Review

Application of Boundary Element Method to Rolling Technology with Special Respect to Flatness and Crown of Plate and Sheet

Junji KIHARA

Department of Metallurgy, Faculty of Engineering, The University of Tokyo, Hongo, Bunkyo-ku, Tokyo, 113 Japan.

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This article introduces the concept and formulation of the boundary element method, and its application to rolling technology with special respect to the flatness problem in flat rolling. The possibility of utilization of the boundary element method in the inverse analysis is also discussed, and the residual stress analysis in the rolled sheet and estimation of the rolling pressure distribution along the width direction are presented as numerical examples. With use of the boundary element methods the effect of the ratio of sheet width to barrel length is discussed. With increase in the ratio, the crown of the rolled sheet decreases. The effect of the roll bending force on the tension distribution in the rolled sheet is also simulated.

KEY WORDS: boundary element method; inverse analysis; residual stress; rolling pressure distribution; crown; flatness; sheet; plate; strip.

1. Introduction

The boundary element method is one of the most useful general numerical methods, like the finite element method. The boundary element method has been applied to all fields in analysis of the boundary value problems.

Now, rolling technology requires the analysis of several types of boundary value problems, *e.g.* both in the deformation of and heat conduction in rolls and rolled products, and many sorts of numerical equations have been applied to the rolling technology except for some numerical examples performed by the author and his colleagues.

The reason this method has not frequently been applied to this field is thought to be as follows. The development of numerical methods for rolling technology began brilliantly with the work of von Karman, when he originated a new field of applied mathematics called engineering mathematics. The slab method, which was proposed by him, obtained its theoretical base by Orowan, and many researchers have used it successfully to estimate the rolling load or torque in flat rolling. The application of either the elasto-plastic or the rigid-plastic finite element method to calculate the displacement field, including the free surface displacement in the rolled products and also the rolling energy has demonstrated that the numerical aspect of rolling technology considerably. For calculation of the deformation of rolls, bending and flattening, a useful numerical technique based on Green's theorem has been developed. On the whole, the qualitative understanding of rolling technology is possible with these well developed numerical methods, and there has not been enough pressure to necessitate more powerful numerical techniques.

The author, however, supposed that the application of a more accurate, integrated and reasonable numerical method would much advance rolling technology. The boundary element method is such a powerful numerical technique,^{1,2)} and

has been used to investigate the residual stress analysis of a rolled sheet, the estimation of the deformation of a roll, the inverse analysis of the thermal property of a cooled billet and the estimation of the temperature distribution in the work roll in cold rolling, all of which are important in rolling technology.

In this article, the principle and formulation of the boundary element method will be introduced with respect to its feasibility or utility in so-called inverse analysis for those rolling people who are yet unfamiliar to the boundary element method. The inverse analysis is useful in correctly measuring and evaluating rolling parameters. Next, some numerical examples of analysis using the boundary element method will be summarized with special respect to the flatness problems in sheet or plate rolling.

2. On the Boundary Element Method

2.1. Formulation

The method is based on both the reciprocal theorem and the principle of virtual work. It involves solving a linear simultaneous equation deduced by discretising approximation from the boundary integration equation of continuum mechanics (1). Details of the deduction will be presented below.

An illustration is the case of elasticity. The region of a body which is loaded or given displacement with constraint is denoted by R . The surface of the body is denoted by B . $u(o;i)$ is the displacement at point o and direction i . $u(o;i,j)$ is the derivative of $u(o;i)$ for j direction. $u(i)$, *etc.* represent the displacement field *etc.* $D(ijkl)$ represents the elastic modulus. Then, the stress at o , denoted by $st(o;ij)$ is represented by

$$st(o;ij) = D(ijkl)u(o;k,l)$$

The strain energy W for the whole body is expressed as

$$W = \int_R u(;i,j) D(ijkl) u(;k,l) dR \quad \dots \dots \dots (1)$$

Now, $u^*('h';mi)$ represents the displacement field in i direction at a general point due to the unit point force to m direction at point h , and we consider the integral W^* , namely,

$$W^* = \int_R u^*('h';mi,j) D(ijkl) u(;k,l) dR.$$

Here, $u^*('h';mi)$ must fulfill the next relationship.

