Automatic Repair of Defects in Grain Boundary Images

Tomoyuki TAKEUCHI

Fifth Research Group, National Research Institute for Metals. Now at 9-501, 891-2, Mamedo-cho, Kohoku-ku, Yokohama, Kanagawa-ken, 222 Japan.

(Received on November 29, 1989; accepted in the final form on February 16, 1990)

A computer program for automated repair of defects in thinned grain boundary images has been developed. The basic data which is necessary to reproduce the geometrical shape of line images are the coordinates of characteristic points and the location of neighboring points connected by lines. Recognition of geometrical structures containing defects is done using the lists of characteristic points, from which a set of data of the distances and angles in the line image surrounding defects is prepared. Repair of grain boundary images consists of erasing undesirable lines and drawing lines between characteristic points. The repair operation involves rewriting of the contents of the data lists. The image reproduced from the repaired data lists is the network of triple points. The rules and method used in the repair operations are similar to those used during manual repair using visual observation. We found that line images containing more than 100 end points as defects could be repaired. With no abnormal structure in the input grain boundary image, the repaired image of the triple point network could be used for recognition of the shape of individual grains. The repair program could be used in a fully automated grain size analyzer.

KEY WORDS: image analysis; microstructure; grain size; line image; data conversion.

1. Introduction

Metallic and ceramic materials are, usually, composed of fine crystalline aggregates. The sizes and shapes of grains, and their distributions are essential factors in determining the mechanical properties of a material. Therefore, automation of measurements of grain size and related quantities is necessary for material quality control. Simple images of microstructures consist of a network structure of grain boundaries observed on a cross-section of polycrystalline single phase materials. Ordinary grey images of grain boundaries may be converted into line images by the use of image processing techniques such as binarization and line thinning. These line images generally contain many defects, such as undesirable lines and missing segment of grain boundaries. The number of these defects can be reduced by suitable selection of the threshold value for binarization, and by contraction and expansion of lines before the final thinning process. Nevertheless, complete elimination of defects is very difficult. In such circumstances, current grain size analyzers require a manual operation step for the repair of defects in the line images, involving human judgments. In order to develop fully automated grain size analyzers, these manual operations must be replaced by an intelligent automatic system.

The purpose of the present research is to develop a computer program for the repair of defects in thinned line images. The system is composed of two parts:

- 1) determining the characteristic point data of the input images and recording them in data lists, and

- 2) repair of the defects by processing these lists.

2. Outline of the Process

The line images for the repair are prepared by the following processes:

- 1) preparation of specimen for the observation,
- 2) operation of microscope and photography,
- 3) input of grey level images,
- 4) image binarization,
- 5) removal of unnecessary features, and
- 6) line thinning.

These preparation processes must be performed carefully to obtain satisfactory images for grain size analysis. This paper, however, does not discuss production of the line images, but the subsequent processes, in which the manual operations usually used to repair defects are substituted by a computer program.

The idealized pattern of the cross-section of single phase polycrystalline materials is a network which is composed of curved lines between adjacent grains (grain boundary) and triple points where 3 lines meet. The triple point network is a necessary condition which should be satisfied in grain boundary images. Geometrically complete images of triple points networks can be obtained by shrinking and expansion of lines by ordinary image processing techniques. This processing, however, erases all branched lines, and many lines which are seen to be grain boundaries by visual observation are lost. Sakaue's thinning program¹⁾ is an improvement on this respect. The other extreme case is where every branch with an end point is elongated and connected with a line on the opposite side, which provide a complete image of the triple point network. Real grain bound-

ary images are intermediate between the above 2 extreme cases.

During the process of visual observation, the information used to determine the method of repair is the line shape recognized in the wide region around the defect. The basic data necessary for a computer to judge the geometrical shape of line images, are the distance between 2 characteristic points which define the line, and the angle between 2 lines going from one characteristic point to its 2 neighboring characteristic points. The shapes of line images are represented as combination of these data. The basic data necessary for this purpose are data lists in which are recorded the coordinate of each characteristic point and the identifiers of neighboring characteristic points connected by lines.

Accordingly, the repair system for thinned grain boundary images consists of 3 segments of computer program as follows: The first involves obtaining the data which are necessary to reproduce the basic shape of the input line image, storing them as data lists. The second consists of execution of the repairs (erasing and connection of lines) through rewriting the content of the lists. The third is the recognition of individual grains from the complete lists describing the triple point network.

