Mathematical Model of Blast Furnace*

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Synopsis

Mathematical model for the upper part from the tuyere level of blast furnace has been proposed in this paper. Overall reaction rates of indirect reduction of iron ore by CO and H₂, carbon solution, direct reduction of molten wustite by coke, decomposition of limestone, and reaction between coke and steam have been incorporated in this model. And also, on the basis of the assumption that the equilibrium of the reaction between CO and H₂O holds in the gas phase in the furnace, the change in moles of H₂ caused by this reaction has been taken into account in this model.

This model consists of nine differential equations and two algebraic equations, and Runge-Kutta-Gill method has been employed in obtaining numerical solutions for this model. Where, initial conditions necessary to conduct the calculation have been determined by taking the overall material and heat balances around the whole furnace.

On the basis of this model, longitudinal distributions of temperatures of gas and solid particles, molar fractions of CO, CO₂, H₂, and H₂O, fractional reduction of iron ore, fractional decomposition of limestone, volume rate of flow and density of gas, bulk density of solid particles, and gaseous pressure under practical operating conditions in blast furnace have been determined quantitatively by digital computer.

Calculated results have been in good agreement with the trends illustrated in terms of observed data by others. Consequently, it has been found that this model may be available for predicting the situations in blast furnace.

I. Introduction

In order to carry out the efficient operation and the appropriate process control of blast furnace, it is necessary to derive the mathematical model, from which the variations of the situations in the furnace associated with the changes of operating conditions can be predicted precisely.

Recently, positive attempts to develop the mathematical model expressing the complex phenomena which take place in blast furnace have been made in connection with the computer control of the furnace. And, many theoretical investigations¹⁾⁻⁶⁾ on the modeling for the blast furnace operation were presented, and as a result of these efforts, statistical model resting on the previous operating data and overall balance model coupled with thermodynamic calculations were proposed in the past.

Namely, Bogdandy¹⁾ developed the mathematical model which was taken into account of reduction rate of hematite pellets by H₂ and regarded blast furnace as countercurrent moving bed. Ridgion,²⁾ Wartmann,³⁾ Staib,⁴⁾ Beer,⁵⁾ and Flierman et al.⁶⁾ proposed the reaction-zone model in which blast furnace was divided up some reaction zones and then theoretical calculations were performed.

Also, the experimental results relating to the longitudinal distributions of temperature and com-

positions of blast furnace gas or of fractional reduction of iron ore in the upper part from the tuyere level were presented.

Mathematical model which was taken into account of six overall reaction-rates of indirect reduction of iron ore by CO, solution-loss reaction, direct reduction of molten wustite by solid coke, decomposition of limestone, indirect reduction of iron ore by H₂, and reaction between coke and steam, and combined with the change in moles of H₂ in reaction between CO and steam was proposed by the author, et al.¹¹⁾⁻¹⁴⁾

In addition, on the basis of the reaction rate of desulphurization of pig iron and on the transient alterations of level and temperature of molten materials in the hearth, mathematical model for the desulphurization in the hearth was developed, and the direct reduction of silica in slag in the upper zone from the tuyere level was analysed theoretically,¹⁵⁾ but this study awaited more theoretical developments.

This report is the summarization of five papers (in Japanese)¹¹⁾⁻¹⁴⁾ on the mathematical model of the upper part from tuyere level mentioned above. Longitudinal profiles of temperatures of gas and solid particles, molar fractions of CO, CO₂, H₂, and H₂O, fractional reduction of iron ore, fractional decomposition of limestone, volume rate of flow and density of gas and gaseous pressure, and bulk density of solid particles can be determined by this model, and all these variables are averaged over the cross section of the furnace.

By the application of this model to the actual operating conditions in the existing blast furnaces in Japan, the situations in the furnaces have been estimated quantitatively.

II. Overall Reaction Rate

1. Indirect Reduction of Iron Ore by CO

In the reaction described as Eq. (1), overall reaction rate R_1^* per unit volume of bed in the blast furnace is defined as Eq. (2).

$$R_1^* = -dn_{\text{CO}}/V_B d\theta \dots\dots\dots(2)$$

Assuming that three steps of diffusion through gas film surrounding the particle, intraparticle diffusion and chemical reaction on the reaction interface occur steadily and in succession during reaction, we can

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represent R_1^* as Eq. (6) in terms of Eqs. (3)–(5).

Rate of gas film diffusion :

$$\pi d_p^2 \varphi^{-1} N k_{f1} \cdot 273(x - x_o)/22.4 t \dots\dots\dots(3)$$

Rate of intraparticle diffusion :

$$\pi d_p^2 \varphi^{-1} N D_{s1} \cdot 273(x_o - x_i)/22.4 t (r_o - r_i) r_o \dots\dots\dots(4)$$

Chemical reaction rate :

$$\pi d_p^2 \varphi^{-1} N k_1 (1 + 1/K_1)(r_i/r_o)^2 \cdot 273(x_i - x_e)/22.4 t \dots\dots\dots(5)$$

$$R_1^* = \frac{\pi d_p^2 \varphi^{-1} N}{\left\{ \frac{1}{k_{f1}} + \left(\frac{d_p}{2} \right) \left\{ (1 - f_s)^{-1/3} - 1 \right\} / D_{s1} \right.} \cdot \frac{273(x - x_e)/22.4 t}{\left. + \left\{ (1 - f_s)^{2/3} k_1 (1 + 1/K_1) \right\}^{-1} \right\}^{-1}} \dots\dots\dots(6)$$

Numerical value of N varies with operating conditions, and it is determined as 4.49×10^4 l/m³(bed) from operating data of blast furnace (A) (see Table 1). The value of k_{f1} can be evaluated from Eq. (7).¹⁶⁾

$$Sh = 2.0 + 0.55(Re)^{1/2} \cdot (Sc)^{1/3} \dots\dots\dots(7)$$

where, $Sh = k_{f1} d_p / D_{CO}$, $Re = d_p u \rho / \mu$, $Sc = \mu / \rho D_{CO}$. Now, we assume that the additivities may be held for both ρ and μ which are contained in Eq. (7), and μ may be calculated from Licht-Stecherts' equation,¹⁷⁾ then the value of D_{CO} may be obtained in terms of D' and D'' which can be determined from Andrussov's equation,¹⁸⁾ as follows :

$$D_{CO} = \frac{1 - x}{(y/D') + (z'/D'')} = 2.592 \times 10^{-6} T^{1.78} \dots\dots\dots(8)$$

From Eq. (9), the value of D_{s1} for intraparticle diffusion through the blanket of metallic iron phase surrounding unreacted core of iron ore particle can be obtained.

$$D_{s1} = D_{CO} \cdot \varepsilon_p \cdot \xi \dots\dots\dots(9)^{19)}$$

where,

$$\varepsilon_p = 0.53 + 0.47 \varepsilon_p \dots\dots\dots(10)^{20)}$$

for

$$0.15 \leq \varepsilon_p \leq 0.5 : \xi = 0.238 \varepsilon_p + 0.04 \dots\dots\dots(11)^{20)}$$

The value of D_{CO} in Eq. (7) may be determined from Eq. (8) for range of low temperature (under 848°K), and from Eq. (12) for range of high temperature (over 848°K).

$$D_{CO} = 2.592 \times 10^{-6} T^2 \dots\dots\dots(12)$$

Whereas, the value of D_{CO} in Eq. (9) for ranges of low and high temperatures can be determined by making use of t in place of T in Eqs. (8) and (12), respectively.

Fig. 1 is a plot of Arrhenius equation and from the data observed by others^{21)–28)} a straight line has been obtained. The value of k_1 in Eq. (6) can be estimated from Eq. (13) which has been derived from this straight line. The activation energy may be determined as 6.9 kcal/mol (CO).

$$k_1 = 347 \cdot \exp(-3460/t) \dots\dots\dots(13)$$

Fig. 1. Rate constant of the indirect reduction of iron oxide by CO gas

Relating to the equilibrium constant K_1 in Eq. (6), Eqs. (14)–(18) have been obtained according to the calculation based on thermodynamic data.²⁹⁾

For range of low temperature ($t < 848^\circ K$) and

$$\text{for } f_s < 0.111 : K_1 = \exp(4.91 + 6235/t) \dots\dots\dots(14)$$

$$\text{for } f_s > 0.111 : K_1 = \exp(-0.7625 + 543.3/t) \dots\dots\dots(15)$$

For range of high temperature ($t > 848^\circ K$) and

$$\text{for } f_s < 0.111 : K_1 = \exp(4.91 + 6235/t) \dots\dots\dots(16)$$

$$\text{for } 0.111 < f_s < 0.333 : K_1 = \exp(2.13 - 2050/t) \dots\dots\dots(17)$$

$$\text{for } f_s > 0.333 : K_1 = \exp(-2.642 + 2164/t) \dots\dots\dots(18)$$

In Fig. 2, empirical data are compared with numerical results concerning the relations between f_s and θ which have been obtained by integrating Eq. (2). It is found from Fig. 2 that experiments illustrate the validity of preceding expressions relating to k_{f1} , D_{s1} , k_1 , and K_1 .

