Mathematical Modelling of Heat Transfer Phenomena in Continuous Casting of Steel

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A steady-state three-dimensional heat flow model based on the concept of artificial effective thermal conductivity has been developed. On the basis of available literature information, boundary conditions to the governing heat flow equation have been applied, and the equation was solved *via* the control-volume based finite difference procedure. The model is sufficiently general and can be applied to various geometrical shapes of relevance to continuous casting of steel. Sensitivity of the predicted results to various numerical approximations including grid configurations, as well as to other modelling parameters such as axial conduction, mushy zone modelling procedure, choice of value of K_{eff} have been extensively studied. It has been shown that assumptions and numerical procedures influence the computed results significantly. Finally, numerical predictions have been compared with three sets of experimental measurements reported in literature on shell thickness in industrial casters. In contrast to some earlier claims, these indicated only poor to moderate agreement between model predictions and experimental results.

KEY WORDS: solidification; continuous casting; steel; mathematical modelling; effective thermal conductivity; heat transfer.

1. Introduction

Heat transfer in the liquid region of a continuously cast section is relatively more complex than in the solidified region due to flow induced in the liquid pool *via* momentum of the pouring stream, by the buoyancy driven natural convection, and also by the electromagnetic stirring, if present. Therefore, in addition to conduction, transfer of heat from the liquid to the solidified region takes place through bulk convection and turbulence. Exact modelling of heat transfer in the solidifying casting consequently requires prior knowledge of flow of liquid steel in the molten pool. This calls for the solution of Navier–Stokes equations for the turbulent flow in conjunction with an appropriate equation of thermal energy transport.

To avoid the inherent complexities associated with the numerical solution of the turbulent fluid flow equations, convective as well as turbulent transport of heat have been either completely ignored¹⁾ or the pool region assumed to be well mixed²⁾ in the earlier studies on heat transfer in continuous casting of steel. A somewhat realistic attempt was made subsequently by Mizikar,³⁾ who took into account the enhanced heat transfer in the liquid due to convection and turbulence by artificially increasing the thermal conductivity of the liquid pool. This implies that the governing equations describing heat flow in any section of strand will be a

pure conduction type equation with a spatially variable thermal conductivity. It has been generally argued^{4,5)} that such an adjustment of thermal conductivity values within the liquid pool is not likely to affect the overall prediction of the rate of solidification and the temperature fields in continuously cast products, since convection predominantly affects the rate at which superheat is removed from the liquid steel, and typically, superheat constitutes only a small fraction of the latent heat of solidification of the liquid steel.

It is however, important to recognize here that the numerical value of K_{eff} was deduced by Mizikar³⁾ through forced fitting the model predictions with those experimentally measured on a slab casting machine. Consequently, so far there is no conclusive evidence in literature to establish that a large thermal conductivity value assigned arbitrarily to the liquid pool can indeed accommodate the effect of fluid motion and turbulence on heat transfer. Despite such an apparent limitation, the concept as introduced originally by Mizikar has been frequently applied by numerous investigators^{5–11)} to investigate various thermal phenomena of relevance to continuous casting.

The general nature of agreement between experimental and computed results, as indicated in these studies,^{8,12)} appears to vary considerably. Based on some theoretical arguments, Mazumdar¹²⁾ attempted to explain the possible reasons for mismatch between

theory and experiment and attributed this to the basic assumption of the modelling procedure itself (*e.g.*, prescription of an unique effective thermal conductivity value throughout the liquid pool to account for the effect of turbulent convection phenomena on heat transfer). In contrast, Brimacombe and coworkers^{8,9)} attributed the cause of such discrepancy to the inherent uncertainties associated with measurements in high temperature industrial systems. More recently Lahiri,¹³⁾ on the basis of an unidimensional heat flow analysis has suggested that the value of effective thermal conductivity in the liquid pool region should be at least 43 times the molecular thermal conductivity of liquid steel. This is however in direct contrast to the equivalent earlier claim that a thermal conductivity value 5–10 times greater than the molecular thermal conductivity is adequate.

Considering such divergent views expressed by the investigators as well as the present world wide interest in an effective alternative approach of modelling, it is naturally important to assess the adequacy of the effective thermal conductivity based model as applied to the mathematical modelling of continuous casting of steel.

