symmetry of material and roll caliber, one sixth portion of the deformation zone is analyzed.

1. Velocity Component \( V_x \)

To determine the velocity component parallel to the rolling direction, two typical velocity fields are assumed and combined.

(1) Velocity Field A

In the velocity field A, it is assumed that spreading of material do not occur. The material drafted by rolls flows only into the rolling direction. Then the contour of free surface is equal to that before rolling (dotted line in Fig. 1).

The velocity component parallel to the rolling direction is determined by;

\[ V_{Ya} = 0 \]

and from the assumption (1) and the constancy of volume;

\[ V_{Ax} \cdot S_X = U \cdot S_0 \]

\[ V_{Ax} = \frac{S_0}{S_X} U \]

where, \( S_0 \): cross-sectional area of material before rolling

\( S_X \): cross-sectional area of material at \( X \) in roll bite

\( U \): speed of material entering the roll bite.

(2) Velocity Field B

In the velocity field B, it is assumed that the material drafted by rolls flows only lateral to the rolling direction. Therefore the velocity component in the rolling direction is constant;

\[ V_{xB} = U \]

(3) Combined Velocity Field

It is assumed that the velocity fields A and B exist simultaneously. The velocity component in the rolling direction is determined by combination of \( V_{Ax} \) and \( V_{xB} \) with a parameter \( a \);

\[ V_x = a \cdot V_{Ax} + (1-a) V_{xB} \]

\[ = \left\{ a \left( \frac{S_0}{S_X} - 1 \right) + 1 \right\} \cdot U \]

Furthermore, the parameter \( a \) is considered to change along the contact arc according to the following formula;

\[ a = a_0 + a_1 \left( 1 - \frac{x}{l_d} \right)^2 \]

where, \( l_d \): the maximum contact length

2. Velocity Component \( V_y \)

The velocity component in the direction parallel to one roll surface \( V_y \) is considered separately in two zones. One is the zone under roll contact area, and the other is the remaining zone. The middle of Fig. 2 shows one sixth portion of deforming material, and the separated zones 1 and 2 are shown in the bottom.

(1) Zone 1

If \( V_{y1} \) is independent of Z-ordinate, constancy of the volume for a small element shown in the top of Fig. 2 leads to

\[ V_x \cdot S_{xr} = (V_x + dV_x) (S_{xr} + dS_{xr}) + h V_{y1} dX, \]

where, \( h \): the height in Z direction

\( S_{xr} \): the area of small element for \( X \)-cross section.

Omission of quadratic differentials results in

\[ V_{y1} = \frac{1}{h} \left( V_x \frac{\partial S_{xr}}{\partial X} + S_{xr} \frac{\partial V_x}{\partial X} \right) \]

By using the notations shown in the top of Fig. 2, \( S_{xr} \) is determined by;

\[ S_{xr} = (h_c + h) y/2 \]

\[ = (2 - h_c - y/\sqrt{3}) y/2 \]

Fig. 2. Notations for definition of velocity components.
\[
\frac{\partial S_{xy}}{\partial x} = \frac{\partial h_c}{\partial x}. \quad \text{........................(12)}
\]

Therefore \(V_{y1} = 0\) at \(y = 0\).

\(2)\) Zone 2

In zone 2, the direction of combined velocity is assumed to be parallel to the boundary \(Y_1(y = \sqrt{3}z)\), which is the interface between two of one sixth portions. Therefore,
\[
V_{y2} = \sqrt{3} V_{z2}. \quad \text{........................(13)}
\]

The volume strain rate is equal to zero;
\[
\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0 \quad \text{........................(14)}
\]

and \(V_{z2}\) is assumed to be independent of \(z\),
\[
\frac{\partial V_{z2}}{\partial z} = \frac{1}{\sqrt{3}} \frac{\partial V_{z2}}{\partial y} = 0. \quad \text{........................(15)}
\]

From Eqs. (14) and (15),
\[
\frac{\partial V_{z2}}{\partial y} = -\frac{\partial V_x}{\partial x} \quad \text{........................(16)}
\]

\[
V_{y2} = -\frac{\partial V_x}{\partial x} + C. \quad \text{........................(17)}
\]

In the zones 1 and 2, from the continuity of velocity component lateral to the boundary, \(V_{y2} = V_{y1}(= W)\);
\[
V_{y2} = W - \frac{\partial V_x}{\partial x} (y - W) \quad \text{.....................(18)}
\]

\(3)\) Velocity Component \(V_z\)

Also \(V_z\) is considered separately for two zones.