Each term in the denominator of right hand side of Eq. (6) represents the resistances of three processes of gas film diffusion, intraparticle diffusion and chemical reaction, respectively. It has been found that the resistance of gas film diffusion is very small in comparison with others and so it may be negligible in blast furnace.

Fig. 2. Comparisons of theoretical curves with experimental data on the indirect reduction of iron oxide by CO gas

2. Solution-Loss Reaction

In the reaction described as Eq. (19), overall reaction rate R_2^* per unit volume of bed in blast furnace is defined as Eq. (20).

$$R_2^* = -dn_{\text{CO}_2}/V_B d\theta \dots\dots\dots(20)$$

Assuming that diffusion in pore and chemical reaction on the interior surface of coke particle arise simultaneously and these processes and the diffusion through gas film surrounding the coke particle occur steadily and in succession during reaction, and combining Eq. (21) with (22) gives R_2^* as Eq. (23).

Diffusion rate through gas film :

$$(\pi d_c^2 N_c \varphi_c^{-1}) k_{f2} \cdot 273(y - y_s)/22.4t \dots\dots\dots(21)$$

Diffusion rate in pore with chemical reaction :

$$E_f m_c N_c k_2 \cdot 273 y_s / 22.4t \dots\dots\dots(22)$$

In Eq. (22), reverse reaction has been neglected.

$$R_2^* = \frac{\pi d_c^2 \varphi_c^{-1} N_c \cdot 273 y / 22.4t}{(1/k_{f2}) + (6/d_c \rho_c E_f k_2)} \dots\dots\dots(23)$$

The numerical value of k_{f2} may be evaluated by substituting $Sh = k_{f2} d_c / D_{\text{CO}_2}$, $Re = d_c u \rho / \mu$, $Sc = \mu / \rho D_{\text{CO}_2}$ into Eq. (7). Value of D_{CO_2} is defined as diffusion coefficient of CO_2 in ternary system of $\text{CO}-\text{CO}_2-\text{N}_2$ and given as Eq. (24).

$$D_{\text{CO}_2} = 2.236 \times 10^{-6} T^{1.78} \dots\dots\dots(24)$$

The value of k_2 in Eq. (23) may be determined from Eq. (25) proposed by Heynert, et al.³⁰⁾

$$k_2 = 2.99 \times 10^3 \exp(-80\,000/Rt) \dots\dots\dots(25)$$

And, effectiveness factor E_f in Eq. (23) can be calculated from Eq. (26) in terms of Thiele modulus m which is defined by Eq. (27).

$$E_f = 3(m \cdot \coth m - 1)/m^2 \dots\dots\dots(26)$$

$$m = (d_c/2) \sqrt{\rho_c k_2 / D_{s2}} \dots\dots\dots(27)$$

In following Eq. (28), the value of D_{CO_2} is determined by using t instead of T in Eq. (24), and the value of 0.45 has been employed for ε_c and the value of ξ_c has been evaluated approximately by taking ε_c in place of ε_p in Eq. (11).

$$D_{s2} = D_{\text{CO}_2} \cdot \varepsilon_c \cdot \xi_c \dots\dots\dots(28)$$

3. Direct Reduction of Molten Wustite

Wustite formed by indirect reduction in the shaft of the blast furnace is melted down and molten wustite falls over the surface of coke particles, and during this falling, it may be reduced by solid coke and $\underline{\text{C}}$. Namely, following chemical equations for direct reduction may be described.

Unfortunately, few kinetic investigations of these reactions have been presented hitherto, and because of lack in kinetic information relating to reaction given as Eq. (30), the reaction given as Eq. (29) alone has been considered in this paper.

Regarding to Eq. (29), Shabrin et al.³¹⁾ and Philbrook et al.³²⁾ have reported that this reaction is of the second order with respect to the concentration of wustite in slag. And hence, the expression for the overall reaction rate per unit volume of bed in the furnace may be given as follows :

$$R_3^* = -dn_W / V_B d\theta = k_3 (A/V_B) c_W^2 \dots\dots\dots(31)$$

The activation energy of this reaction has been determined as 53.33 kcal/mol (CO) according to the data obtained by Shabrin et al.³¹⁾ and the frequency factor has been estimated as $4.66 \times 10^4 \text{ m}^4/\text{hr} \cdot \text{kgmol}(\text{CO})$ according to the analysis of the data by Philbrook et al.³²⁾ In terms of these values, the expression for k_3 has been obtained as follows :

$$k_3 = 4.66 \times 10^4 \exp(-53\,300/Rt) \dots\dots\dots(32)$$

On the basis of the assumption that all iron oxides are wustite and molten wustite dissolves in slag homogeneously, c_W in Eq. (31) may be given as follows :

$$c_W = \frac{3F_{mH}(1-f_s)}{(W_s/\rho_s) + 71.85\{3F_{mH}(1-f_s)\}/\rho_W} \dots\dots\dots(33)$$

From the following empirical equation³³⁾ for gas absorption in packed bed, the effective surface area for reaction per unit surface area of coke particles can be written as follows :

$$\beta = \beta'(F_s + F_W)/(F_s + F_W + F_P) \dots\dots\dots(34)$$

where,

$$\beta' = 0.34 (We)^{0.67} / (Fr)^{0.50} (a_i d_c)$$

$$We = d_c \rho_F u_F^2 / \sigma, \quad Fr = u_F^2 / g d_c$$

If we neglect the effect of composition and temperature of slag, fractional reduction of wustite, flow rate of charge materials and cross-sectional area of the furnace on the values of σ and u_F , then the approximate value of β may be determined as 0.078. And so, available specific-surface area in Eq. (31) can be described as follows :

$$(A/V_B) = 0.078 \{6(1-\varepsilon)/\varphi_c d_c\} \dots\dots\dots(35)$$

4. Decomposition of Limestone

Overall reaction rate for chemical equation given as Eq. (36) may be defined as Eq. (37).

$$R_4^* = -dn_L / V_B d\theta \dots\dots\dots(37)$$

R_4^* can be described as Eq. (41) in terms of Eqs. (38)—(40) which express the rates of three steps oc-

curing in succession during reaction.

Rate of reaction taking place on reaction interfaces :

$$4\pi r_i^2 \varphi_L^{-1} N_L k_4 R' t (c'_e - c'_i) / K_4 \dots\dots\dots (38)$$

Rate of intraparticle diffusion through the blanket of CaO phase surrounding unreacted core of limestone :

$$\{4\pi r_o r_i / (r_o - r_i)\} \varphi_L^{-1} N_L D_{s4} (c'_i - c'_o) \dots\dots\dots (39)$$

Rate of diffusion through gas film :

$$4\pi r_o^2 \varphi_L^{-1} N_L k_{f4} (c'_o - c') \dots\dots\dots (40)$$

$$R_4^* = \frac{4\pi r_o^2 \varphi_L^{-1}}{(1/k_{f4}) + d_L \{(1-f_i)^{-1/3} - 1\} / 2D_{s4}} \cdot \frac{N_L (c'_e - c')}{\cdot + \{(1-f_i)^{2/3} k_4 R' t / K_4\}^{-1}} \dots\dots\dots (41)$$

The value of k_{f4} in Eq. (41) can be calculated by putting $Sh = k_{f4} d_L / D_{CO_2}$, $Re = d_L u \rho / \mu$, and $Sc = \mu / \rho D_{CO_2}$ in Eq. (7).

In case of determining the value of D_{CO_2} , for range of low temperature ($T < 1173^\circ K$) Eq. (24) may be used as it is, but for range of high temperature ($T > 1173^\circ K$), because of the intense endothermic reaction, it may be estimated by making use of $(T+t)/2$ instead of T in Eq. (24).