Some assessment of the mathematical model studies have been attempted by earlier investigators.^{8,9,12,14)} However, these were quite limited in their scope (*viz.*, unidimensional model, model specific to a particular casting configuration, sensitivity of numerical parameters on computed results were not assessed and so on). In the present study, therefore, the evaluation of mathematical modelling based on effective thermal conductivity has been carried out in a much more comprehensive fashion as compared to those carried out earlier. The salient features of the present investigation are noted below.

(1) A 3-D heat flow model has been considered for the mathematical simulation and a computational procedure developed for solving the same.

(2) The generalized heat flow model developed in this study can be applied to the analysis of practically all continuous casting configurations (billets square and round, bloom, and slab).

(3) The influence of various numerical approximations and parameters on computed results were assessed rigorously. It is to be emphasized here that in no earlier equivalent studies, sensitivity of computed results has been assessed with reference to numerical parameters.

2. Mathematical Formulation

2.1. Assumptions in Modelling

To describe heat flow within the solidifying strand during continuous casting mathematically, the following simplifying assumptions have been incorporated in the present mathematical model.

(1) Influence of fluid turbulence and convection on heat transfer has been taken into account by artificially increasing the thermal conductivity of steel in the liquid pool region.

(2) Solidification is essentially under equilibrium

condition.

(3) Solidification front is flat (*i.e.* planar) with respect to the adjacent liquid.

(4) The cast section is of fixed dimension throughout the process.

(5) Meniscus surface is essentially flat.

(6) Density and specific heat of steel are invariant.

(7) Except for the latent heat release, heat effects associated with other phase transformation reactions (*e.g.*, δ -ferrite $\rightarrow$ austenite, austenite $\rightarrow$ pearlite *etc.*) have been neglected.

(8) Effect of segregation, mould oscillation, bending of strand *etc.* have been ignored.

(9) Due to symmetry of heat flow in square sections (*e.g.*, billet), only a quadrant of its cross-section has been considered for the heat flow analysis. Whereas, for the continuous casting of slab, a semi-infinite geometry has been assumed and heat flow through the narrow face has been ignored. Similarly, for cylindrical billet, axial symmetry has been assumed (*i.e.*, $\partial T/\partial \theta = 0$).

2.2. Governing Heat Flow Equation

Heat balance over a small volume element in the system (**Fig. 1**), in Cartesian coordinate under steady state conditions, can be represented in terms of the following partial differential equation:

$$\rho C U_o \left(\frac{\partial T}{\partial Z} \right) = \frac{\partial}{\partial Z} \left(K_{\text{eff}} \frac{\partial T}{\partial Z} \right) + \frac{\partial}{\partial X} \left(K_{\text{eff}} \frac{\partial T}{\partial X} \right) + \frac{\partial}{\partial Y} \left(K_{\text{eff}} \frac{\partial T}{\partial Y} \right) + S \quad \dots\dots\dots (1)$$

The term on the L.H.S. of Eq. (1) represents heat

Fig. 1. Schematic of a typical continuous casting operation and a three dimensional volume element in the casting strand illustrating the concept of heat balance applied to derive Eq. (1).

flow in the axial (*i.e.* withdrawal) direction (Z) due to the bulk motion (U_o) of the descending strand. The first term on R.H.S. is a conduction term in axial direction, whereas the second and the third terms represent conduction of heat along the transverse plane (*viz.*, X and Y co-ordinates respectively). Since the exact influence of axial conduction on overall heat flow has so far been not demonstrated explicitly, therefore as seen from Eq. (1), the axial conduction term has been included in the present study.

2.3. Procedure for Modelling Effect of Latent Heat Release

The mushy zone, where the latent heat is released, involves several complex phenomena and consequently, to describe 'S' mathematically some idealizations have been made. Thus, in the present study the latent heat release in the mushy zone has been estimated from the distribution of solid fraction (γ_s) assuming equilibrium solidification of steel^{11,12} and represented according to:

$$S = \rho U_o \Delta H_f \frac{\partial \gamma_s}{\partial Z} \quad \text{.....(2)}$$

In Eq. (2) ($\partial \gamma_s / \partial Z$) represents change in solid fraction (γ_s) with progress of solidification in a volume element (see later) during its descent in the axial direction. The solid fraction can be estimated from the relevant portion of iron-carbon equilibrium diagram. At any temperature (T) between the liquidus and solidus temperatures, for a given initial carbon (C_o) content in steel, solid fraction (γ_s) can be estimated by applying Lever rule to the iron-carbon diagram.

2.4. Boundary Conditions

The relevant boundary conditions applied to solve Eq. (1) are summarized below. As stated earlier, for billet casting, only one quadrant of the cross-section has been considered (Fig. 2).