$$D(ijkl) u^*('h';mk,lj) + Dd(h;mi) = 0 \quad \dots \dots (2)$$

$Dd(h;mi)$ is Dirac's delta function and 0 at any point other than h , which is called the source point. The function fulfills the equation:

$$\int_R Dd(h;mi) dR = Dk(mi)$$

where $Dk(mi)$ is Kronecker's delta function. Then, $u^*('h';mi)$ is the well-known Kelvin fundamental solution in elasticity.³⁾ There is the same sort of fundamental solution in other continuum systems.²⁾ With the application of partial integration for W^* , this is represented as

$$W^* = \int_B D(ijkl) u(;k,l) n(j) u^*('h';mi) dB \\ - \int_R D(ijkl) u(;k,lj) u^*('h';mi) dR,$$

and because of the principle of virtual work,

$$W^* = \int_B D(ijkl) u(;k,l) n(j) u^*('h';mi) dB,$$

where $n(j)$ represents the j component of the outer normal of the surface. Here, $D(ijkl) u(;k,l) n(j)$ is the traction $p(;i)$ acting on the surface. Then,

$$W^* = \int_B p(;i) u^*('h';mi) dB.$$

On the other hand, by the reciprocal theorem,

$$W^* = \int_R u(;i,j) D(ijkl) u^*('h';mk,l) dR,$$

and by partial integration,

$$W^* = \int_B D(ijkl) u^*('h';kl) n(j) dB \\ - \int_R D(ijkl) u^*('h';mk,lj) u(;i) dR \\ = \int_B p^*('h';mi) u(;i) dB + Dk(oh;mi) u(o;i)$$

where $Dk(oh;mi)$ is unity when o coincides with h and m coincides with i . We obtain the next relationship.

$$Dk(oh;mi) u(o;i) + \int_B p^*('h';mi) u(;i) dB \\ = \int_B u^*('h';mi) (;1) dB \quad \dots \dots \dots (3)$$

If the source point h exists at the inner region of the body, the displacement $u(h;m)$ can be estimated by the surface integral of the displacement and the traction on the surface.

$$u(h;m) = \int_B u^*('h';mi) p(;i) dB \\ - \int_B p^*('h';mi) u(;i) dB \quad \dots \dots \dots (4)$$

The displacement in this region is represented perfectly by the boundary values.

If the source point h lies on the boundary, the integration of $\int_B p^*('h';mi) u(;i) dB$ must be performed by the application of the Cauchy integration theorem because of the singularity of $p^*('h';mi)$ at point h .

First, the boundary is discretised into enough small boundary elements for the distribution of $u(;i)$ or $p(;i)$ to be assumed as constant or, at most, linear on the boundary element. We obtain a linear simultaneous equation such as

$$H(bh;mi) u(b;i) = G(bh;mi) p(b;i) \quad \dots \dots (5)$$

where

$$H(bh;mi) = 0.5 Dk(bh;mi)$$

$$+ \int_{BE(b)} p^*('h';mi) f dB$$

$$G(bh;mi) = \int_{BE(b)} u^*('h';mi) f dB.$$

BE(b) is a discretised boundary element where the representative point b belongs, and f is the multiple of the interpolation function by the Jacobian of the local coordinate in each element to the global one depending on the element model. f is unity for the constant element model. Point b is the geometrical center of the element when the constant element model is adopted, and is the nodal point connecting several elements when the linear element model is adopted. The integration on each element is evaluated by the Gauss numerical integration unless the representative point coincides with the source point. Otherwise, namely in the case of a self element, the integration must be done carefully by use of some special technique to maintain accuracy.

After solving all boundary parameters, we can calculate the strain or stress in the region by the relation (4) and the definition of strain or stress. Considering the obtained and prescribed boundary parameters as the constants, the strain $sr(o;ij)$ is estimated by the next equations.