\(1)\) Zone 1

As the volume strain rate is zero,
\[
\frac{\partial V_x}{\partial x} + \frac{\partial V_{y1}}{\partial y} + \frac{\partial V_{z1}}{\partial z} = 0. \quad \text{........................(19)}
\]

From Eq. (10)
\[
\frac{\partial V_{y1}}{\partial y} = \frac{1}{h} \left( V_x \frac{\partial}{\partial x} \left( \frac{\partial S_{xy}}{\partial x} \right) + \frac{\partial S_{xy}}{\partial x} \frac{\partial V_x}{\partial x} \right) + \frac{1}{h^2} \frac{\partial^2 h}{\partial x^2} \left( V_x \frac{\partial S_{xy}}{\partial x} + S_{xy} \frac{\partial V_x}{\partial x} \right) \quad \text{........................(20)}
\]

Therefore, from Eqs. (10) to (12),
\[
\frac{\partial V_{y1}}{\partial y} = \frac{1}{h} \left( V_x \frac{\partial h_c}{\partial x} + h_c \frac{\partial V_x}{\partial x} + \frac{1}{\sqrt{3}} V_{y1} \right). \quad \text{........................(21)}
\]

By integrating Eq. (19) by using Eq. (21) and from the boundary condition (\(V_{z1} = V_{y1}/\sqrt{3} \) at \(z = y/\sqrt{3}\)),
\[
V_{z1} = -\left( V_x + \frac{1}{h} \left( V_x \frac{\partial h_c}{\partial x} + h_c \frac{\partial V_x}{\partial x} + \frac{1}{\sqrt{3}} V_{y1} \right) \right) \times \left( z - \frac{y}{\sqrt{3}} \right) + \frac{1}{\sqrt{3}} V_{y1}. \quad \text{........................(22)}
\]

The value of \(V_{z1}\) is determined within the limit of \(y/\sqrt{3} \leq z \leq \sqrt{3} h_c\).

\(2)\) Zone 2

From Eqs. (11) and (18),
\[
V_{z2} = \frac{1}{\sqrt{3}} W - \frac{1}{\sqrt{3}} \frac{\partial V_x}{\partial x} (y - W). \quad \text{........................(23)}
\]

The value of \(V_{z2}\) is determined within the limit of \(y/\sqrt{3} \leq z \leq \sqrt{3} A/4\).

\(4)\) Neutral Point

The velocity components are determined in the above procedure. To determine the speed of material entering the roll bite, \(U\) in Eq. (7), the co-ordinates of the neutral point are assumed on the contact area in terms of a parameter \(\beta\).
\[
x_N = \beta \cdot l_s
\]
\[
y_N = 0
\]
\[
z_N = h_c (x = x_N) \quad \text{........................(24)}
\]

On this point, the velocity of material is equal to the roll speed \(V_r\). Because of \(V_y = 0\) at \(y = 0\),
\[
V_r = V_{x_N} \cos \theta_N - V_{z_N} \sin \theta_N \quad \text{........................(25)}
\]

where, \(\theta_N\): the neutral angle. Therefore,
\[
\sin \theta_N = 2 \cdot (l_d - X_N)/D_r \quad \text{........................(26)}
\]

where, \(D_r\): the diameter of roll.

From Eqs. (7), (22) and (25), the velocity \(U\) is determined.

\(2)\) Calculation

Function \(\Phi\) (Eq. (2)) is calculated by the velocity components assumed above, in terms of three parameters \((a_0, a_1, \text{and } \beta)\).

\(1)\) Energy Consumption Rate for Deformation

The first term of Eq. (2) represents an energy consumption rate for deformation of the material. The equivalent strain rate is determined by
\[
\varepsilon_{eq} = \left[ \frac{2}{3} (\varepsilon_x^2 + \varepsilon_y^2 + \varepsilon_z^2 + \varepsilon_{xy}^2 + \varepsilon_{yz}^2 + \varepsilon_{xz}^2) \right]^{1/2}.
\]

\(2)\) Energy Consumption Rate on Velocity Discontinuity

The second term of Eq. (2) represents an energy consumption rate on the surface of velocity discontinuity. There are two surfaces of the velocity discontinuity in this velocity field. One is the x-cross section at the entry of roll bite, and the other is the interface between zones 1 and 2.

The values of velocity discontinuity are \(V_{y2} + V_{z2}\) and \(V_{z1} - V_{z2}\), respectively.

\(3)\) Energy Consumption Rate on Contact Surface

The third term of Eq. (2) represents an energy consumption rate on the contact surface between roll and material. From the assumption (5), the shearing stress on the velocity difference between roll and ma-
material on the contact surface is $|\sqrt{V_x^2 + V_z^2} - V_r|$.

4. Energy Consumption Rate by Tension

The remaining terms of Eq. (2) represent energy consumption rates by front and back tension. From the determined velocity field, the velocity of material at the entry and exit of roll bite is $U$ and $U'$, respectively, where $S_i$ is the cross-sectional area at the exit of roll bite. The tensile forces are given as the rolling condition.

5. Numerical Procedure

The procedure for numerical solution by computer is shown in Fig. 3. First, reasonable values of parameters ($\alpha_0$, $\alpha_1$, and $\beta$) are assumed. Next, the profile of material in the roll bite is determined by the assumed velocity field. Then the function $T_b$ (Eq. (2)) is calculated by numerical integration. Calculation is repeated with variation of the values of parameters. A set of parameters, which minimize $\Phi$, should be the correct answer.