The value of D_{s4} is determined from Eq. (42)

$$D_{s4} = D_{CO_2} \cdot \varepsilon_{vL} \cdot \xi_L \dots\dots\dots (42)$$

where, the value of ε_{vL} can be estimated from following equation.

$$\varepsilon_{vL} = 0.702 \varepsilon_{pL} + 0.298 \dots\dots\dots (43)$$

According to the experimental measurements, the data of ε_{pL} in Eq. (43) have been obtained as 0.20.

For calculation of ξ_L , the following empirical equation which Curie³⁴⁾ obtained by experiments for packed bed of various solid particles has been employed.

$$\xi_L = (\varepsilon_{vL})^{0.41} \dots\dots\dots (44)$$

The expression for k_4 in Eq. (41) has been obtained as follows :

$$k_4 = 5.47 \times 10^6 \exp(-40\,000/RT) \dots\dots\dots (45)$$

In Eq. (45), activation energy has been estimated according to the report by Slonin³⁵⁾ and Hütting et al.,³⁶⁾ and a frequency factor has been determined from empirical data by Britton, et al.³⁷⁾

According to the data of the pressure of decomposition by Perry, et al.,³⁸⁾ K_4 in Eq. (41) has been represented as follows :

$$\log K_4 = -8\,202.5/t + 7.0099 \dots\dots\dots (46)$$

In order to check the validity of preceding expressions relating to k_{f4} , D_{s4} , k_4 , and K_4 , comparisons of experimental data³⁹⁾ and relations between f_i and θ which have been obtained by integrating Eq. (37) are shown in Fig. 3.

Fig. 3. Comparisons of theoretical curves with experimental data on the decomposition of limestone (Key, *: assumed value)

5. Indirect Reduction of Iron Ore by H_2

In regard to the indirect reduction of hematite sphere by H_2 , the author et al.²⁰⁾ have considered a chemical equation given as Eq. (47) and represented R_5^* as Eq. (48).

$$R_5^* = \frac{\pi d_p^2 \varphi^{-1} N}{(1/k_{f5}) + (d_p/2) \{(1-f_s)^{-1/3} - 1\} / D_{s5}} \cdot \frac{273(w-w_e)/22.4t}{\cdot + \{(1-f_s)^{2/3} k_5 (1 + 1/K_5)\}^{-1}} \dots\dots\dots (48)$$

The value of k_{f5} in Eq. (48) can be determined by putting $Sh = k_{f5} d_p / D_{H_2}$, $Re = d_p u \rho / \mu$, and $Sc = \mu / \rho D_{H_2}$ in Eq. (7). The values of D_{H_2} and D_{s5} can be determined as follows :

$$D_{H_2} = 3.960 \times 10^{-6} T^{1.78} \dots\dots\dots (49)$$

$$D_{s5} = D_{H_2} \cdot \varepsilon_v \cdot \xi \dots\dots\dots (50)$$

Where, ε_v and ξ may be estimated from Eqs. (10) and (11), respectively, and D_{H_2} in Eq. (50) is evaluated by taking t in place of T in Eq. (49).

The values of k_5 and K_5 are expressed by drawing a distinction between range of low temperature and range of high temperature. Namely :

$$\text{for } t < 848^\circ K: k_5 = 102.78 t \cdot \exp(-14\,900/RT) \dots\dots (51)$$

$$K_5 = \exp(8.883 - 8\,475/t) \dots\dots\dots (52)$$

for

$$t > 848^\circ K: k_5 = 82.50 t \cdot \exp(-15\,300/RT) \dots\dots (53)$$

$$K_5 = \exp(1.0837 - 1\,737.2/t) \dots\dots (54)$$

For range of low temperature (unstable region of FeO), McKewan⁴⁰⁾ and Otake et al.⁴¹⁾ have recognized the effect of adsorption of H_2O in the case of reduction of magnetite. But according to the experimental results of reduction of magnetite by Ulrich et al.⁴²⁾ and those of hematite by Ito et al.⁴³⁾ for

range of high temperature, it may be assumed that there is no effect of adsorption. Now, in the shaft of the furnace, partial pressure of H_2 is low and unstable region of FeO is narrow, and so, the effect of adsorption may be neglected.

6. Reaction between Coke and Steam

By means of the assumption that the reaction expressed by Eq. (55) may be of the irreversible first-order, R_6^* can be expressed as Eq. (56).

$$R_6^* = \frac{\pi d_c^2 \varphi_c^{-1} N_c \cdot 273 v / 22.4 t}{(1/k_{f6}) + (6/d_c \rho_c E_f' k_6)} \quad \dots\dots\dots(56)$$

The value of k_{f6} in Eq. (56) can be determined by putting $Sh = k_{f6} d_c / D_{H_2O}$, $Re = d_c u \rho / \mu$, and $Sc = \mu / \rho D_{H_2O}$ in Eq. (7).

The value of D_{H_2O} can be calculated as follows :

$$D_{H_2O} = 1.500 \times 10^{-6} T^{1.78} \quad \dots\dots\dots(57)$$

And, an effectiveness factor E_f' in Eq. (56) can be evaluated by taking Thiele modulus m' which is defined as Eq. (58) instead of m in Eq. (26).

$$m' = (d_c/2) \sqrt{\rho_c k_6 / D_{s6}} \quad \dots\dots\dots(58)$$

D_{s6} in Eq. (58) can be estimated from following equation.

$$D_{s6} = D_{H_2O} \cdot \varepsilon_c \cdot \xi_c \quad \dots\dots\dots(59)$$

In Eq. (59), ε_c and ξ_c can be estimated in the same way as those for D_{s2} , and D_{H_2O} can be evaluated by employing t in place of T in Eq. (57). And thus determined D_{s6} may be described as follows :

$$D_{s6} = 6.620 \times 10^{-2} (1.500 \times 10^{-6} t^{1.78}) \quad \dots\dots\dots(60)$$

With respect to k_6 , Eq. (61) has been given by modifying the empirical equation which Kunugi, et al.⁴⁴⁾ obtained according to the reaction-interface model.

$$k_6 = 4.83 \times 10^4 t \cdot \exp(-17311/t) \quad \dots\dots\dots(61)$$

7. Reaction between CO and Steam

It is mentioned that the reaction expressed as Eq. (62) occurs fairly fast in the presence of iron oxide and coke particles owing to the catalytic actions of these particles.^{45),46)} Hence it has been assumed that the reaction between CO and H_2O in the shaft of the blast furnace is in the state of chemical equilibrium in gas phase.

The value of K_7 of this reaction can be indicated as follows :

$$K_7 = yw/xv = \exp(4355/T - 3.8207) \quad \dots\dots\dots(63)^{47)}$$

Since the sum of each molar rate of flow of H_2 and H_2O is constant according to this reaction, we have

$$F(w+v) \equiv \varphi : \text{const.} \quad \dots\dots\dots(64)$$

From Eqs. (63) and (64), we obtain R_7^* as follows :

$$R_7^* = J(Fw)/22.4(\pi D_z^2/4) \Delta z \quad \dots\dots\dots(65)$$

where $J(Fw)$ can be evaluated by assuming that the equilibrium of this reaction holds in gas phase at each position of $(z + \Delta z)$ and z from top of the bed.

III. Theoretical Analysis in Upper Zone from Tuyere Level

1. Mathematical Model for Bed between Top and Tuyere Level

On the basis of the assumptions that the steady state exists in the furnace and that volume rate of flow of particles and fractional void in the bed are retained unchangeably, Eqs. (66)–(76) are obtained by taking the heat, material and force balances for gas and solid particles around the differential height of bed dz at an arbitrary position z from top of the bed as illustrated in Fig. 4. (With regard to the derivations of these fundamental equations, see (REFERENCE 12))

$$\frac{dT}{dz} = \frac{A_z(q_1 + 22.4Cq_2T + q_3) + \pi D_z U(T - T_{we})}{\rho F(G + TdC/dT)} \quad \dots\dots\dots(66)$$

$$q_1 \equiv (1 - \alpha')(-\Delta H_i^c)R_i^* + (1 - \beta')(-\Delta H_2^c)R_2^* \\ + (1 - \gamma')(-\Delta H_4^c)R_4^* + (1 - \delta')(-\Delta H_5^c)R_5^* \\ + (1 - \varepsilon')(-\Delta H_6^c)R_6^* + (1 - \zeta')(-\Delta H_7^c)R_7^*$$

$$q_2 \equiv (1.2507\xi + 0.7261\tau)R_i^* + 0.5246R_2^* \\ + 1.9768R_4^* + 0.7143R_5^* + 0.5364R_6^*$$