$$sr(o;ij) = \{ u(o;i,j) + u(o,j,i) \} / 2 \quad \dots \dots (6),$$

$$u(o;i,j) = \int_{BE(b)} u^*('o';mi,j) f dB p(b;m)$$

$$- \int_{BE(b)} p^*('o';mi,j) f dB u(b;m) \quad \dots \dots (7)$$

and

$$st(o;ij) = D(ijkl) \int_{BE(b)} u^*('o';mk,l) f dB p(b;m)$$

$$- D(ijkl) \int_{BE(b)} p^*(o'; mk, l) f dB u(b; m) \dots \dots \dots (8)$$

Here we should like to represent the coefficients appearing in Eqs. (7) and (8) as follows:

$$SR(o; mij) = \int_{BE(b)} \{ u^*(o'; mi, j) + u^*(o'; mj, i) \} f dB / 2,$$

$$DR(o; mij) = \int_{BE(b)} \{ p^*(o'; mi, j) + p^*(o'; mj, i) \} f dB / 2,$$

$$ST(o; mij) = D(ijkl) \int_{BE(b)} u^*(o'; mk, l) f dB$$

and

$$DT(o; mij) = D(ijkl) \int_{BE(b)} p^*(o'; mk, l) f dB.$$

The finer the boundary discretised, the more the degree of freedom is saved compared to the finite element method for the same degree of accuracy of the result. The scale of the data storage cannot be so saved as expected in the boundary element method, because all boundary parameters relate to each other and H and G matrices are full matrices. It is well known that, in the finite element method, parameters must relate with several parameters connected by the pertinent elements and that only one rigidity matrix is necessary. When general numerical methods were introduced, the boundary element method was not so highly evaluated as the finite element method, because the computation speed was much slower and too much time is necessary to calculate the components of the full matrices. Recently, however, with the great advances in hardware this disadvantage has disappeared, and the above advantages seems to be certified.

The other important and different feature of the boundary element methods from that of the finite element method is that the boundary parameters are the traction, not the nodal force as in the latter method, and the displacement is the same. This is sometimes a problem when a hybrid system composed of the two methods is adopted.

2.2. Modification of the Formulation

Formulation of boundary element modelling is done in the cartesian coordinate space. In rolling technology, however, the shape of the region is frequently axisymmetrical as is that of a roll. It is convenient that the formulation can be modified to enable calculation of a non-axisymmetrical problem of a axisymmetrical body. In the cylindrical coordinate, $p(b; i)$ and $u(b; i)$ in the cartesian system are converted to $p(b; l)$ and $u(b; l)$ of the cylindrical expressions with use of transformation matrix $T(r/x)$. Thus,

$$p(b; l) = T(l/i) p(b; i).$$

The same relationship is valid between $u(b; l)$ and $u(b; i)$, so that boundary element simultaneous equation can be obtained for the parameters in the cylindrical system. $T(x/r)$ is

the inverse form of $T(r/x)$, and $T(i/l)T(l/j) = Dk(ij)$. Using $T(x/r)$,

$$p(b; i) = T(i/l) p(b; l)$$

and

$$u(b; i) = T(i/l) u(b; l).$$

Substituting these relationships to Eq (5),

$$H(bh; mi) T(i/l) u(b; l) = G(bh; mi) T(i/l) p(b; l) \dots \dots \dots (9)$$

is obtained, where the components of H and G matrices are estimated on the cylindrical or conical boundary elements (4).

2.3. Formulation of Inverse Analysis by Application of the Boundary Element Method

The boundary element method appears to be an excellent numerical calculation engine for the system of inverse analysis, because, as mentioned, all surface variables are related to each other by the H and G full matrices. In rolling technology, the surface traction distribution on the contact surface is of the great interest. In estimating traction, we must determine the deformation of both the elastic tool and the plastic material on the basis of the surface compatibility and the equilibrium of traction. For simplicity we often assume that the deformation of the tool is negligible, and estimate only the deformation of the material.

Other than the interaction between the tool and the material, the residual stress in the worked material is also of interest. One mean of determination is to cut off part of the worked material, measure the deformation generated, calculate the traction released on the cutting surface based on an assumption of redistribution of the residual stress and evaluate the residual stress distribution.

In any way, some assumptions are necessary when the problem is expected to be solved only in the side of the plastic material or the elastic body.

If the measurement of the displacement or the strain is possible on the boundary free from traction, that can be utilized for the inverse analysis of the traction and displacement on the contact surface or the residual stress in a worked material without any assumption other than boundary element modelling.