3. Preparation of the Data Lists of Characteristic Points

Input pictures are thinned line images produced with the use of a program developed by Sakaue.¹⁾ They are displayed on a computer graphic screen. The lines have been assured 8-neighbor connection. Five kinds of characteristic point are necessary for the reproduction the geometrical structure of the input image. Their names are expressed as 4 figure integers as follows:

- (1) Triple point:
(3 lines meet) 3001-3999
- (2) Quadruple point:
(4 lines meet) 4001-4099
- (3) Border point:
(intersects with border) 1001-1299
- (4) End point:
(line terminates inside) 2001-2499
- (5) Curved point:
(direction changes) 5001-5999

Curved points are defined as the point where the line direction changes of more than or equal to 45° . The line direction at the current pixel point is defined by the line connecting the previous pixel point and the next pixel point in the 8 neighbors. This definition corresponds to an approximation of a circle by an octagon. The most significant digit of the integers (names) represents the kinds of characteristic points, and 3 digits following this correspond position in the file.

As an example of the content of the list of characteristic points, the case of a triple point is shown in Fig. 1. The first column is an integer which indicates the state of the point in the processing sequence. For

STATE VARIABLE	PROCESS CONTROL
X-COORDINATEe	POSITION
Y-COORDINATEe	
n1	NEIGHBOUR POINTS
n2	
n3	

Fig. 1. Content of triple point data list.

example, the number '0' means that the point is erased in the repair processing. The second and third columns are the coordinates of the point expressed by integers from 0 to 399, respectively. The data from the fourth column on are the names of the neighboring characteristic points connected by lines. *n1* is the name of the preceding characteristic point. *n2* and *n3* are arranged in clockwise order of the directions of the connecting lines. In the case of a quadruple point *n4* is added. Border points and end points have only one neighboring point. In the case of an end point, *n2* and *n3* are used as data to show the state of lines in the area in front of the end point.

The size of images was 384×384 pixel square. Eight rows in the border region of 400×400 pixel square are omitted, since some disorder appears there due to the thinning process. The memory value of a pixel point on a line is 1 and elsewhere 0.

Determination of the coordinates and the names of the neighboring points is carried out using the fact that the grain boundary image is composed of a connected network. The network is searched using a cursor tracing the line image. The starting point is the upper left corner (8, 8), and it moves along the border line in a clockwise direction until it meets a line, or a pixel point with a value 1. This point is the entrance to the network. A flow chart of the operation is shown in Fig. 2. The details of the operation are as follows:

- (1) Searching neighbor:

The cursor searches the value of 8 neighboring pixel (0 or 1), and points with the value 1 are recorded. If two points are contact among the 8-neighbor points (square), they are counted as one point. The path number on the 8 neighbor points is in a range of 1-4.

- (2) Way number=2:

The cursor is judged to be on a continuous line.

- (3) Curved point:

The angle between the line direction at the point of the cursor and starting direction at the last characteristic point assured to be 45° or greater.

- (4) New characteristic point:

The characteristic point is determined as new point or previously recorded point, by the comparison of the coordinates of the point with those already recorded.

- (5) Recording:

A name is given to the new characteristic point according to its character and its order of occurrence.

Fig. 2.

Flow chart of the operation to acquire the data of the characteristic points in a network line images.

Subsequent processes are as follows:

- (a) Write the identifier in the corresponding column in the list of the preceding characteristic point.
- (b) Write the coordinates of the present point in the list of the new characteristic point, and the identifier of the preceding point in the column $n1$. In the case of triple point (or a quadruple point) the directions of the path recorded in step (1) are written on columns $n2$ and $n3$ in a clockwise order. The direction of the next move is the value in $n2$.

(6) Append:

This occurs when the indicator arrives at a point which is already recorded, and the list processing is as follows:

- (a) Write the identifier of the present point in the vacant column of the preceding point.
- (b) In the case of triple point, write the name of the previous point in column $n3$.
- (c) In the case of a quadruple point, the direction arrived for is compared with the data in $n3$ and $n4$, and the identifier of the previous point is written in the corresponding column. The direction of the next move is determined by the direction of the column where an identifier is not recorded.

(7) Advance possible:

Determine whether there is a possible move when the previous operation was (5) or (6). If there is a direction recorded in the column of $n3$ (or $n4$), it is the direction to move. In the case of an end point or a border point, no move is possible.

(8) Advance:

Move the cursor one pixel in the specified direction.