Fig. 2. Schematic of grid distribution in (a) one quarter of a square billet (b) the central vertical plane and (c) a transverse plane.

- (1) at the meniscus ($Z=0$),
 - (a) inside the pouring stream,

$$0 \leq X \leq r_s \text{ and/or } 0 \leq Y \leq r_s \quad T = T_o \quad \text{.....(3)}$$
 - (b) outside the pouring stream,

$$r_s < X \leq a/2 \text{ and/or } r_s < Y \leq b/2 \quad q_z = 0 \quad \text{.....(4)}$$

- (2) at the outflow boundary ($Z=L$),

$$0 \leq X \leq a/2 \text{ and/or } 0 \leq Y \leq b/2 \quad \partial T / \partial Z = 0 \quad \text{.....(5)}$$

- (3) at the axis of symmetry and/or central plane,

$$X=0, 0 \leq Y \leq b/2, 0 \leq Z \leq L \quad \partial T / \partial X = 0 \quad \text{.....(6)}$$

$$Y=0, 0 \leq X \leq a/2, 0 \leq Z \leq L \quad \partial T / \partial Y = 0$$

- (4) at the cast surface,

$$X=a/2, 0 \leq Y \leq b/2, 0 \leq Z \leq L \quad q_s = -K \frac{\partial T}{\partial X} \Big|_{X=a/2}$$

$$Y=b/2, 0 \leq X \leq a/2, 0 \leq Z \leq L \quad q_s = -K \frac{\partial T}{\partial Y} \Big|_{Y=b/2} \quad \text{.....(7)}$$

The boundary condition (1) originated from the fact that the temperature inside the pouring stream was assumed to be the same as the pouring or casting temperature (T_o) of the steel. Outside the pouring stream, melt surface assumed to be covered with an insulating slag layer. Therefore, correspondingly heat flux across the meniscus can be assumed to be zero. Far away from the meniscus, the outflow boundary has been considered and the normal temperature gradient at this boundary was prescribed to be zero (*i.e.* B.C. (2)). Across the central plane, due to symmetrical heat flow in all directions, zero normal temperature gradients was assumed (*i.e.* B.C. (3)).

At the cast surface, the Savage-Pritchard correlation¹⁵ has been adopted and consequently, q_s (Eq. (7)) in the mould region ($0 \leq Z \leq L_m$, $L_m \leq L$) has been expressed as:

$$q_m = [2.67 - 0.33 \sqrt{Z/U_o}] \times 10^6 \quad \text{.....(8)}$$

In the secondary cooling zone ($L_m < Z \leq L_s$, $L_s < L$), heat extracted from the surface of the casting is predominantly by impinging water sprays. Therefore, the surface heat flux (q_s) can be expressed by the following expression:

$$q_s = h_s (T_s - T_w) \quad \text{.....(9)}$$

Similarly, in the radiation cooling zone ($L_s < Z \leq L$) below sprays, surface heat flux can be represented via the following expression:

$$q_s = \sigma \epsilon (\theta_s^4 - \theta_a^4) \quad \text{.....(10)}$$

The governing equation together with the boundary conditions summarized above represent the complete mathematical description of heat flow in continuous casting.

3. Numerical Solution Procedure

Due to the nonlinearity of (1) latent heat source

term, (2) surface boundary conditions (Eqs. (8)–(10)), and (3) variable thermophysical properties (*i.e.* thermal conductivity), numerical methods were to be employed to obtain the required solution. Towards this, a control volume based finite difference procedure^{16,17)} has been adopted in the present study. In this procedure calculation domain has been divided or discretized into a number of non-overlapping control volumes (Fig. 2). Around all such control volumes, the governing equation has been integrated numerically to yield a system of algebraic equations (*i.e.*, discretization equations).¹⁶⁾

3.1. The Computer Program

For numerical solution of the present problem, a general computer program in FORTRAN77 and in double precision has been developed. The program is so written that computations in three dimension (3-D) as well as in 2-D and 1-D can also be performed by manipulating certain key parameters. Furthermore, an interesting feature of the present computer program has been that the same program can also be applied for computations in Cartesian as well as in the cylindrical polar coordinate systems. The transformation from one coordinate system to another or from one geometry to another can be illustrated by considering a general

form of the heat conduction equation presented in a preceding section *viz.*,

$$\rho C U_o \left(\frac{\partial T}{\partial Z} \right) = \frac{\partial}{\partial Z} \left(K_{\text{eff}} \frac{\partial T}{\partial Z} \right) + \frac{1}{\alpha} \frac{\partial}{\partial X} \left(\alpha K_{\text{eff}} \frac{\partial T}{\partial X} \right) + \lambda \frac{\partial}{\partial Y} \left(K_{\text{eff}} \frac{\partial T}{\partial Y} \right) + S \dots\dots\dots(11)$$

in which, α has been defined as an index of coordinate system and λ as index of coordinate dimension.