On the contact surface or the cut surface through a worked material, both the displacement and traction are unknown parameters in principle. The boundary element equation, therefore, is impotent because of the excess unknown parameters. The shortage of equations for the unknown parameters is the multiple of the number of representative points (the number of the discretised element on the contact or the cut surface) with the number of the dimension of the space (for example, 2 in 2 dimension space). The impotence will be cured by supplying the auxiliary equations relating the measured parameters to the boundary parameters. The district where the displacement $u(b; i)$ is prescribed is B(1), that where the traction $p(b; i)$ is prescribed is B(2), and the district where neither parameter is unknown is denoted as B(3). The representative points of B(1), B(2) and B(3) are represented as $(b1.)$, $(b2.)$ and $(b3.)$, respectively. The boundary element equation can be written as

$$\begin{aligned}
 & G(b1.h;mi) p(b1.;i) - H(b2.h;mj) u(b2.;i) \\
 & - H(b3.h;mi) u(b3.;i) + G(b3.h;mi) p(b3.;i) \\
 & = H(b1.h;mi) u(b1.;i) - G(b2.h;mi) p(b2.;i) \\
 & = F(h1. + h2.;m) \dots \dots \dots (10),
 \end{aligned}$$

where (h1.) and (h2.) are the points corresponding to (b1.) and (b2.), respectively. As mentioned above, the impotence of the equation to solve the boundary parameters especially on the contact surface B(3) must be cured by the supplement of some auxiliary relationships, examples of which are that between the boundary parameters on B(3) and the surface strains on B(2), that between the total load or total work of the tool and the surface traction on B(3) and the difference of the displacement on B(3) which is estimated from the shape of the deformed material relating to the traction distribution on the same boundary B(3).

The such auxiliary equation is represented in general by

$$AU(bh;mi) X(b;i) = AF(h;m) \dots \dots \dots (11),$$

the concrete form of which, of course, differs by the respective case. The simultaneous equation is composed by combining Eq.(10) with Eq.(11).⁴⁾

If there are excess relationships available, this allows obtaining of a solution group of some scale and discussion on the reliability or reproducibility of the system from a statistical point of view.

The boundary element method is a useful technique for modelling and simulation of the process.⁵⁾ Therefore, this method is also available for a so-called curve-fitting method which is a traditional inverse analysis.⁶⁾

3. Practical Examples

3.1. Preface

The numerical examples which the author will present here have been published in conference proceedings, institute articles or textbooks. Because of copyright restrictions, only concepts and schematic features of the results will be given. Details of the results can be found in the original published articles listed at the end of this review.

3.2. Estimation of the Residual Stress Distribution in a Rolled Sheet

Residual stress is anticipated in a rolled sheet. If it is distributed at an intensity exceeding a critical level which is determined by the buckling condition, the shape of the sheet will be waved. Even though the sheet may not be buckled, blanks cut from it will have a poor shape because of relief by deformation due to the rearrangement of residual stress distribution. The determination of this distribution is important in order to evaluate the rolling conditions resulting from the rolling parameters other than the estimation of the quality of the sheet itself.

The sheet illustrated in Fig.1 is cut along the line A-A', and the strain due to the cutting is measured at points near this line A-A'. Half of the sheet bounded by A-A' and the other boundary is in a state of plane stress. There is no B(1) boundary, but B(2), a free boundary, and B(3), the A-A' line. The direction A-A' is taken as the width direction of the sheet and coincides with the direction of the y axis, and the rolling direction coincides with the x direction. The cutting

along A-A' is thought of as the loading $p(h3.;i)$ on the line; that is, $str(h3.;ij)$ represents the residual stress on the line, and

$$\begin{cases} p(h3.;x) = -str(h3.;xx) \\ p(h3.;y) = -str(h3.;xy) \end{cases} \dots \dots \dots (12).$$

Reversely, the $-p(h3.;i)$ is loaded on the A-A' line, and after calculation of all the boundary parameters, we can obtain the residual stress distribution in the half sheet using Eq.(8).

In this case, there is no B(1) boundary, and $p(b2.;i)$ is zero as mentioned above. We adopt the relation between the stress in the region and the boundary parameters as the auxiliary equation. Therefore, the equation to estimate the parameters on the A-A' line is:

$$\begin{cases} H(b2.h;mj)u(b2.;i) - G(b3.h;mi)p(b3.;i) \\ \quad + H(b3.h;mi)u(b3.;i) = F(h2.;m) \\ -DT(oh2.;mij)u(h2.;i) + ST(oh3.;mij)p(h2.;i) \\ \quad - DT(oh3.;mij)u(h3.;i) \\ \quad = st(o,mj) \end{cases} \dots \dots \dots (13)$$

The three strain components at points o are measured. These strain components are measured by losette gages, and converted to the corresponding stress components. Therefore, the least number of o points is $2/3$ that of the representative points of the boundary element model on the A-A' line.