$$q_3 \equiv 6(1 - \varepsilon)h_p(T - t)/\varphi d_p$$

$$dt/dz = A_z(q_3 + C_s tq_5 + q_4)/\rho_b F_s \{C_s + t(dC_s/dt)\} \quad \dots\dots\dots(67)$$

$$q_4 \equiv \alpha'(-\Delta H_i^c)R_i^* + \beta'(-\Delta H_2^c)R_2^* + \gamma'(-\Delta H_4^c)R_4^* \\ + \delta'(-\Delta H_5^c)R_5^* + \varepsilon'(-\Delta H_6^c)R_6^* + \zeta'(-\Delta H_7^c)R_7^*$$

$$q_5 \equiv (16 + 12\xi)R_i^* + 12R_2^* + 44R_4^* + 16R_5^* + 12R_6^*$$

$$df_s/dz = A_z(R_i^* + R_6^*)/3F_s c_{H_2O} \quad \dots\dots\dots(68)$$

$$df_t/dz = A_z R_4^*/F_s c_{L_0} \quad \dots\dots\dots(69)$$

$$\frac{dx}{dz} = \frac{22.4A_z\{(\xi + \xi x)R_i^* + (x - 2)R_2^* + xR_4^* + (x - 1)R_6^*\}}{F} \quad \dots\dots\dots(70)$$

$$\frac{dy}{dz} = \frac{22.4A_z\{(\xi y - \tau)R_i^* + (y + 1)R_2^* + (y - 1)R_4^* + yR_6^*\}}{F} \quad \dots\dots\dots(71)$$

$$\frac{dw}{dz} = \frac{22.4A_z\{\xi w R_i^* + w R_2^* + w R_4^* + R_5^* + (w - 1)R_6^*\}}{F} \quad \dots\dots\dots(72)$$

$$\frac{d\rho_b}{dz} = \frac{-A_z\{(16 + 12\xi)R_i^* + 12R_2^* + 44R_4^* + 16R_5^* + 12R_6^*\}}{F_s} \quad \dots\dots\dots(73)$$

$$dp/dz = f_k(1 - \varepsilon)G^2 p^\circ T/g_c \varepsilon^3 d_p \rho T^\circ p \quad \dots\dots\dots(74)$$

$$f_k \equiv 1.75 + 150(1 - \varepsilon)/Re, \quad Re \equiv d_p G/\mu,$$

$$\mu = 4.960 \times 10^{-8} T^{3/2}/(T + 103)$$

for Eq. (19), $\Delta H_2^\circ = 40.8 \times 10^3 \text{ kcal/kgmol (CO}_2\text{)}$

for Eq. (29), $\Delta H_3^\circ = 31.13 \times 10^3 \text{ kcal/kgmol (CO)}$

for Eq. (36), $\Delta H_4^\circ = 42.5 \times 10^3 \text{ kcal/kgmol (CO}_2\text{)}$

Reaction of Eq. (47) is classified into three processes as follows:

for Eq. (55), $\Delta H_6^\circ = 31.5 \times 10^3 \text{ kcal/kgmol (CO)}$

for Eq. (62), $\Delta H_7^\circ = -9.84 \times 10^3 \text{ kcal/kgmol (CO}_2\text{)}$

Assuming that blast furnace gas consists of gaseous mixture of CO, CO₂, H₂, H₂O, and N₂ and that additivity concerning each component holds, we can express mean specific heat as follows:

$$C = (S_1 + S_2 T - 2.00 \times 10^5 y / T^2) / M \dots\dots\dots(78)$$

where, $S_1 \equiv 6.50 + 0.60x + 4.00y + 0.12w + 0.66v$

$$S_2 \equiv (1.00 + 0.20x + 1.40y - 0.19w + 1.58v) \times 10^3$$

$$M \equiv 28 + 16y - 26w - 10v$$

Now, C_s is assumed to be a linear function of t , approximately between 150°C—1 600°C. That is, C_s at 150°C has been determined by taking account of the compositions of charge materials at the top and C_s of condensed phase at 1 600°C has been determined by using the specific heats of slag, pig iron, and coke at 1 600°C and these flow rates at tuyere level. And, assuming that there exists a linear relation between t and C_s , we can obtain the values of C_s for existing blast furnaces according to these operating conditions as follows:

$$\text{for B.F. (A), } C_s = 0.1897 + 3.147 \times 10^{-5} t \dots\dots\dots(79)$$

$$\text{for B.F. (D), } C_s = 0.1913 + 3.464 \times 10^{-5} t \dots\dots\dots(80)$$

Particle-to-fluid heat transfer coefficient h_p has been determined from Eq. (81) proposed by Ranz.⁵⁰⁾

$$Nu = 2.0 + 0.60(Re)^{1/2} \cdot (Pr)^{1/3} \dots\dots\dots(81)$$

where, $Nu = h_p d_p / k$, $Re = d_p G / \mu$, $Pr = C_p \mu / k$,

$$G = \rho F / A_z$$

Mean diameter of solid particles d_p of the burdens is evaluated as follows:

$$d_p = \sum X_i / \sum (X_i / d_{pi}) \dots\dots\dots(82)$$

And, overall heat transfer coefficient for the process of heat loss from the furnace wall is described as follows:

$$U = \frac{W_w C_w (T_{we} - T_{wi})}{A_w (T_m - T_0) / \ln \{ (T_m - T_{we}) / (T_0 - T_{we}) \}} \dots\dots\dots(83)$$

$$\text{where, } A_w = \pi D_b (L_s / \sin \omega_1 + L_b / \sin \omega_2 + L_a) \\ - \pi \{ L_s^2 / (\sin \omega_1 \cdot \tan \omega_1) + L_b^2 / (\sin \omega_2 \cdot \tan \omega_2) \}$$

3. Temperature of Solid Particles at the Top of Furnace

For the sake of simplification of the analysis, the problems have been reduced to that of one dimensional heat conduction through bed and thus Eq. (84) which represents the variation of temperature of solid particles on large bell with residence time has been obtained as follows:

$$t = T_0 - (T_0 - t_f) \operatorname{erf} (z_b / 2 \sqrt{\nu \theta}) \dots\dots\dots(84)$$

where, thermal diffusivity $\nu \equiv k_e / \rho_b C_{s0}$.

Now, we designate θ_0 as time intervals in charging operations, and then mean temperature t_m of solid particles which fall from large bell to the top of the bed at every θ_0 can be expressed as follows:

$$t_m = T_0 - \gamma (T_0 - t_f) \dots\dots\dots(85)$$

where, $\gamma \equiv \{ 5 \operatorname{erf} (0.1127 l_0 / 2 \sqrt{\nu \theta_0}) + 8 \operatorname{erf} (0.5 l_0 / 2 \sqrt{\nu \theta_0}) + 5 \operatorname{erf} (0.8873 l_0 / 2 \sqrt{\nu \theta_0}) \} / 18$, $l_0 \equiv V_s / A_B$, $A_B = \pi l \{ (D_H - l \cot \omega_B) \operatorname{cosec} \omega_B + (D_H + l \cot \omega_H) \operatorname{cosec} \omega_H \}$

And, the value of l can be obtained as a real root of Eq. (86) which satisfies the condition of $0 < l \leq D_H \cdot \tan \omega_B / 2$.

$$\pi l^2 (\cot \omega_B + \cot \omega_H) \{ D_H / 2 - l (\cot \omega_B - \cot \omega_H) / 3 \} - V_s = 0 \\ \dots\dots\dots(86)$$

After falling, the rising of the temperature of solid particles at the top level of the bed takes place and the variation of this temperature can be described as follows:

$$t = T_0 - (T_0 - t_m) \exp (-\sigma' \theta), \sigma' \equiv 6 h_p / d_p \rho_p C_{s0} \dots\dots\dots(87)$$

From Eq. (87), temperature of solid particles at the top level t_0 which has been averaged during the intervals of charging operations can be expressed as follows:

$$t_0 = T_0 - \lambda (T_0 - t_m) \dots\dots\dots(88)$$

where, $\lambda \equiv \{ 5 \exp (-0.1127 \theta_0 \sigma') + 8 \exp (-0.5 \theta_0 \sigma') + 5 \exp (-0.8873 \theta_0 \sigma') \} / 18$

Hence from Eqs. (85) and (88), we have

$$t_0 = T_0 - \gamma \lambda (T_0 - t_f) \dots\dots\dots(89)$$

Consequently, t_0 can be determined from Eq. (89) in terms of temperature of solid particles fed into the top of blast furnace t_f and temperature of top gas T_0 .