(1) if $\alpha=1$ and $\lambda=1$, Eq. (11) becomes a 3-D heat conduction equation in cartesian coordinate, applicable to heat flow during billet casting

(2) $\alpha=1$ and $\lambda=0$; give 2-D heat conduction equation in Cartesian coordinate, applicable to slab caster

(3) $\alpha=r=X$ and $\lambda=0$; yield heat flow equation in 2-D cylindrical polar coordinate system, applicable to round/axisymmetric billet caster.

As mentioned already, all the above three types of typical casting geometries have been simulated computationally in the present study. Flow chart of the computer program is shown in Fig. 3. As summarized in the subsequent sections, relevant data of actual continuous casting operation were taken from literature for carrying out numerical computations, and their subsequent validation with experimental measurements.

4. Results and Discussion

A variety of grid configurations were employed in order to arrive at practical grid independent solutions. However, it was found that increasing the grid numbers beyond 25×40 produce no significant change in the computed results. Consequently, from a practical point of view, 25×40 grid system was chosen and was considered to produce results that were effectively independent of nodal configurations. This corresponds to a grid spacing of 3 mm in the transverse and 50 mm in the axial direction respectively. On the basis of these computational trials, therefore, similar grid spacings (*e.g.* transverse $\cong 3$ mm and axial $\cong 50$ mm) as those corresponding to 25×40 grid systems were employed in all subsequent calculations reported. The implications of grid independent solutions will be discussed again when comparisons between predictions and experiments are made in subsequent sections.

Figures 4(a) and 4(b) respectively show the sample variations of shell thickness and midface surface temperature with distance below the meniscus for some of the grid configurations applied. These show that 25×40 and 25×80 produced almost identical estimates of both shell thickness as well as surface temperature. However, minor differences exist between 25×20 and 25×80 grids or for 16×80 and 25×40 grids. It is interesting to note that numerically predicted results are relatively more sensitive to the number of grid points along the transverse directions, which is evident from the differences indicated between predictions derived *via* 16×80 and 25×80 grid systems.

Influence of choice of values of K_{eff} on computed results has been discussed later in detail. For all other

Fig. 3. Flow chart of computer program for the model based on effective thermal conductivity concept.

Fig. 4. Influence of grid configurations on predicted (a) shell thickness and (b) strand surface temperature (for data set 4, Table 1).

calculations including variation of grid spacing, K_{eff} has been taken as $7K$ as originally arrived at by Mizikar.³⁾ The justification for this choice has also been substantiated later.

In the solid region however, K_{eff} was considered to be equal to K . Similarly in the mushy zone, the continuum approach has been considered and the mixture rule viz., $[K_{\text{mush}} = \gamma_s K + (1 - \gamma_s) K_{\text{eff}}]$, where γ_s is the fraction of solid] was applied to estimate the relevant effective thermal conductivity value.

In the subsequent sections, first of all, influence of various numerical approximations on the predicted results have been rigorously assessed. Next, computations were carried out on actual continuous casting conditions and compared with experimental results reported in literature.

4.1. Arithmetic Mean vs. Harmonic Mean Approximations

Owing to a non-uniform distribution of thermal conductivity (K_{eff}) in the calculation domain, the role of arithmetic and harmonic mean interpolation techniques has been studied computationally. It is important to note here that while thermal conductivity values are known only at the grid points, calculation of discretization equation coefficients require thermal conductivity values to be known at the mid positions between the nodal points¹⁶⁾ (i.e. at the control volume faces, Fig. 5). Consequently, it is necessary that proper interpolation techniques are applied to estimate K_{eff} at the control volume face from those of the adjacent nodal points.