By sensitive analysis, the measuring point must not be too distant, that is, it must be less than 3 times that of the discretising distance on the A-A' line apart from that line. This is explained by St. Venant's principle of elasticity.

On the other hand, the point must not be too near the line, because the accuracy of stress determination by Eq.(8) becomes poorer when the points are closer to the line than the discrete length. The accuracy of the calculation of stress or strain near the boundary can suffer by ill treatment, in which Gaussian numerical integration or another inappropriate integral method is adopted to evaluate the components relating to the nearest element severely affected by the singularity of $P^*(h';mi,j)$. If we give a rigid displacement to the body, neither strain nor stress increment occurs. If this result is used to determine the component of $DR(hh;mmj)$ or $DT(hh;mmj)$ where neither (hh) nor (mm) means summation

Fig. 1. The cutting line A-A' and components of traction and stresses.

but (hh) means that the component is the self element and (mm) means the component is the point force in the same direction as that of the displacement or traction, we obtain much more accurate numerical results, and the problem is solved. Unfortunately, this has not yet been applied to the analysis of the residual stress distribution.⁴⁾

As a whole, we can estimate the unknown displacement and traction acting on the contact plane or along the cutting line using Eq.(13) with the aid of strain measurement in the region for the 2-D problem, and on the free B(2) boundary for the 3-D problem.

3.3. Determination of the Line Pressure of Roll Separation in Plate Rolling

The effect of the intensity and pattern of distribution of line pressure, which is the rolling load per unit width on the profile of the rolled sheet, is of much interest to understand the flatness problem in sheet or plate rolling. An inverse analytical technique using the boundary element method was adopted to evaluate the line pressure distribution from the measurement of the crown of the rolled plate.⁶⁾

The barrel length of the work roll is 5 490 mm, and its diameter is 1 186 mm with an initial crown of 0.50 mm. The barrel length of the back-up roll is 5 390 mm, 100 mm shorter than that of the work roll. The author guesses that this allows to increase the effect of the work roll bender. The diameter is 2 360 mm. The interaction between the work roll and the back-up roll is modelled as follows:

Along the length of contact, which is the length of the back-up roll in this case, the pressure distribution is assumed to be uniform, and the width of the contact is assumed to be the same as that calculated by Hertzian theory. Neither bending effect nor constraint action is given to the work roll by the back-up roll. The back-up roll supports only the sum of the roll separating force and the bending force.

The work roll is modelled as a simple cylinder without any neck. The effect of bender force is modelled as the shear force acting on the side plane, and that of the bending moment is modelled as the bending stress (normal stress) distribution on the plane. The bender force is applied to each neck at a point 140 mm away from the end of the work roll and the force is 2 400 kN in all cases.

The wear crown is expressed in the same manner as the initial one, and this is the value at the center. The thermal crown is expressed by the two point values, one at the center of the plate and the other at the edge. The real crown is estimated by the addition of the wear crown to and subtraction of the thermal crown from the initial one. The crown curve is modelled to be sinusoidal.

The initial and the reduced mean thickness of plates, those widths, the total rolling load, the thermal crown and the wear crown are prescribed. Above these, the displacement of the axis of the work roll evaluated by the conventional method is also given.

The magnitude of frictional stress is usually about one order less than that of normal pressure, and the frictional stress changes its direction along the arc of contact. Therefore, the effect of the stress on the roll crown is assumed to be negligible with comparison to that of the line pressure.

The boundary element modelling of the work roll will be described below. The axi-symmetrical system is adopted as described in Sec.2.2. The length of the arc of contact in each case is taken as the length of the discretised element to

rolling direction at the contact area. The length of the element at the contact area between the work roll and the back-up roll to rolling direction is taken to be equal to the contact length calculated by Hertzian theory. The roughness of discretisation to rolling direction increases with distance from the arc of contact or the contact area with the back-up roll, and the maximum angle length of the element is 0.6 radian. The number of elements in the barrel is 12, and the discretisation along the width is finer near the edge of the plate. The degree of freedom in the boundary element equation is 531. The sort of element is a linear one. The Young's modulus and Poisson's ratio are 210 GPa and 0.3, respectively.