(9) Back to preceding point:

When the previous operation was (5) or (6), the cursor jumps to the previous characteristic point. Otherwise, in the case of triple point, the indicator

jumps to the point recorded in column $n1$. In the case of a quadruple point, it jumps to the point recorded in $n3$ or $n4$ depending previous moves. This is done according to the description recorded in the state variable column of the quadruple point, and detail is omitted.

(10) Entrance:

When the cursor arrives back at the entrance, the list of characteristic points in the network is usually complete. If not, processing goes to step (7).

Thinned grain boundary images are, usually, constructed by many networks. Search of the next network is done according to the flow chart in Fig. 3. After a network has been acquired, the cursor moves along the border in a clockwise direction. If it meets a line starting at the border, we judge whether it is already recorded one or not. If that is a new border point, acquisition of the network data as illustrated in Fig. 2 is performed again. This operation is continued until the starting point is reached again. All networks connected to the border line have then been acquired.

The next operation is a search in front of the end points. The indicator moves in the triangle region defined within $\pm 30^\circ$ from the extrapolated line in the forward direction until finding a line. The first point found in this search can be classified to some of the followings: an end point, a point on a line, or a point on the border. If the point is on a line, the cursor moves along the line and finds a characteristic point. If the point has not been recorded in the data lists, it is the entrance of a new network, and acquisition of the network data is performed. If the point is already recorded in the list, its identifier is recorded in column $n2$ as reference data for the repair of defects. When the point is not an end point, another characteristic point on the opposite side of the line is recorded in column $n3$. When the extrapolated line meets the border of the picture, the coordinates x and

Fig. 3.

Flow chart for acquisition of a general image by finding isolated networks.

y are recorded in columns $n2$ and $n3$. When these operations have been carried out on all end points, the data lists are complete.

The above operations may leave some isolated lines or small networks unlisted. They are abandoned as lines unrelated to grain boundaries. If some problems for the grain size analysis remain, they should be solved as general problems related to procedures such as specimen preparation, input of images, binarization, and/or thinning.

4. Repair of Defects

Repair of defects in line images reproduced from the data lists is performed by rewriting the contents of the data lists of characteristic points prepared in the previous section. This processing consists of a local operation determined by the local geometrical data of a defect, and a global operation which is determined by the shape of the network in the wide region around a defect.

4.1. The Local Operation Procedure

The first processing step is to divide a quadruple point into two triple points. In the thinned image of grain boundaries, quadruple points are exceptional and appear with less than about 1 % of the frequency of triple points. There is also some geometrical complexity in the neighbor relationship compared with triple points. Considering of this operation, description of the repair program becomes simple. The procedure is as follows. Firstly, we acquire the directions of the 4 neighboring points using the coordinates recorded in the data lists, and find the minimum angle between the neighboring lines. Then we select the x - or y -direction which is the nearest to the direction of minimum angle. The new position is the point which is displaced one unit distance from the quadruple point in the selected direction. Two triple points are placed at the new position and at the posi-

tion of quadruple point, and they are added to the list of the triple points. The triple point which has the neighboring point recorded in the column $n1$ of the quadruple point is called A and the other point is called B. The data of the new triple points are then as follows: the neighboring points $n1$ and $n2$ values of A are $n1$ and $n2$ of quadruple point, $n3$ is the identifier of the triple point at B. The identifier in $n1$ of the triple point at B is the identifier of the triple point at A, and $n2$ and $n3$ are $n3$ and $n4$ values for the quadruple point. Finally the identifier of the quadruple point recorded in the list of neighboring 4 points are rewritten using the identifier of the corresponding new triple points.

A standard length D is necessary for decisions made during the repair process. D is defined as the length of a side line of the image, 400, divided by the square root of the total number of characteristic points in the line image. Three kinds of local operation are applied to all end points, based on the data recorded in the columns $n2$ and $n3$.

(1) The first operation is applied when 2 end points are mutually the nearest end point found in the forward direction. If the distance between them is less than D , the point pair is connected by a line, which is regarded as the missing grain boundary. The operation on the data lists consists of erasing of the pair of end points, and substituting with 2 curved points. The data of the end points are transferred to the pair of curved points. Finally the identifier of the end point of each neighboring point is rewritten to be the identifier of the corresponding new curved point.

(2) The second operation is applied when the columns $n2$ and $n3$ in the end point list contain data corresponding to coordinates on the border of picture. If the junction of the branch with an end point is a triple point, and the angle of a stem bend at the triple point is less than 150° , and the distance between the end point and the border line is less than

D, a new border point is placed at the coordinates recorded in $n2$ and $n3$, and a new curved point is added to substitute for the end point. Finally, the end point identifier in the neighboring point list is replaced by the identifier of the new curved point.

