Note

A Note to the Single-size Approximation of Mist Flow in a Nozzle

1. Introduction

Nozzle flow of suspension of small solid or liquid particles in a gas is of great technical importance for various engineering applications. Hence, we have established the system of governing equations for the nozzle flow of a gas-particle mixture in case where a continuous distribution of the particle size is present. Thereby, it has been demonstrated that the system of equations completely determines all of the flow properties in the flow field if the inlet condition and the nozzle configuration are prescribed,¹⁾ and that for prescribed pressure profiles along the whole nozzle length variations of the nozzle geometry lead to the establishments of the conditions for optimum nozzle performance.²⁾

In the present paper, we wish to investigate the effect of a continuous distribution of particle size on the thermodynamic properties of gas-particle mixtures. In reality, it would be a very interesting and important problem to find the mixture with particles of only a single size corresponding to that with the actual size distribution on the same loading ratio condition. The present problem is equivalent to finding such a particle size that the single-size approximation gives a good fit to the case of the actual mixture. Furthermore, the effect of phase-transformation of very small particles cooled below the freezing point on the nozzle performance is briefly discussed from a practical point of view.

2. Numerical Experiments

2.1. Computational Conditions

For the numerical calculation, a gas-particle mixture composed of air and water-particles is treated. The physical constants as well as the reference conditions adopted here are as follows:

Particle material density $\bar{\rho}_{mp} = 1\,000$ (kg/m³),

Gas specific heat at constant pressure

$\bar{C}_{pg} = 1\,004$ (J/kg·K),

Specific heat of particle $\bar{C}_{pp} = 4\,187$ (J/kg·K),

Gas constant $R = 286.86$ (J/kg·K),

Gas temperature at reservoir $\bar{T}_0 = 323$ K,

Gas viscosity at reservoir $\bar{\mu}_0 = 2.07 \times 10^{-5}$ (Pa·s).

Again, the general expression of gas viscosity is given as a function of gas temperature $\bar{T}$ by

$$\bar{\mu} = \bar{\mu}_0 (\bar{T}/\bar{T}_0)^{0.6} \quad \dots\dots\dots(1)$$

Let us now assume that the minimum and the maximum radii of the particle contained in the mixture are specified as $\bar{r}_{p,\min} = 1.0$ (μm) and $\bar{r}_{p,\max} = 50$ (μm), and that the distribution function of particle size $\bar{r}_p$ is given by

$$\bar{\phi}_0(\bar{r}_p) = \bar{J} \bar{r}_p \exp(-\bar{r}_p^2/250) \quad \dots\dots\dots(2)$$

in which $\bar{J}$ is the normalizing factor and it has been found to be $\bar{J} = 1/124.58$ (μm⁻²). Also, it is convenient, in the later, to define the average radius $\bar{l}_{p0}$ at reservoir by

$$\bar{l}_{p0}^3 = \int \bar{\phi}_0(\bar{r}_p) \bar{r}_p^3 d\bar{r}_p \quad \dots\dots\dots(3)$$

At the present case, it has been followed that $\bar{l}_{p0} = 17.396$ (μm).

Next, let it be assumed that the pressure $\bar{p}_0 = 10 \times 10^5$ (Pa) at the reservoir and $\bar{p}_E = 1.0 \times 10^5$ at the nozzle exit, and that the pressure profile is expressed as

$$\bar{p} = \frac{\bar{p}}{\bar{p}_0} = -0.46 \frac{g(x)}{[g(x)]^2 + 1} + 0.54; \quad g(x) = k_1 x + k_2 \quad \dots\dots\dots(4)$$

in which $x = \bar{x}/\bar{L}_*$ ($\bar{L}_*$: nozzle radius at throat = 1.0 mm), $k_1 = 0.325347$ and $k_2 = 0.026096$. It is noted that the position at $\bar{x} = 0$ corresponds to that of the throat for gas-only flow and the position at $\bar{x} = 10 \bar{L}_*$ corresponds to that of the nozzle exit.

2.2. Results

For the single-size approximation, the size distribution function of particles in the mixture can be replaced by

$$\bar{\phi}_0(\bar{r}_p) = \bar{\delta}(\bar{r}_p - \bar{r}_p) \quad \dots\dots\dots(5)$$

where, $\bar{\delta}$: the Dirac delta-function.

However, it is beyond the range of possibility mathematically to find $\bar{r}_p$ such that the single-size approximation gives a best fit to the gas-phase flow properties of the mixture with the actual size distribution $\bar{\phi}_0(\bar{r}_p)$. It seems to be somewhat reasonable that we put

$$\bar{r}_p = \bar{l}_{p0} \quad \dots\dots\dots(6)$$

We should like to stress that the numerical experiments have been performed on the basis of the theoretical model described in Ref. 2).