First the estimation for displacement of the axis in the work roll is given in each case by the usual traditional method. If the information can be utilized as an auxiliary relationship between the surface variables of the work roll, the problem becomes very simple, and the line pressure distribution can be easily solved by the boundary element equation associated with the discretised form of Eq.(4). Trials of this nature were made, but the results were illegal, because the tensile tractions appeared at the arc of contact.

Alternatively, as auxiliary information the difference of axial displacement $ud(b3.c;r)$, which corresponds to the working crown of the work roll, is thought to be useful. Here ($b3.$) means the representative point at the arc of contact, (c), which is one of the ($b3.$)s, means the center of the barrel and $ud(b3.c;r)$ is the difference of the axial displacement at ($b3.$) from that at center. This information is gained by measuring the crown of the plate. Another auxiliary relation is necessary, because the number of $ud(b3.c;r)$ data is short by one. The relation between the line pressure distribution and total rolling load is adopted as an auxiliary relation. In this case the B(1) region is absent as in the previous section, the roll separating force is denoted as P and the equation to estimate the line pressure distribution is;

$$\left\{ \begin{array}{l} H(b2.h;mi)T(i/l)u(b2;l) + H(b3.h;mi)T(i/l)u(b2;l) \\ - G(b3.h;mi)T(i/l)p(b3;l) = F(h2.;m) \\ u(b3;r) - u(c;r) = ud(b3.c;r) \\ p(b3.;r)BE(b3) = -P \\ p(b3.;l) = Dk(lr)p(b3.;r) \end{array} \right. \dots \dots \dots (14)$$

Three cases of plate rolling are analyzed, and the rolling conditions for each case are listed below.

Case 1:

reduction	: from 15.38 to 13.61 mm
width of plate	: 4 900 mm
roll separating force	: 61 750 kN
thermal crown	: 0.12 mm at edge, 0.36 mm at center
wear crown	: 0.10 mm

Case 2:

reduction	: from 6.71 to 5.82 mm
width of plate	: 2 438 mm
roll separating force	: 40 660 kN
thermal crown	: 0.42 mm at edge, 1.26 mm at center
wear crown	: 0.19 mm

Case 3:

reduction	: from 29.20 to 26.58 mm
width of plate	: 3 200 mm

roll separating force : 24 460 KN
 thermal crown : 0.42 mm at edge, 1.26 mm at center
 wear crown : 0.20 mm

Here, the value of the crown corresponds to change in the diameter of the roll. The given crown of the plate in each case is illustrated in Fig.2. The shape of the crowns differ greatly; especially in case 2 the so-called edge drop is smeared into the whole crown. The other cases reveal a clear edge drop and a minus total crown.

The calculated results of the rolling pressure, by which the length of the arc in contact is multiplied to obtain the line pressure, are shown in Fig.3. Regardless of the different features of crown shape, the mean rolling pressure is distributed in about the same manner. One notable fact obtained by this analysis is that the crown and its relation to the manner of edge drop are critically affected by the ratio of the plate width to the barrel length.

In the analysis here, the pressure distribution along the arc of contact is assumed to be constant, and this is thought as reasonable because the ratio of the length of the arc of contact to the mean thickness of the plate is ranged from 1.4 to 3.5, and the rolling pressure would not drastically change.

3.4. Simulations of the Profile Formation and Residual Stress Generation in Strip Rolling

In the previous section, the effect of the ratio of the plate

Fig. 2. The deviation of thickness along the width.

Fig. 3. The rolling pressure distribution along the width.

width to barrel length of the work roll was described as important in forming the crown of the plate. Using the boundary element model, the effect of the width-barrel ratio was evaluated in strip rolling in a 2-high mill.⁷⁾

The diameter of the roll is 400 mm, and the barrel length is 1 700 mm. The shape of the roll is modelled to be a simple cylinder, and the center of the roll chock is thought to locate 140 mm apart from the edge of the roll. The supporting load and its associating bending moment are modelled as the shear traction at the end (previously referred as "edge") plane and the bending normal stress distribution on the plane respectively. The length of the arc in contact is 10 mm, and the mean rolling pressure is 1 GPa.