(3) The third operation occurs when the neighboring point recorded in column $n1$ of the end point is a triple point. If the length of the branch is less than D , and the stem angle is greater than 160° , the end point and its branch are erased. The triple point is substituted by a curved point. Finally, the identifier of the erased triple point recorded in the list of the pair of neighbor points is rewritten using the identifier of the new curved point.

Those local operations are carried out very quickly by comparison with a global operation described in the next section. Through these local operations the number of end points can be reduced by as much as one half.

4.2. The Global Operation Procedure

The purpose of this operation is to carry out the general procedure for the judging, whether the end point can be connected to a suitable point on the opposite side. This judgment and any commands should be expressed using necessary rules similar to those used in the repair of line images by visual observation.

Fig. 4 illustrates the procedure. (a) is a part (120×120) of an input line image. (b) is the image reproduced from the data lists. The small circles A–G are end points, which are the objects of repair. The end points F and G can be connected by the

local operation described in the previous section. As an example of a global operation, we deal with a closed loop containing 4 end points A, B, C, and D.

In the global operation, we consider the shape of the closed loop surrounding an end point. In the first step, we seek candidate points which could be connected with the starting end point. These are selected from the other end points or curved points which are concave towards the inside of the loop. The search is carried out by successive movements over the neighboring characteristic points on the loop. The search starts from an end point and is controlled by the following rules:

(1) At a triple point, the cursor moves to the neighbouring point on the left hand side of the search direction.

(2) At an end point, the cursor returns to the previous point.

(3) At a border point, the cursor moves to the neighboring border point in an anticlockwise direction on the border.

(4) The search continues until it comes back to the starting point.

The sequence of the identifiers of the 3 neighboring points of a triple point in the data lists is arranged in a clockwise manner during the process of list preparation as described in Chap. 3. This is a necessary condition for efficient recognition of the closed loop surrounding the end point, and the characteristic points constructing the closed loop are easily found simply by referring the identifiers of neighboring points in the data lists. The basic parameters necessary for recognition of the shape of the loop

(a) Input image
(b) Image reproduced from data lists
(c) Repaired image 1
(d) Repaired image 2

Fig. 4.
Illustration of the procedures for recognition of the geometrical shape of a closed loop containing defects, and its repair.

are the length and the direction of a line connecting two points. The accuracy in numerical calculation of the length is sufficient within the minimum pixel interval, and the accuracy of angle determination is 1° . For the high speed calculation, look up tables of functions required are precalculated, and the calculations are done using the tables.

Lengths and angles which are useful for deciding connections between characteristic points, are recorded during the course of progression round a closed loop. They are selected from the lengths and directions of the lines connecting neighboring points, and the distances between the current point and the starting end point. Points which are possible candidates for connection with the starting end point are those which had the minimum distances during progression round the loop. The minimum points are usually other end points or curved points concave toward the inside. A list of minimum points is thus prepared during progression round the loop.

The details of this operation are illustrated with reference to the loop containing 4 end points A–D in Fig. 4(b). When the order of points in the row is A, B, C, and D, the operation starts from A. The closed loop is traced in the order A *a* B *a* b c d C D *d* e f g h b a A. Points which have minimum distance from A, such as g, D, etc., are recorded, and g is selected as the connecting point from A according to the rule described below. The next loop starts from B and ends as . . . g A a B. The operation for B is similar to that for A, and it is repeated for C and D.

There are various methods for selecting the connecting point from the list of candidate points. We describe rules adopted in the program used in practice, in the next section. When the candidate point for connection is an end point, the surrounding network has the shape illustrated in Fig. 5. The lengths and angles used during the connection procedure for the 2 end points are as follows.

- a*: the distance from the end point to the junction of the branch,
- ϕ : the angle between 2 lines supporting the branch,
- b*: the distance from the candidate point to the junction of the branch,
- ψ : the angle between 2 lines supporting the branch,
- d*: the distance from the end point to the candidate point, and
- θ : the angle between the line extrapolated from the end point and the line connecting the two end points.

Fig. 5. An example of the geometry around defects.

The end point and the candidate point are connected if one of the following conditions is satisfied:

- (1) $(\theta < 90)$ and $(\phi < 120)$ and $(\psi < 120)$ and $(d/2 < a+b)$, or
- (2) $(\theta < 90)$ and $(\phi < 90)$ and $(\psi < 90)$ and $(d/5 < a+b)$.