4. Temperatures of Gas and Coke (Molten Materials) at Tuyere Level

In this section, the equation which we calculate the temperature of coke (or molten materials) at tuyere level t_c in terms of t_f , T_0 , T_b , T_{wi} , and T_{we} ,

the equation which we determine the theoretical flame temperature at tuyere level T_t in terms of t_c and T_b , and the equation which we evaluate the mean temperature of gas at tuyere level T_m over the cross section of the furnace in terms of t_c and T_t have been given. (Informations for derivation of these equations are given in (REFERENCE 12))

$$t_c = (\sqrt{P_2^2 - P_1 P_3} - P_2)/P_1 \dots\dots\dots(90)$$

where,

$$\begin{aligned} P_1 &\equiv n_1^2 - n_4 \times 10^{-3}, & P_2 &\equiv n_1 n_2 - 1.465 n_4, & P_3 &\equiv n_2^2 - n_3, \\ n_1 &\equiv A(m_1 - \gamma s_1)/(1 - \gamma) s_1, & n_2 &\equiv B + A(s_2 - m_0 t_0)/(1 - \gamma) s_1, \\ n_3 &\equiv B^2 - AD, & n_4 &\equiv A(c' + d'), & s_1 &\equiv C_1 \rho_1 F_1, & s_2 &\equiv C_0 \rho_0 \cdot \\ &F_0 T_0 + W_w C_w (T_{wc} - T_{wi}) + \sum F_{mi} L_{fi} - \sum F_{mj} (-\Delta H_j^\circ) - Q_s, \\ m_0 &\equiv A_0 G_{s0} C_{s0}, & m_1 &\equiv A_1 G_{s1} C_{s1}, & A &\equiv \{0.50 + 1.20(b' + c') \\ &+ 1.005 d'\} \times 10^{-3}, & B &\equiv 3.25 + 6.60(b' + c') + 6.61 d', & D &\equiv \\ &E - F + G - H + J, & E &\equiv -115\,596 a' - 52\,832(b' + c') + \\ &31\,382 d', & F &\equiv (6.50 + 8.27 F') T_b + (0.50 + 0.129 F_1) \times 10^{-3} \cdot \\ &T_b^2 + 1.877 \times 10^5 \cdot F' / T_b, & F' &\equiv a' + b' + c', & G &\equiv (2a' - d') \cdot \\ &(7.16 T_v + 1.29 \times 10^{-3} T_v^2), & H &\equiv (13.24 a' + 5.86 b') t_{oil} + \\ &(0.81 a' + 2b') \times 10^{-3} t_{oil}^2 + 2.2 \times 10^5 b' / t_{oil}, & J &\equiv 298 J_1 + \\ &88.804 J_2 + 3.3557 J_3, & J_1 &\equiv 7.19 a' + 0.93 b' - 2.00 c' - \\ &3.13 d', & J_2 &\equiv -1.641 a' + 0.929 b' - 0.07 c' + 1.285 d', \\ &J_3 &\equiv 187.7 a' + 407.7 b' + 297.7 c' + 110 d', & a' &\equiv 7.089 \times \\ &10^{-3} \beta_H O_b, & b' &\equiv 1.181 \times 10^{-3} \beta_C O_b, & c' &\equiv 1.266 \{0.21 + r_{O,b} \\ &-(5.6 \beta_H + 0.933 \beta_C) \times 10^{-3} \cdot O_b\}, & d' &\equiv 1.575 V_b + 14.18 \\ &(\beta_H O_b) \times 10^{-3} \end{aligned}$$

Now assuming that the burning zone in front of the tuyere is a closed system for heat transfer, we can express Eq. (91) in consideration of the heat of reaction and the sensible heat.

$$T_t = \{ \sqrt{B^2 + A(C - D)} - B \} / A \dots\dots\dots(91)$$

where, $C \equiv (c' + d')(2.93 t_c + 1.0 \times 10^{-3} t_c^2)$

And, T_m can be described as follows:

$$T_m = (1 - \gamma) T_t + \gamma t_c \dots\dots\dots(92)$$

where, $\gamma \equiv (1 - \kappa)(2R_c/D_1)^2$, $\kappa \equiv 4[R_c^{3/2} - 3R_c/\delta + 6\sqrt{R_c}/\delta^2 - 6\{1 - \exp(-\delta\sqrt{R_c})\}/\delta^3]/\delta R_c^2$, $\delta \equiv 24\pi L_1(1 - \varepsilon_1)h_{p1}R_c^{3/2}/\varphi_c d_{c1}C_1\rho_1 F_1$

Thus, t_c and T_m at tuyere level can be obtained in terms of Eqs. (90)–(92). But in these calculations, it is necessary to know t_0 , T_0 , ρ_0 , and F_0 . Whereas, t_0 can be determined from Eq. (89) provided that T_0 is known. From Eqs. (75) and (76), we have

$$\rho_0 = 1.2507 + 0.7261 y_0 - 1.1614 w_0 - 0.4471 v_0 \dots\dots\dots(93)$$

$$F_0 = 47.4 F_b / (1 - x_0 - y_0 - w_0 - v_0) \dots\dots\dots(94)$$

Hence, if x_0 , y_0 , w_0 , v_0 , and T_0 are known, then t_c and T_m can be obtained. The compositions of top gas may be determined from overall material balance (see **IV-1**) and T_0 may be obtained from overall heat balance (**IV-2**). Heat of fusion $\sum F_{mi} L_{fi}$, heat of reaction $\sum F_{mj} (-\Delta H_j^\circ)$, and heat of formation of slag Q_s which are involved in s_2 of Eq. (90) may be expressed later (see **IV-2**).

IV. Calculation of Overall Balance

1. Overall Material Balance

According to the overall material balance taken around the whole furnace (between top and hearth), F_0 , x_0 , y_0 , z_0 , w_0 , W_P , and W_S are obtained as follows:

$$W_P \alpha_{Fe} + W_D \delta_{Fe} = W_0 \gamma_{Fe} + W_C \zeta_A \varepsilon_{Fe} + W_{Mn} \eta_{Fe} + W_{Fe} \dots\dots\dots(95)$$

$$\begin{aligned} F_{mCc} + W_{oil} \beta_C / (12) + F_{mCL} \\ = W_P \alpha_C / (12) + F_0 (x_0 + y_0) / (22.4) + F_{mCD} \dots\dots\dots(96) \end{aligned}$$

$$F_0 z_0 = (0.79)(60 F_b) \dots\dots\dots(97)$$

$$F_0 w_0 / (22.4) = (60 F_b) V_b / (18\,000) + W_{oil} \beta_H / (2) \dots\dots\dots(98)$$

$$\begin{aligned} 1.5 F_{mH} + 2 F_{mM} + F_{mL} + 0.21 (60 F_b) / (22.4) \\ + (60 F_b) V_b / (2) (18\,000) + F_{mO_2A} + W_P \{ \alpha_{Si} / (28) \\ + \alpha_{Mn} / (2) (54.9) \} = F_0 (x_0 / 2 + y_0 + v_0 / 2) / (22.4) \\ + F_{mO_2D} \dots\dots\dots(99) \end{aligned}$$

$$x_0 + y_0 + z_0 + w_0 = 1 \dots\dots\dots(100)$$

$$\begin{aligned} W_S = 56.1 F_{mCaO} + 60.1 F_{mSiO_2} + 40.3 F_{mMgO} \\ + 70.9 F_{mMnO} + 102.0 F_{mAl_2O_3} + 79.9 F_{mTiO_2} \\ - W_P \{ 60.1 \alpha_{Si} / (28) + 70.9 \alpha_{Mn} / (54.9) \} \dots\dots\dots(101) \end{aligned}$$

Eq. (91) is obtained from Fe balance and from this equation W_P is determined. Eqs. (96)–(99) are derived from C, N₂, H₂, and O₂ balances, respectively. F_0 , x_0 , y_0 , z_0 , and w_0 can be determined by solving Eqs. (96)–(100) simultaneously. Eq. (101) is the condensed-phase balance and W_S is obtained from this equation.