Young's modulus of the roll is 210 GPa, and Poisson's ratio is 0.3. The flattening displacement of the surface of the roll is defined by the standard as no displacement occurs at the center of the end plane.

The ratio of the strip width to barrel length is varied from 0.16 to 0.74. The flattening displacement at 20 mm in from the edge of the strip and that at the center of the sheet are plotted in Fig.4. The displacement at the point near the edge of the strip increases with the increase in ratio, while that at the center remains constant. This simulation result corresponds well to the fact mentioned in the previous section.

Discussion of the comparison between the displacement of the generator line at the back side and that of the axis reveals that the usual work roll used in strip rolling is like a bulky body rather than a simple beam.

Now, the boundary element model of a roll is also utilized to simulate the effect of bending force and rolling reduction on the residual stress formation in the strip.⁸⁾ For simplicity, so-called tension feedback is modelled as a simple Tresca like rolling pressure condition:

$$p(b3.;r) = -p + st(b3.;x) \dots \dots \dots (15),$$

where $p(b3.;r)$ corresponds to negative rolling pressure at the point $b3.$ in the arc of contact, p is the rolling pressure in the case of no tension in the strip, and $st(b3.;x)$ is the tension in strip in the rolling direction at the point $b3.$ and at the same time the residual stress after rolling. Here, front and back tensions at $b3.$ are assumed to be equal.

Fig. 4. The comparison of the effect of width-barrel ratio on the surface indentation at center and near the edge of the strip.

Details of the model of the mill are as follows:

The barrel length of both the work and the back-up roll is 1 800 mm. The diameter of the work roll is 400 mm without crown, and that of the back-up roll is 1 100 mm. The back-up roll only is modelled to support the roll separating force with uniform distribution of pressure along the contact area with the work roll and without any bending action as illustrated in the previous section. Bender action is also modelled in the same as in the previous section. The elasticity of the rolls are the same as that in the other illustrations.

The width of the rolled strip is 1 500 mm, and its mean initial thickness is 1 mm. It is assumed that no tension occurs at a 20 % thickness reduction, so the initial crown of the strip is modelled to fulfill this condition. The p in Eq.(15) is determined so that the rolling load is 10 MN, and the rolling load is modelled to be proportional to the mean reduction. The deformation of the strip is modelled to be in the ideal plane strain condition.

The sequence of the simulation is as follows:

The initial crown of the strip, *i.e.* the thickness distribution of the strip, is calculated to be compatible with the work roll crown by the boundary element method under the condition of uniform rolling pressure distribution along the width.

The effect of either the work roll bender or the change in reduction on the work roll crown is calculated by the boundary element technique.

The tension generated in the strip due to the deviation from constancy of the crown ratio between the initial one and that after rolled is calculated on the assumption that the integration yields no tensile force and that the magnitude is proportional to the discrepancy of the reduction from its mean value. The proportionality is thought to be a plane strain tensile modulus of elasticity, but it is not difficult to set a proper value in order to converge the iterative calculation reflecting the tension feedback effect. Only the proportion itself is important, and the magnitude must be controlled from the convergency of iteration.

The crown of the roll is again calculated on the basis of the updated boundary condition, namely the updated distribution of the rolling pressure. The crown is reflected to that of the rolled strip, the tension distribution and subsequently the pressure distribution are calculated. The iteration stops at a certain level of convergency of tension distribution.

After the convergence of iteration, the residual stress distribution is evaluated by multiplication of the plane-strain elastic modulus, in this case 230 GPa, on the deviation of elongation.

Figure 5 shows the effect of the work roll bending force applied on the neck 140 mm away from the end of the roll on the residual stress distribution.

If we can create more realistic models of the tension feed-back and at the same time the proper modulus distribution corresponding to the feedback model, more reliable result of the residual stress distribution will be possible.

4. Concluding Remarks

There are few examples of applications of the boundary element method to analysis the mechanics in the metal work-

Fig. 5. Examples of simulation of the effect of bending force on the residual stress in the rolled strip.

ing processes, except those done by the present author and his colleagues. Therefore, in this article it is impossible to introduce such works done by other people.

Judging from the level of analysis attained, the method has great potential to produce reliable inverse analytical systems to test the situation of rolling diagnostically and provide excellent simulation models of rolling practice.

The author is hopeful that many people will apply the boundary element method in investigating rolling technology, and through discussion with him will develop the technology itself.

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