When the candidate point is an curved point, they are connected if the following condition is satisfied:

- $(d < a \times 2)$ and $(\phi < 120)$.

When the candidate point does not satisfy the above rules, similar process is done for the next closest candidate point. This is repeated until a connecting point is found. If there is no point which satisfies the connection rules, the line from the end point to the junction of the branch is erased. When all the end points are connected or erased by the above procedure, the data lists of characteristic points are complete, and the image reproduced from the data lists is the triple point network.

Figs. 4(c) and 4(d) are examples of final pictures reproduced from the data lists repaired by the same method except the step (3) in the local operation (Sec. 4.1). In the repair of Fig. 4(c), the condition for erasing of short branch was

- (Stem angle > 160) and (Branch length $< D$)

In the repair of Fig. 4(c), the condition for angle was omitted and short branches with end points were erased using only the length condition. The reason for the difference in Figs. 4(c) and 4(d) was that the branches A and C in Fig. 4(b) were erased during the local operation in the procedure for (c).

The constants in the connection conditions in the global operation were determined on the basis of a qualitative judgment made during repair by visual operation. The effect of the change in the value of the constant on the final picture is small. The evaluation of the final picture should be made by the comparison with the original image under the microscope or its microphotograph. There seems to be no absolutely correct answer for the repaired grain boundary images. The final picture could possibly be evaluated by marking under examination, similar to the case of repair by visual observations.

5. Recognition of Individual Grains

Individual grains in the repaired images are recognized for the characteristic points which compose a grain in the complete triple points network. By the use of the identifier list of characteristic points in a grain and the coordinates of each point recorded in the lists of characteristic points, we can get complete geometrical information about the shape of every grain. Grain lists are prepared by the following procedures.

Starting from a border point on the left of the upper border line, the cursor moves to the first border point. It moves to the neighbor point recorded in the lists. On the triple point it moves to the neighboring point on the left hand side. The names of the characteristic points passed in this process are recorded in the order of the progression. On arrival

at a border point, the next movement is to the nearest point on the border line in an anticlockwise direction. When the border point is not the start point of the grain, the cursor continues the advance until coming back to the starting point of the grain. The name list of the first grain is complete.

Next, the cursor move backwards the previous path until finding a border point which was passed only once or a triple point which was passed once or twice. This point is the starting point of the next grain, and the process described above is repeated. If starting point cannot be found in the grain, the cursor jumps to the previous grain, and search for the starting point of the next grain. By repeating these operations, the cursor finally comes back to the starting grain. The grain lists are then completed.

6. Practical Example

The procedures described in Secs. 3 and 4 are programmed in C language run on a personal computer, NEC PC-9801VX (the performance is similar to IBM PC/AT). All the procedures were arranged in one program, whose total executable file size was 190 kB. Fig. 6(a) shows the input image prepared by the simple thinning program developed by Sakaue.¹⁾ In this processing, small regions of less than 50 dots in area after binarization were eliminated before the thinning process.

The number of characteristic points acquired from the input image (Fig. 6(a)) was as follows: 63 border points, 286 triple points, 1 quadruple point, 109 end points, and 442 curved points. The number of triple points listed in the array was 305, due to the addition of new triple points during the repair process. The number of triple points surviving in the repaired image shown in Fig. 6(b) was 234. The number of the border points listed in the array was 67, and the number surviving in the final list was 58. The number of grains on the border line in Fig. 6(b) is 56, the number of inside grains is 91, and the total number

is 147. The maximum number of characteristic points (including curved points) making up a grain in Fig. 6(b) was 43. There were 5 grains which had more than 30 characteristic points. The processing time of Fig. 6 from the start of the program to the end of acquiring data of all characteristic points was 20 s. The time from the start of repair to the end of grain counting was 20 s.

The above operations were carried out on 40 thinned grain boundary images, which were prepared by standard or simple methods of Sakaue's thinning program¹⁾ on 20 gray level images recorded on disks. The gray level images were input by Ito²⁾ from microphotographs (400 times magnification) of annealed ultra low carbon steel sheet. The original pictures were prepared as test samples for joint research projects in the Grain Size Group of the Committee on Material Evaluation by Image Analysis, Basic Research Asso. on Specific Subjects of The Iron and Steel Institute of Japan. All of the input images could be converted to networks of triple points, even if there were some abnormal and complex areas in the thinned images. The results showed that the time for the conversion of a line image into data lists was proportional to the total number of characteristic points, and the repair time was proportional to the number of end points.