2. Overall Heat Balance

The value of T_0 can be determined from Eq. (102) which is derived from overall heat balance taken around the whole furnace. And thus, t_0 can be obtained from Eq. (89) in terms of T_0 .

$$\begin{aligned} T_0 = \{ (60 F_{bw}) C_{bw} \rho_{bw} T_b + W_{t0} C_{s0} t_0 - W_P C_{sP} t_P \\ - W_S C_{sS} t_S + Q_s - \sum F_{mi} L_{fi} \\ + \sum F_{mj} (-\Delta H_j^\circ) - Q_w \} / (F_0 \rho_0 C_0 + W_D C_{sD}) \dots\dots\dots(102) \end{aligned}$$

Assuming that iron ore becomes molten Fe, limestone becomes molten CaO, and the other materials charged become molten oxides through the melting processes, and considering that the additivity on the heat of fusion of each component holds, we can describe Eq. (103).

$$\begin{aligned} \sum F_{mi} L_{fi} = (3.7 F_{mFe} + 3.6 F_{mSiO_2} + 19.0 F_{mCaO} \\ + 26.0 F_{mAl_2O_3} + 18.5 F_{mMgO} \\ + 13.0 F_{mMnO}) \times 10^3 \text{ (kcal/hr)} \dots\dots\dots(103) \end{aligned}$$

Assuming that the reaction coupled Eq. (19) with Eq. (1) is one of direct reductions (i.e. (1/3) Fe₂O₃ + C = (2/3) Fe + CO) and the individual heat of reactions of Eqs. (55) and (47) is negligibly small, we have

$$\begin{aligned} \sum F_{mj} (-\Delta H_j^\circ) = [5.45 F_{mWi} - 2.12 F_{mH} - 7.12 F_{mM} \\ - 31.13 F_{mWd} - 42.5 F_{mL} + 29.2 \cdot (0.21)(60 F_b) / (11.2) \\ - (28.4)(60 F_b) V_b / (18\,000) - \{ 157.8 \alpha_{Si} / (28) \\ + 5.1 \alpha_C / (12) \} \cdot W_P] \times 10^3 \text{ (kcal/hr)} \dots\dots\dots(104) \end{aligned}$$

Now, we employ Eq. (105)⁵¹⁾ for the heat of formation of slag Q_s and Eq. (106)⁵¹⁾ for the heat loss from furnace wall Q_w .

$$Q_s = 314(56.1F_{mCaO}) - 17W_s \quad (\text{kcal/hr}) \dots (105)$$

$$Q_w = (400D_b + 1\,000D_1 + 100) \times 10^3 \quad (\text{kcal/hr}) \dots (106)$$

V. Results Obtained by Mathematical Model

1. Profile of Blast Furnace and Operating Data

Numerical calculations which are based on the practical operating conditions of the existing blast furnaces (A) and (D) in Japan have been performed in terms of digital computer (HITAC 5020E).

Profiles of these blast furnaces are illustrated in Fig. 5. Principal operating conditions and data which are applied to the present model are given in Tables 1 and 2, respectively. Necessary data to calculate the overall material balance (IV-I) are given in Table 3. Results shown in Table 4 have been obtained by solving Eqs. (92)–(100) under the conditions indicated in Table 2.

Observed values are also shown in Table 4, and

it is found that the calculated data are comparatively coincident with observed data. And it may be considered that the calculated ratio of the flow rate of top gas to the volume rate of blast is approximately equal to the empirical ratio reported hitherto. Ap-

Table 2. Calculated data necessary to develop the mathematical model

Index	B.F. (A)	B.F. (D)
F_b (Nm^3/min)	2942	2837
A_0G_{s0} (kg/hr)	314.1×10^3	290.3×10^3
F_s ($m^3(\text{bed})/\text{hr}$)	271.7	223.8
d_p (m)	2.97×10^{-2}	2.59×10^{-2}
ρ_{b0} ($kg/m^3(\text{bed})$)	1156	1297
$\Sigma F_{mi}L_{fi}$ (kcal/hr)	19.23×10^6	18.32×10^6
F_{mH} (kgmol/hr)	1224	1060
C_{H0} (kgmol/ $m^3(\text{bed})$)	4.505	4.852
C_{L0} (kgmol/ $m^3(\text{bed})$)	0.4281	0.5846
θ_0 (hr)	0.148	0.155
T_{wi} ($^{\circ}\text{C}$)	26	28
T_{we} ($^{\circ}\text{C}$)	32	32

Table 1. Operating conditions of blast furnaces (A) and (D)

Index	B.F. (A)					B.F. (D)				
Blast factors										
Volume (Nm ³ /min)	3027					2962				
Temperature (°C)	1093					922				
Pressure (Kg/cm ²)(g)	1.929					1.795				
Humidity (g/Nm ³)	22.5					35.5				
Oil injected (g/Nm ³)	41.93					0				
Charge factors										
Mass, volume and mean diameter of charge materials	Mass (t)	Vol. (m ³ (bed))		Dia. (cm)		Mass (t)	Vol. (m ³ (bed))		Dia. (cm)	
Sinter	21.00	12.22		1.85		26.284	14.934		1.73	
Ore	12.40	4.98		2.18		4.738	2.051		2.11	
Metal, pellet	0.20	0.10		—		0	0		—	
Limestone	1.70	1.04		3.31		2.026	1.213		1.85	
Mn ore	0.45	0.24		—		0.100	0.387		2.19	
Silica	0	0		—		0.203	0.119		1.51	
Coke	10.10	21.10		5.30		11.006	15.951		5.63	
Sum	45.85	39.68				44.957	34.655			
Frequency of charge (times/day)	162					155				
Tor gas										
Temperature (°C)	209					209				
Pressure (Kg/cm ²)(g)	0.4					0.596				
CO, CO ₂ , and H ₂ (vol%)	23.6	18.4		4.4		23.6	18.6		2.7	
Pig iron										
Production (t/day)	3495					2995				
Temperature (°C)	1450					1377				
Si, Mn, P, S (wt%)	0.58	0.76	0.184	0.038		0.76	1.00	0.122	0.028	
Frequency of tapping (times/day)	8					7				
Slag										
Production (t/day)	1000					1030				
CaO, MgO, SiO ₂ , Al ₂ O ₃ , and S (wt%)	39.54	4.24	33.50	15.90	0.98	43.4	6.29	34.8	14.4	0.77

plying the calculated data of Table 4 and the operating data of Tables 3 and 5, we can calculate the overall heat balance (IV-2), and the values of T_0 and t_0 obtained have been shown in Table 6.

2. Results and Considerations of Situations in Furnace

Results obtained by applying the present model to the existing blast furnaces have been illustrated in

Table 3. Operating data necessary to take the overall material balance

Index	B.F. (A)	B.F. (D)
F_b (Nm ³ /min)	2942	1413
V_b (g(H ₂ O)/Nm ³ (blast))	23.2	15.3
W_c (kg(coke)/hr)	68.0×10^3	69.4×10^3
W_D (kg(dust)/hr)	4222	2059
W_L (kg(CaCO ₃)/hr)	11.226×10^3	12.7×10^3
W_{Fe} (kg(Fe)/hr)	1456	0
W_{Mn} (kg(Mn ore)/hr)	2927	4376
W_o (kg(ore)/hr)	230.63×10^3	195.5×10^3
W_{oil} (kg(oil)/hr)	7615	0
α_C	0.0459	0.0478
α_{Fe}	0.9407	0.9346
α_{Mn}	0.0076	0.0100
α_{Si}	0.0058	0.0076
β_C	0.86	—
β_H	0.118	—
γ_{Fe}	0.5911	0.5901
γ_W	0.1122	0.0997
δ_C	0.3199*	0.2780
δ_{Fe}	0.3376*	0.3426
δ_W	0.0473*	0.0436
δ_H	0.430*	0.4416
ε_{Fe}	0.0630	0
ε_H	0.0945	0
ζ_A	0.0825	0.127
η_{Fe}	0.1701	0.1920

* assumed value

Table 4. Calculated results based on the overall material balance

Index	B.F. (A)	B.F. (D)
Composions of top gas		
CO, x_0	0.239 (0.236)	0.224 (0.236)
CO ₂ , y_0	0.184 (0.184)	0.198 (0.186)
N ₂ , z_0	0.521 (0.536)	0.548 (0.550)
H ₂ , w_0	0.056 (0.044)	0.027 (0.029)
Volume rate of flow of top gas		
$F_0 \times 10^{-3}$ (Nm ³ /hr)	2.689	2.453
Ratio of F_0 to F_b		
$F_0/60F_b$	1.53	1.42
Production of pig iron		
$W_P \times 10^{-3}$ (kg/hr)	145.8 (145.6)	124.3 (124.8)
Production of slag		
$W_S \times 10^{-3}$ (kg/hr)	49.0 (41.7)	48.8 (42.9)