In the present study, K_{eff} at control volume faces have been computed via (1) arithmetic mean and (2) harmonic mean approximation procedures.¹⁶⁾ For a given control volume face 'e' midway between the nodes P and E (Fig. 5), arithmetic mean approximation gives the following value of conductivity at control volume face:

$$K_{\text{eff},e} = \frac{1}{2} (K_{\text{eff},P} + K_{\text{eff},E}) \quad \text{.....(12)}$$

In contrast, harmonic mean approximation for the same provides:

$$K_{\text{eff},e} = \frac{2K_{\text{eff},P} \times K_{\text{eff},E}}{[K_{\text{eff},P} + K_{\text{eff},E}]} \quad \text{.....(13)}$$

Fig. 5. A typical three dimensional control volume in Cartesian coordinate system.

Fig. 6. Effect of arithmetic mean and harmonic mean approximation techniques for interpolation of K_{eff} on the predicted shell thickness (for data set 2, Table 1).

Figure 6 presents the variation of computed shell thickness derived via the two above mentioned approximation procedures. Despite wide variation of thermal conductivity in the calculation domain, these show practically negligible differences and indicate that the two procedures provide practically identical estimates.

4.2. Lower Order vs. Higher Order Interpolations for Estimating the Strand Surface Temperatures

As shown in Fig. 2, at the cast surface, there is no grid point and hence cast surface temperature has to be calculated from the predicted internal temperature fields via some suitable interpolation techniques. Similarly, in

Fig. 7. Boundary control volumes considered for (a) lower order and (b) higher order interpolation methods for estimating casting surface temperature.

Fig. 8. Effect of lower order and higher order interpolation techniques on the predicted midface temperature of strand (for data set 2, Table 1).

the secondary cooling zone, the boundary condition at the cast surface has been prescribed *via* the heat transfer coefficient (h_s) and the spray water temperature (T_w). In order to evaluate the corresponding surface heat flux (q_s), surface temperatures at various positions in the spray cooling zone have to be estimated first from the computed internal temperature field during each iteration using either the lower order or the higher order boundary treatments.¹⁸⁾

In the lower order interpolation procedure, surface temperature or surface heat flux is estimated on the basis of a single nodal point temperature adjacent to the domain boundary (Fig. 7(a)). Thus, on the basis of such considerations,

$$q_s = \frac{K_s}{\delta} (T_{n-1} - T_s) \quad \text{or,} \quad T_s = T_{n-1} - \frac{\delta}{K_s} q_s \quad \dots (14)$$

On the other hand, the higher order interpolation takes into account two successive internal nodes adjacent to the cast surface boundary (Fig. 7(b)) for estimation of the relevant surface temperature or heat flux¹⁸⁾ as follows:

$$q_s = 4 \frac{K_s}{\delta} (T_{n-1} - T_s) - 3q_{n-1} \quad \dots (15)$$

$$\text{in which,} \quad q_{n-1} = \frac{K_i}{\delta} (T_{n-2} - T_{n-1}) \quad \dots (16)$$

Figure 8 presents temperature variations at the midface along the axial direction, as estimated *via* the two procedures. The higher order interpolation method is seen to predict somewhat higher surface temperature

Fig. 9. Influence of axial conduction term in the governing heat flow equation on the predicted shell thickness.

(about 4%) throughout the strand as compared to lower order interpolation. Therefore, the former has been adopted in the present work since it is physically more realistic than the lower order method.

4.3. Influence of Axial Conduction on the Computed Results

In the previous studies,^{1-4,7-12)} conduction of heat in the axial direction has been assumed to be negligible. It has been generally considered that relatively higher withdrawal rate of casting and lower thermal conductivity of steel makes the term $U_o \partial(\rho CT)/\partial Z$ in Eq. (1) dominant in comparison to the corresponding conduction term along the same (*i.e.*, axial) direction. However, in the present study the axial conduction term has been incorporated in the governing equation to test directly the validity of such an assumption. Thus, to assess the sensitivity of the axial heat conduction term in Eq. (1), calculations were carried out by ignoring axial conduction altogether as well as considering axial conduction in the numerical calculation procedure. Results thus obtained are summarized in Fig. 9 which show that there is no influence of the axial conduction term on the predicted shell thickness at distance less than 2.8 m from the meniscus. However, the difference between the two estimates below a strand distance of 2.8 m (Fig. 9), have resulted from the application of a boundary condition of the type $K \partial T / \partial Z = 0$ at $Z = L$ for computation considering axial conduction term. In contrast to this, when axial conduction of heat is neglected, no such condition at the outflow boundary is required.