Fig. 1 shows the nozzle geometry along the nozzle axis, that is, the variation of nozzle radius $L(= \bar{L}/\bar{L}_*)$, $\bar{L}$ =dimensional nozzle radius) with $x(= \bar{x}/\bar{L}_*)$ on the condition that the particle-to-gas mass flow rate ratio $\nu = 1$ (ν is commonly called the loading ratio). It can be seen from this figure that the single-size approximation gives a close agreement to the result of the case

Fig. 1. Variation of L with x on the condition that $\nu=1$.

where the distribution of particle size is taken into consideration.

Figs. 2(a) to 2(c) give the variation of the density $\rho (= \bar{\rho}/\bar{\rho}_0$; the ratio of local density to that at reservoir), temperature $T (= \bar{T}/\bar{T}_0)$ and velocity V (the ratio of local velocity to $\bar{a}_0$; $\bar{a}_0$ =sonic velocity at reservoir) of gas-phase along the nozzle axis on the condition that $\nu=1$. The single-size approximation may exhibit a good applicability to the gas-phase flow properties.

3. Discussion and Conclusions

The numerical results for $\bar{\phi}_0(\bar{r}_p) = \delta(\bar{r}_p - l_{p0})$ have been demonstrated above. It is reasonable to consider that the accuracy of the single-size approximation is dependent upon the form of the corresponding actual distribution function $\bar{\phi}_0(\bar{r}_p)$. However, in the present calculation, the single-size result gives a satisfactory approximation to the nozzle configuration as well as the flow properties of gas-phase in comparison with the size-distributed result.

First, it is worthwhile to discuss the particle-size effect on the flow properties and the nozzle configuration, because the sensitivity of L , ρ , T and V to particle size distribution function $\bar{\phi}_0(\bar{r})$ seems to come from the significant difference according to the selection of different single-size of particles contained in the mixture. Then, the effects have been examined for the gas-particle mixtures with only a single-size of particles as $\bar{\phi}_0(\bar{r}_p) = \delta(\bar{r}_p - \bar{r}_p)$. Although the numerical results are not exhibited here, the single-size which satisfies Eq. (6) has been confirmed to give a best-fit approximation.

Second, we wish to discuss whether or not the single-size approximation is applicable to all of the cases. It should be emphasized that the single-size approximation is valid only to determining the flow properties of gas-phase. While the behavior of particles with various size remains unknown. The problem of phase-transformation from liquid particles to solid ones should be checked, because the gas temperature may fall below the freezing point at the nozzle exit. Fig. 3 indicates the variation of particle temperature

Fig. 2. Variation of density (a), temperature (b) and velocity (c) of gas-phase on the condition that $\nu=1$. Note that ρ obtained by the single-size approximation overlaps the numerical result of the case where the particle size distribution is taken into account.

$T_p(r_p)$ ($= \bar{T}_p(\bar{r}_p)/\bar{T}_0$ and $r_p = \bar{r}_p/l_{p0}$) along x , with r_p as a parameter, on the same condition as the previous case except the pressure profile (see the same figure). It can be seen from the figure that water-particles with a smaller size attain a lower temperature state at a nozzle exit. In the present calculation, the particles of 1 (μm) are markedly cooled below the freezing point, although the fraction of volume occupied by such small particles is slight. Therefore, one should bear in mind that the volume expansion of

Fig. 3.

Relation between $T_p(r_p)$ and x .

Note that these results are calculated in accord with the pressure profile indicated in this figure, which is different from the profile of Eq. (4), and that each size is made dimensionless as $r_p(i) = [\bar{r}_{p,\min} + 4.9i]/\bar{r}_{p0}$ ($i = 0-10$).

particles can lead to choking of nozzle outlet in the special case where the greater part of particles contained in a mist flow is very small in size. Or, even though the volume fraction occupied by very small particles is relatively small, there may be some cases where such dusty ices play a role of nucleation of phase-transformation of supersaturated liquid vapor in a mist, and rapidly increase in size.

In conclusion, we note that although the single-size approximation gives a good fit to the nozzle configuration as well as the flow properties of gas-phase of the case of the actual mixture, the size distribution is desired to be taken into account for the establishment of conditions for optimum nozzle performance.

REFERENCES

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Natsuo NATTA

Department of Mineral Science and Technology,
Faculty of Engineering,
Kyoto University,
Yoshida-honmachi, Sakyo-ku, Kyoto,
Kyoto-fu, 606 Japan.

Masahiro NAKAMURA

Graduate School, Kyoto University,
Yoshida-honmachi, Sakyo-ku, Kyoto,
Kyoto-fu, 606 Japan.