7. Discussion

In the method described here, preparation of data blocks of characteristic points was carried out by tracing the network of thinned grain boundary images, and the repair of defects was done by list processing. The conversion of a line image into data lists of characteristic points is usually carried out by vectorization,³⁾ in which the characteristic points (except curved points) are extracted by template matching. Our program was very complicated in comparison to the vectorization method, but the processing time is more than 10 times shorter. The data lists

(a) Input image

(b) Processed image

Fig. 6. An example of the repair of the thinned microstructure of ultra low carbon steel.

prepared by vectorization represent a line image using a set of lines with given length and direction, and can only reproduce the structure of original line image approximately. The lists produced by our method have information about the relative direction of the neighboring points around triple and quadruple points, in addition to the vector data. This is useful for recognition and comparison of the shape of line images, and the speed of pattern recognition becomes increases.

The defects in line images of grain boundaries are the cause that fully automated grain size measurements are not possible. The repair process developed in our work gives an automated grain size counting capability as good as visual observation, as tested by Mishima,⁴⁾ when a thinned grain boundary image is prepared well. If there are no abnormal patterns in the line image and almost all the lines correspond to actual grain boundaries, the network pattern of triple points prepared by this procedure can be used for the grain size analysis. Moreover, the combination of the data list of characteristic points in the final image and the identifier lists of the characteristic points of each grain, make it possible to extract information about the geometrical shapes of individual grains and their distribution in materials.²⁾

In measurements of grain size made in copper alloys and austenite stainless steels, twin boundaries are usually omitted. In list processing, twin boundaries can easily be detected by the straightness of the line and the shape of the triple point on both side of the twin boundary.

In the case of 2 phase alloys, the 3 kind of grain boundaries, which are distinguished by the phases on either side of the boundary, should be recognized in the thinning process. If this is possible, corresponding lists of characteristic points can be made, and more complex measurement and analysis becomes possible for real 2 phase materials. It is expected that the microstructure of materials can be treated as images reproduced from the data lists of idealized images. In the near future, it should be possible to store the microstructure of materials as the data lists in material databases.

8. Conclusions

A computer program for the automated repair of defects in thinned grain boundary images has been

developed. The method and its performance are as follows:

(1) The geometrical structures of line images containing defects are recognized with using data lists of characteristic points, in which the coordinates and the names of the neighboring points are recorded.

(2) Repair of the line images consists of erasing undesirable lines and drawing lines between pairs of characteristic points. The decision rules are similar to those used in visual observations. The repair operation comprises rewriting the contents of the data lists. All end points in original images are eliminated in the images reproduced from the repaired data lists.

(3) The program was written in C language and executed on a personal computer, NEC PC9801VX. When the total number of the characteristic points in the input image is about 1 000, and the number of defects (end point) is about 100, repair may be complete in 40 s.

(4) If there is no abnormal structure in the input image, the repaired image of triple point network may be used for recognition of the shape of individual grains. The repair program can be used as part of fully automated grain size analysis.

Acknowledgments

The present work formed a part of a joint research project organized by the Committee on Material Evaluation by Image Analysis in Basic Research Association on Specific Subjects, ISIJ. The author expresses his thanks for the support of ISIJ, and for the cooperation of Prof. K. Ito of The University of Tokyo, Dr. K. Sakaue of Electrotechnical Laboratory, and Dr. Y. Mishima of Tokyo Institute of Technology.

REFERENCES

- 1) K. Sakaue: *ISIJ Int.*, **30** (1990), 475.
Report of the Committee on Material Evaluation by Image Analysis, ISIJ, ed., ISIJ, Tokyo, (1989), 2.4.2, 30.
- 2) K. Ito: *ISIJ Int.*, **30** (1990), 490.
Report of the Committee on Material Evaluation by Image Analysis, ISIJ, ed., ISIJ, Tokyo, (1989), 2.4, 26, and 2.4.4, 48.
- 3) H. Tamura, ed.: *Introduction to Image Analysis*, Souken Shuppan, Machida, Tokyo, (1985), 224.
- 4) Y. Mishima: *ISIJ Int.*, **30** (1990), 496.
Report of the Committee on Material Evaluation by Image Analysis, ISIJ, ed., ISIJ, Tokyo, (1989), 2.4.5, 52.