4.4. Influence of Modelling Procedures for Heat Flow in the Mushy Zone

In some of the earlier studies,^{3,8,12)} from the view point of heat flow, mushy zone has been considered as a solid region and thus, the effective thermal conductivity ($K_{\text{eff}} = 7K$) value was assigned only to the completely liquid region. Lait *et al.*⁸⁾ studied the influence of incorporating the mushy zone in the liquid region (*i.e.* $K_{\text{eff}} = 7K$ assigned in the regions $T \geq T_{\text{sol}}$) as well. About 6% overall increase in the shell thickness for plain

Fig. 10. Influence of mushy zone treatment on predicted (a) shell thickness and (b) midface temperature (for data set 2, Table 1).

low carbon steel billet has been reported from such consideration. Some other investigators assumed the thermal conductivity of mushy zone to vary with fraction of liquid at various positions in this zone¹¹⁾ as:

$$K_{\text{mush}} = K\gamma_s + (1 - \gamma_s)K_{\text{eff}} \dots\dots\dots (17)$$

In the present study both the previously mentioned approaches have been considered to assess directly the influences of specific modelling procedures applied to the mushy zone on overall predicted heat transfer rates. In another set of computation, $K_{\text{eff}} = 7K$ (i.e. without treatment) has been assigned throughout the region above the liquidus temperature (i.e. $T \geq T_{\text{liq}}$) of steel and mushy zone has been ignored.

The results derived on the basis of the above two approaches have been presented in Figs. 10(a) and 10(b). There, it is at once evident that the respective estimates of shell thickness as well as midface temperatures are somewhat different (about 4–5%) indicating essentially that the modelling procedure applied to the heat flow in mushy zone is critical to the overall accuracy of the numerically predicted results.

4.5. Influence of Mould Heat Flux Expression on Computed Results

Based on several measurements on industrial billet and slab casters, the mould heat flux correlations have been shown to be within a scatter of ± 40 –50%.⁹⁾ Calculations have been consequently carried out by arbitrarily changing the average mould heat flux value up to ± 40 %, and results thus obtained are presented in Fig. 11. It is evident from the figure that the actual limit of confidence of the mould heat flux has an important bearing on the overall predicted results and thus, is expected to affect the overall accuracy of numerical computation considerably.

4.6. Sensitivity of Computation to the Choice of Effective Thermal Conductivity Values

In view of a wide range of K_{eff} values suggested in the literature^{3,9,13)} as has been mentioned already, it is worthwhile to assess the sensitivity of computation to the choice of an appropriate value of the K_{eff} in the liquid pool. In this connection, different values of K_{eff} were arbitrarily selected (viz., 1K, 7K, 12K and 30K) and numerical computations were carried out for a slab caster.¹⁹⁾

Fig. 11. Influence of arbitrarily varying the mould heat flux on the predicted shell thickness (for data set 2, Table 1).

Fig. 12. Influence of different effective thermal conductivity values on predicted shell thickness of a slab caster (data set 3, Table 1).

Figure 12 presents estimates of shell thickness for various K_{eff} values. This shows that the shell thickness does not vary much if the K_{eff} is changed from 1K to 7K. However, there is significant influence on shell thickness when K_{eff} value was arbitrarily increased to 12K. It was also observed that a K_{eff} value equal to 30K, did not give any solidification at all in the mould region. Equivalent results were also obtained for a square billet caster.

The trend in results as presented in Fig. 12 can be explained as follows. The overall heat flux at any location in the transverse direction, say X , can be expressed by the routine gradient expression (*viz.*, $q_X = -K_{\text{eff}} dT/dX$). For Fig. 12, heat flux at the boundary is the same for all the curves. Therefore the internal temperature distribution would depend on the magnitude of assumed K_{eff} value. For lower K_{eff} values (*e.g.* 1K), relatively large temperature gradients and for higher K_{eff} (*e.g.* 12K) values more uniform and hence less temperature gradient are to be expected within the liquid metal pool. This effect led to decrease of shell thickness with increasing value of k_{eff} . Figure 13 presents typical temperature profiles at $K_{\text{eff}} = 1K$ and 12K. It clarifies and confirms the explanation presented above.

K_{eff} is an adjustable parameter and there is little information based on which this can be assigned an appropriate value (only scope is trial and error) consequently, the most popular and widely applied value (*viz.*, $K_{\text{eff}} = 7K$) has been adopted in the present study for computations.