() in the right column: observed value

Fig. 6 for B.F. (A) and Fig. 7 for B.F. (D). Ordinate indicates the distance from stock line to tuyere level and abscissa shows the average for whole cross section

Table 5. Operating data necessary to take the overall heat balance

Index	B.F. (A)	B.F. (D)
T_b (°K)	1366	1195
t_{oil} (°K)	350	350
t_P (°K)	1723*	1750*
t_S (°K)	1753*	1780*
Compositions of ore		
T•Fe	0.5911	0.59005
FeO	0.1122	0.1025
SiO ₂	0.0542	0.06428
CaO	0.0454	0.05538
MgO	0.0088	0.01549
MnO	0.0036	0.0029
Al ₂ O ₃	0.0217	0.01964
TiO ₂	0.0056	0.0012
Comp. of coke		
C	0.8942	0.8730
SiO ₂	0.0399	0.0525
CaO	0.0066	0.0035
MgO	0.0025	0.0019
MnO	0.00065	0.0010
Comp. of Mn ore		
T•Fe	0.1701	0.1493
SiO ₂	0.1040	0.2702
CaO	0.0081	0.0083
MgO	0.0029	0.00497
MnO	0.3944	0.3004
Al ₂ O ₃	0.0654	0.04313
Comp. of limestone		
SiO ₂	0.0143	0.0282
CaO	0.5457	0.5318
Comp. of dust		
T•Fe	0.3486	0.3426
FeO	0.0536	0.0436
C	0.2750	0.2780
SiO ₂	0.06672	0.0779
CaO	0.04529	0.0306
MgO	0.00875	0.0093
MnO	0.01732	0.0108
Al ₂ O ₃	0.03489	0.0201
TiO ₂	0.00425	0.0012

* assumed value

Table 6. Calculated results based on the overall heat balance

	B.F. (A)	B.F. (D)
λ	0.9796	0.9808
γ	0.4577	0.4480
T_0 (°K)	572	575
t_0 (°K)	451	455

Fig. 5.
Profiles of the blast furnaces (A) and (D)

Fig. 6.
Analytical results obtained by the present mathematical model in the blast furnace (A)

of T , t , f_s , f_l , x , y , w , F , ρ , ρ_b , p , and v .

It has been found that the profiles of these variables demonstrated in Figs. 6 and 7 show the similar aspects as those of empirical results on T , x , y , and w obtained by Shürmann et al.,⁷⁾ on T , x , and y given by Korshikov¹⁰⁾, and on y observed by Lazarev et al.⁹⁾

Solid particles fed into the top of the bed are rapidly heated and temperature difference between gas and solid particles decreases progressively. Both

solid particles and gas have nearly the same temperature at the zone from the upper part of shaft to that of bosh. From the upper part of bosh, the temperature difference between gas and solid particles increases along with approach to the tuyere level. This is due to the fact that endothermic effects and C_s increase at the zone near the tuyere level.

Now, the step change in the plot of t is due to the assumption that the melting process of iron ore takes

Fig. 7.
Analytical results obtained
by the present mathemat-
ical model in the blast
furnace (D)

place at 1 400°C. Up to this change, f_s has been calculated through overall reaction rate of indirect reduction, and after this change, that of direct reduction has been employed. In Figs. 6 and 7, it is found that the plots of f_s vs. z exhibit the similar features, and the reduction of Fe_2O_3 to Fe_3O_4 is completed at the level below by several meters from stock line. It may be considered that the reaction of Eq. (47) plays an insignificant role in blast furnace; that is, because of small values of heat of reaction and w , the effects of this reaction on the variations of temperature and f_s are not important. It appears that the decomposition of limestone proceeds rapidly at the zone of temperature between about 1 000° and 1 300°K.

Reactions presented by Eqs. (36), (19), (29), and (55) are all the highly endothermic reactions and the number of moles of gas is increased by these reactions. Particularly, because of high activation energy for the reactions of Eqs. (19) and (29), these reactions play an important role in the high-temperature region. Namely, these reactions exert the large influence on the variations of temperature and F . The value of F remains constant over a length of shaft, and this is due to the fact that the carbon-deposition has not been taken into account in this study.

Reaction of Eq. (1) becomes predominant in the range of temperature under about 950°C, but above this temperature, reaction of Eq. (19) becomes predominant, and consequently, the maximum of x comes out, and this lies on the upper part of boshes in both furnaces. It is found that the level of maximum of x is almost coincident with the level where the increase in the value of y begins. Now, it is essential to note that CO_2 generates progressively even in the lower zone than the level where y begins to increase, but it is consumed rapidly by the reaction of Eq. (19).

The existence of two minima in the profile of w

is seen at the zone from the top of the bed to the upper part of shaft and at the zone from the lower part of shaft to the upper part of bosh, respectively. On this basis, it may be considered that the reaction of Eq. (62) proceeds forward and exhibits exothermic effect in the former zone, and it proceeds reversely, in the latter zone.

The value of ρ designates the density of gaseous mixture at standard state, and so, the actual value of ρ is in need of modification with the values of T and p in the furnace. The value of ρ_b represents not the true value but the apparent value, and this is due to the assumption that F_s remains constant in the bed from the top to the tuyere level. A profile of v is demonstrated in Fig. 7 alone.

VI. Conclusion

Mathematical model incorporated with the overall rate of reactions given by Eqs. (1), (19), (29), (36), (47), and (55), and the change in moles of H_2 caused by reaction given by Eq. (62) has been proposed here. Initial conditions necessary to conduct the numerical calculation have been determined by the overall material and heat balances which have been taken around the whole furnace.

Calculated results relating to the longitudinal distributions of temperature and molar fractions of blast furnace gas have been in good agreement with the features illustrated in terms of the observed data by other investigators.

Carbon-deposition reaction and direct reduction of molten wustite by C dissolved in liquid iron have not been taken into account because of the lack in fundamental data. And it has been assumed that average diameter and volume rate of flow of solid particles are constant in the furnace and the melting point of iron ore is 1 400°C.

This model includes these defects mentioned above, but it may be useful for predicting the situations in blast furnaces which could not be found hitherto but for actual measurement.