6. Detail Profile

If the velocity field obtained above is used, two types of errors arise. One is the fact that the directions of roll and combined vector of velocity components $V_x$ and $V_z$ are different on the contact area between roll and material, except on the line of $y=0$. The other is the fact that the contour of free surface at the gap between two rolls is not similar to that before rolling; barrel occurs in calculation. This is not consistent with the assumption (2). However, differences are less than 1.0%. Therefore, these errors are negligible.

III. Experimental Procedure

To verify the theoretical results, rolling experiments are done.

Figure 4 shows the outline of an experimental 3-roll rolling mill. It consists of three disc type rolls, which are set at an angle of 120° to each other with the axis of rolls on one plane. Each roll is driven individually by a DC motor. In this experiment, roll speeds are fixed constant. Roll separating force and roll torque are measured for each roll by a load cell and a torque gauge. They are stored automatically in a personal computer.

Pure Pb and Pb-2% Sb alloy are used as the model material of hot steel and hot aluminum. The shape of hexagon is defined by the shape factor $l/A$, where $l$ is the length of the shorter side, and $A$ is the length of one side of a regular triangle made by extending longer sides. The magnitude of draft is defined by the size ratio $D_0/d_0$, where $D_0$ and $d_0$ are the diameters of inscribed circle on the specimen and roll caliber, respectively (Fig. 5).

Rolling experiments are carried out by varying $l/A$ and $D_0/d_0$. Table 1 shows the experimental condition.
IV. Result and Discussion

Theoretical and experimental results are compared. In theoretical calculation, the yield stress \( k \) is 9.60 MPa for pure Pb and 15.97 MPa for Pb-2\%Sb alloy. These values are determined as a mean yield stress from a compression test. Shear factor \( m \) is fixed at 1.0 on assumption of sticking friction. Theoretical calculation and experiment are done without tension.

In the following figures, plots show the experimental results and solid lines show the theoretical results.

1. Roll Torque

Figure 6 shows the variations of roll torque \( T_q \) with the size ratio \( D_0/d_0 \) by taking \( A \) as the parameter, when rolling material is pure Pb and \( l/A \) is 0.25. The theoretical result slightly overestimated for a small size ratio and underestimated for a large size ratio, comparing to the experimental result. This may arise from the assumption of constant yield stress without taking into consideration of material work-hardening; the actual yield stress is slightly higher for a large size ratio and lower for a small size ratio than the assumed value. However, theoretical and experimental values are in good agreement. The characteristics of roll torque to size ratio for each size of specimen A are well simulated.

Figure 7 shows a similar result by taking the shape factor \( l/A \) as the parameter, when the material is Pb-2\%Sb alloy and \( A \) is 40 mm. Similarly to Fig. 6, the theoretical result slightly overestimated for a small size ratio and underestimated for a large size ratio, comparing to the experimental result.

Though it is thought that a improved selection of the yield stress is necessary, the present energy method can simulate the effects of size ratio, size and shape factor of material, and type of material on roll torque satisfactorily.

2. Reduction in Cross-sectional Area

Figure 8 shows the variation of reduction in cross-sectional area \( R_e \) with the size ratio \( D_0/d_0 \) by taking \( A \) as the parameter, when the material is pure Pb and \( l/A \) is 0.25. Theoretical and experimental results are very close. It is thought that the assumption of \( m = 1.0 \) is valid.

Figure 9 shows a similar result by taking \( l/A \) as the parameter, when the material is Pb-2\%Sb alloy and \( A \) is 40 mm.

Materials are thought to affect hardly the deformation behavior in this theory because of the assumption (4). Therefore, theoretical reduction in cross-sectional area can be determined independently of material. In comparison of the circles in Fig. 8 with the triangles in Fig. 9, which are the results of \( A \) is 40

| Roll shape  | Flat  |
| Dia. \( D_r \) (mm) | 150   |
| Speed \( V_r \) (m-min\(^{-1}\)) | 9.5   |

| Specimen material | Pb, Pb-2\%Sb |
| Shape | Hexagon |
| Size \( A \) (mm) | 40, 50, 60 |
| Shape factor \( l/A \) | 0.16, 0.25, 0.33 |
| Temperature (°C) | 20±3 |
| Lubricant | Magic ink |
mm and \( l/A \) is 0.25, the difference of \( R_e \) between two materials is not clear. It is thought that the assumption (4) is valid in the case of reduction in cross-sectional area.

Therefore this theoretical analysis can adequately simulate the effects of size ratio and shape factor on reduction in area.

V. Conclusion

For theoretical analysis of the rolling characteristics of 3-roll rolling process, the energy method using a simplified velocity field is introduced. Three parameters are used to specify the admissible velocity field and to minimize the function \( \Phi \). Theoretical results on roll torque and reduction in cross-sectional area are very close to experimental results. Therefore, the present energy method can simulate the roll torque and reduction in cross-sectional area of 3-roll rolling.

REFERENCES