4.7. Comparison of Computed Predictions with Experimental Measurements Reported in Literature^{8,20)}

In order to validate the mathematical model and thus to test the adequacy of the model developed, comparisons have been made between predicted results and

Fig. 13. Influence of different effective thermal conductivity values on the predicted temperature field in the transverse section of a slab at the mould exit.

corresponding experimental measurements reported in literature. Figures 14 and 15 present the computed solid shell profiles for two different square billet casters. Relevant data employed in these computations have been obtained from those reported by Lait *et al.*⁸⁾ and are summarized in Table 1. However, as may be noted,

Fig. 14. Comparison between predicted and experimental shell thickness of a typical square billet caster (conditions as in Ref. 8) for 0.14 m sq. billet).

Fig. 15. Comparison between predicted and experimental shell thickness of a typical square billet caster (conditions as in Ref. 8) for 0.133 m sq. billet).

Table 1. Data of continuous casting used in the present computations.

Parameters	Data Set-1 (Ref. 8))	Data Set-2 (Ref. 8))	Data Set-3 (Ref. 19))	Data Set-4 (Ref. 20))
Cast geometry	Square billet	Square billet	Slab	Round billet
Cast size (m × m)	0.14	0.133	0.25	0.115
Mould length (m)	0.51	0.685	0.60	0.50
Casting speed (m s ⁻¹)	0.0254	0.044	0.0125	0.0317
Steel carbon (%)	0.10	0.10	0.14	0.10
Melt superheat* (°C)	25	25	25	25
Solidus temperature (°C)	1 496	1 496	1 496	1 496
Liquidus temperature (°C)	1 529	1 529	1 526	1 529
Spray heat transfer coefficient (W m ⁻² °C ⁻¹)	—	—	—	1 079.45**

* Estimated. ** Source. Ref. 14).

Table 2. Thermophysical properties of steel* used in computation.

Density	(kg m^{-3})	7400
Latent heat of solidification	(J kg^{-1})	272×10^3
Specific heat	($\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$)	682
Thermal conductivity	($\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$)	34.6 and $15.89 + 0.0117^{**}$

* Source: Physical Constants of Some Commercial Steels at Elevated Temperatures, ed. by The British Iron and Steel Research Association, Butterworths Scientific Pub., London, (1953).

** Source: Ref. 3).

some of the useful information, such as melt super heat and spray heat transfer coefficient, are not available in Ref. 8). Therefore a typical superheat value of 25°C has been employed in computation of the results presented in Figs. 14 and 15. Because of uncertainty in the spray heat transfer coefficient values, comparisons have been restricted to the mould region only, although the secondary cooling zone was also included (with an estimated heat transfer coefficient data) in the numerical solution scheme. It is important to note here that, in view of negligible axial conduction in the mould region and somewhat beyond (Fig. 9), inclusion of secondary cooling zone in the computation is not likely to affect the predicted results in the mould region. Figure 14 presents two computed curves for the two boundaries of mushy zone corresponding to $\gamma_s=0$ and $\gamma_s=1$. They also reveal that the overall agreement between the experimental and predicted values is somewhat poor. The agreement between theory and experiment appears to be reasonable only in the upper mould region. The discrepancy between theory and experiments as illustrated in Figs. 14 and 15 can be primarily attributed to the following reasons:

(1) Uncertainty/Error in Experimental Measurements

It is unlikely that gold tracer penetrated up to the solid/mushy zone interface, particularly where the mushy zone thickness was appreciable.¹⁴⁾ So the experimental points may not exactly reflect the solidus position.

(2) Inadequacy of the K_{eff} Concept

It is rather well known that nozzle configurations (single point entry nozzle *etc.*) considerably affect the solid shell growth within the mould.²¹⁾ However, the present model is too simplistic and therefore, does not embody the influence of such critical hydrodynamic variables. As recent study²²⁾ shows that hydrodynamic phenomena play important roles in continuous casting of steel and therefore, model based on the concept of artificial effective thermal conductivity would not have much relevance to the accurate modelling of the associated thermal phenomena. Some other studies also tend to support this opinion.^{14,22)}

In Fig. 16 another set of experimental data for a typical round billet caster reported by Ushijima,²⁰⁾ have been presented along with the present theoretical

Fig. 16. Comparison between the experimental measurement and the present estimates of shell thickness with various grid configurations for a typical round billet caster (data set 4, Table 1).

estimates. Fairly reasonable agreement for the upper pool region (*i.e.* mould) is demonstrated in the plot. Whereas, in the submould region, significant discrepancy between the experimental and predicted data is evident. As mentioned already, discrepancy between prediction and observation can be attributed to the experimental technique (in this case, visual observations) as well as to the mathematical modelling concepts applied. Towards this, it is important to mention here that Asai and Szekely¹⁴⁾ have demonstrated good agreement with the observation reported by Ushijima²⁰⁾ via their fluid flow based heat transfer model. Hence it cannot be stated that the discrepancy illustrated in Fig. 16 is due to heat transfer coefficient value employed in the present investigation.