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Nomenclature

A	effective furnace area for reactions, (m^2)	$F_{mCc}, F_{mCD}, F_{mCL}$	blast, (Nm^3/min) molar rate of flow of C in coke, dust, and limestone, ($\text{kgmol}(\text{C})/\text{hr}$)
A_B	heat-transfer area of packed bed on large bell, (m^2)	$F_{mO_{2A}}, F_{mO_{2D}}$	molar rate of flow of O_2 in ash and dust, ($\text{kgmol}(\text{O}_2)/\text{hr}$)
A_w	total surface area of inner wall from stock line to tuyere level, (m^2)	F_{mH}, F_{mM}, F_{mL}	molar rate of flow of hematite, magnetite, limestone, (kgmol/hr)
A_z	cross-sectional area of furnace at z , (m^2)	F_{mi}	molar rate of flow of component i , ($\text{kgmol}(i)/\text{hr}$)
a_t	specific surface area of coke-bed, ($\text{m}^2/\text{m}^3(\text{bed})$)	F_{mWd}, F_{mWi}	molar rate of flow of wustite formed by direct and indirect reduction, (kgmol/hr)
C, C_s	specific heat of gas and solid particles, ($\text{kcal}/\text{kg}\cdot^\circ\text{C}$)	F_{O_2}	volume rate of flow of enriched oxygen, (Nm^3/min)
C_{bw}	specific heat of wet blast, ($\text{kcal}/\text{kg}\cdot^\circ\text{C}$)	F_P, F_S, F_W	volume rate of flow of pig iron, slag, wustite, ($\text{m}^3(\text{bed})/\text{hr}$)
C_{sP}, C_{sS}	specific heat of pig iron and slag, ($\text{kcal}/\text{kg}\cdot^\circ\text{C}$)	f_t	fractional decomposition of limestone
C_w	specific heat of cooling water, ($\text{kcal}/\text{kg}\cdot^\circ\text{C}$)	f_s	fractional reduction of iron ore
c_{H_0}, c_{L_0}	initial concentration of hematite and limestone, ($\text{kgmol}/\text{m}^3(\text{bed})$)	G, G_s	mass velocity of gas and solid particles, ($\text{kg}/\text{m}^2\text{hr}$)
c_w	concentration of wustite in molten slag, ($\text{kgmol}(\text{FeO})/\text{m}^3(\text{slag})$)	g	acceleration due to gravity, (m/hr^2)
c', c'_e, c'_i, c'_o	concentration of CO_2 in bulk of gas, at equilibrium, at interfacial surface on which reaction takes place, at outer surface of solid particles, ($\text{kgmol}(\text{CO}_2)/\text{m}^3$)	g_c	conversion factor, ($\text{kg}\cdot\text{m}/\text{Kg}\cdot\text{hr}^2$)
D', D''	diffusion coefficient of CO in system CO- CO_2 and CO- N_2 , (m^2/hr)	K_1, K_4, K_5, K_7	equilibrium constant of reaction given by Eqs.(1), (36), (47), (62), respectively, (—, atm, —, —)
$D_{\text{CO}}, D_{\text{CO}_2}, D_{\text{H}_2}, D_{\text{H}_2\text{O}}$	diffusion coefficient of CO, CO_2 , H_2 , and H_2O in blast furnace gas, (m^2/hr)	k	thermal conductivity of gas, ($\text{kcal}/\text{m}\cdot\text{hr}\cdot^\circ\text{C}$)
D_H	diameter of bottom of hopper equipped around large bell, (m)	k_e	effective thermal conductivity, ($\text{kcal}/\text{m}\cdot\text{hr}\cdot^\circ\text{C}$)
D_{s1}, D_{s5}	intraparticle diffusion coefficient of CO and H_2 in reduced iron phase, (m^2/hr)	$k_{f1}, k_{f2}, k_{f4}, k_{f5}, k_{f6}$	gas-film mass transfer coefficient in reaction given by Eq.(1), (19), (36), (47), (55), respectively, (m/hr)
D_{s4}, D_{s2}, D_{s6}	intraparticle diffusion coefficient of CO_2 in phase of CaO, of CO_2 and H_2O in coke, (m^2/hr)	$k_1, k_2, k_3, k_4, k_5, k_6$	rate constant of reaction given by Eqs.(1), (19), (29), (36), (47), (55), respectively, (m/hr, $\text{m}^3/\text{kg}\cdot\text{hr}$, $\text{m}^4/\text{kgmol}(\text{CO})\cdot\text{hr}$, $\text{kgmol}(\text{CO}_2)/\text{m}^2\cdot\text{hr}$, m/hr, $\text{m}^3/\text{kg}\cdot\text{hr}$)
D_z	inner diameter of furnace at z , (m)	L_1	height of burning zone formed in front of tuyere, (m)
d_c, d_L, d_p	average diameter of particles of coke, limestone, iron ore, (m)	l	height of packed bed on large bell, (m)
d_{c1}	diameter of coke at tuyere level, (m)	m_c	mass of a particle of coke, (kg)
F, F_s	volume rate of flow of gas and solid particles, (Nm^3/hr , $\text{m}^3(\text{bed})/\text{hr}$)	N, N_c, N_L	number of particles of iron ore, coke and limestone per unit volume of bed, ($1/\text{m}^3(\text{bed})$)
F_b, F_{bw}	volume rate of dry blast and wet	$n_{\text{CO}}, n_{\text{CO}_2}, n_L, n_W$	number of moles of CO, CO_2 , CaCO_3 and FeO, (kgmol)
		O_b	addition-ratio of heavy oil, ($\text{g}/\text{Nm}^3(\text{blast})$)
		p, p°	pressure of gas and p at standard states, (Kg/m^2)
		R, R'	gas constant, ($\text{kcal}/\text{kgmol}\cdot^\circ\text{K}$, atm· $\text{m}^3/\text{kgmol}\cdot^\circ\text{K}$)
		R_c	radius of coke-core at tuyere level, (m)
		$R_1^*, R_2^*, R_3^*, R_4^*, R_5^*, R_6^*$	overall reaction rate per unit volume of bed in reaction given by Eqs. (1), (19), (29), (36), (47), (55),

	respectively, (kgmol(CO)/m ³ (bed)·hr, kgmol(CO ₂)/m ³ (bed)·hr, kgmol(CO)/m ³ (bed)·hr, kgmol(CO ₂)/m ³ (bed)·hr, kgmol(H ₂)/m ³ (bed)·hr, kgmol(H ₂ O)/m ³ (bed)·hr)	$\alpha_C, \alpha_{Fe}, \alpha_{Mn}, \alpha_{Si}$	mass fraction of C, Fe, Mn, Si in pig iron
R_7^*	change in moles of H ₂ caused by reaction given by Eq.(62), (kgmol(H ₂)/m ³ (bed)·hr)	$\alpha', \beta', \gamma', \delta', \epsilon', \zeta'$	fraction of heat of reaction acquired by solid particles for reaction given in Eqs.(1) and (29), (19), (36), (47), (55), (62), respectively
r_i, r_o	radius of reaction interface and outer surface of particle, (m)	β_C, β_H	mass fraction of carbon and hydrogen in heavy oil
r_{O_2b}	oxygen enrichment ratio = F_{O_2}/F_b	γ_{Fe}, γ_W	mass fraction of Fe and FeO in iron ore
T, t	temperature of gas and solid particles (molten materials), (°K)	$\delta_C, \delta_{Fe}, \delta_H, \delta_W$	mass fraction of C, Fe, Fe ₂ O ₃ , FeO in dust
T°	temperature of gas at standard states, (°K)	ϵ	fractional void in bed
T_b	blast temperature, (°K)	$\epsilon_c, \epsilon_P, \epsilon_{pL}$	porosity of coke, iron ore, limestone
T_v	temperature of steam blown into blast pipe, (°K)	$\epsilon_v, \epsilon_{vL}$	porosity in reducing-iron phase and phase of CaO
T_{we}, T_{wi}	exit and inlet temperature of cooling water, (°K)	$\epsilon_{Fe}, \epsilon_H$	mass fraction of Fe and Fe ₂ O ₃ in ash
t_{oil}	temperature of heavy oil blown into blast pipe, (°K)	ζ_A	mass fraction of ash in coke
t_P, t_S	temperature of pig iron and slag at tapping, (°K)	η_{Fe}	mass fraction of Fe in Mn ore
U	overall heat transfer coefficient based on inner surface area of furnace-wall, (kcal/m ² ·hr·°C)	θ	time, (hr)
u, u_F	superficial velocity of gas and molten materials, (m/hr)	μ	viscosity of blast furnace gas, (kg/m·hr)
V_B	volume of bed in furnace, (m ³)	ξ, ξ_c, ξ_L	labyrinth factor of iron ore, coke, phase of CaO
V_b	addition-ratio of steam, (g/Nm ³ (blast))	ρ, ρ_{bw}	density of blast furnace gas and wet blast, (kg/Nm ³)
V_s	volume of charge-materials per one charge, (m ³ (bed))	ρ_b, ρ_p	bulk and apparent density of solid particles, (kg/m ³ (bed), kg/m ³ (particle))
v, v_0	molar fraction of H ₂ O in bed and at top of bed	ρ_c	apparent density of coke, (kg/m ³ (coke))
$W_c, W_D, W_{Fe}, W_L, W_{Mn}, W_o$	mass rate of flow of coke, dust, metallic iron, limestone, Mn ore, iron ore, (kg/hr)	ρ_F, ρ_S, ρ_W	density of molten materials, slag, wustite, (kg/m ³)
W_{oil}, W_w	mass rate of flow of heavy oil and cooling water, (kg/hr)	σ	interfacial tension, (kg/hr ²)
W_P, W_S	mass rate of flow or tapping rate of pig iron and slag, (kg/hr)	$\varphi, \varphi_c, \varphi_L$	shape factor of iron ore, coke, limestone
W_{t0}	total mass rate of flow of charge materials, (kg/hr)	ω_B, ω_H	angle of inclination of surface of large bell and hopper to horizontal plane
w, w_0, w_e	molar fraction of H ₂ in bed, at top of bed, at equilibrium		
X_i	volume fraction of solid particles having diameter d_{pi}		
x, x_o, x_i, x_e, x_0	molar fraction of CO in bulk of gas, at outer surface of ore, reaction interface, equilibrium and at top of bed		
y, y_0, y_s	molar fraction of CO ₂ in bed, at top of bed, at outer surface of solid particle		
z', z_0	molar fraction of N ₂ in bulk of gas and at top of bed		
z_B	distance from bottom of packed bed on large bell, (m)		

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