As a final point, it is to be mentioned here that none of the earlier studies attempted to examine the issues of sources of uncertainty in the numerical calculation procedure. However, as the present study has indicated that the numerical approximations applied have considerable bearing on the accuracy of the predicted results. Similarly, in no studies reported so far, the issue of grid independent solution has been addressed.

To illustrate this point further, in Fig. 16 predictions based on 12×40 and 16×80 grid systems have also been included in addition to those derived via 25×40 grid system. With reference to Fig. 16, it is important to note that prediction of shell thickness are relatively more sensitive to grid distribution in the transverse/radial direction. This is to be expected since variation of thermal properties are relatively more steep in the transverse direction than are in the longitudinal direction/withdrawal direction. Considering the pouring temperature as a reference, immediately below the meniscus the thermal field derived via a fine grid system is expected to be relatively more close too the reference temperature than those deduced via a sparse grid system. Since a higher temperature field results in a low solidified shell thickness and *vice versa*, consequently, it is seen in Fig. 16 that with the increase in grid configuration in any direction, the shell thickness de-

creases. Similarly, predicted surface temperature derived from internal temperature field is also expected to be somewhat greater for finer grid system, leading to different heat flux ($=h\Delta T$) values at the cast surface for the two grid systems. Consequently, the influence of grid distribution is seen to be relatively more pronounced in the secondary cooling zone than in the mould region.

Figure 16 appears to indicate that as the grid configuration applied became more and more sparse, the prediction tends to come closer to the observation, and this apparently suggests that reasonable to excellent agreement exists between theory and experiments. Nevertheless, as has been discussed already, the numerical results, unless are independent of nodal configuration, have no validity and hence comparison of such results with experiments have in reality, no meaning.

Hence, in Fig. 16 comparison of experimental measurement is meaningful only with curve 3 (24×40 grid), for which the solution was found to be grid independent. This shows that agreement between predictions and experimental data are poor. The authors have already discussed it. It may be due to uncertainties in experimental data or inadequacy of mathematical models based on effective thermal conductivity concept. It is possible that both of these are contributing partly to discrepancy. It is not possible to make any more definite statement without further studies.

5. Conclusion

The following general conclusions can be drawn from the present study:

(1) Details of numerical procedure have considerable effect on the predicted results. It is shown that solutions have to be essentially grid independent or else altogether different conclusions may emerge.

(2) Axial conduction of heat was found to be negligible in comparison to bulk motion of the descending strand.

(3) Model of heat flow in the mushy zone affected the predicted results significantly.

(4) The computed results were relatively insensitive to the choice of K_{eff} within the range of $1K$ to $7K$. However, a higher value of K_{eff} exhibited less solidification, and indeed at K_{eff} larger than $12K$, there was no solidification at all in the mould region.

(5) Finally, numerical predictions have been compared with three sets of measurements reported in literature on shell thickness in industrial casters. In contrast to the earlier claims, these indicated only poor to moderate agreement between model predictions and experimental data.

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Nomenclature

- a : Billet size in the direction of X -axis (m)
 b : Billet size in the direction of Y -axis (m)

- (m)
 C : Specific heat ($\text{J kg}^{-1} \text{ } ^\circ\text{C}^{-1}$)
 h_s : Spray heat transfer coefficient ($\text{W m}^{-2} \text{ } ^\circ\text{C}^{-1}$)
 K : Molecular thermal conductivity of steel ($\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$)
 K_{eff} : Effective thermal conductivity ($\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$)
 $K_{\text{eff},e}$ etc.: Effective thermal conductivity value at one face of a 3-D control volume ($\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$)
 $K_{\text{eff},P}$ & $K_{\text{eff},E}$: Effective thermal conductivity values at nodes P and E , respectively ($\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$)
 K_s : Thermal conductivity adjacent to the cast surface ($\text{W m}^{-1} \text{ } ^\circ\text{C}^{-1}$)
 L : Caster/domain length simulated (m)
 L_m : Mould length (m)
 L_s : Secondary cooling zone length (m)
 q_m : Instantaneous mould heat flux (W m^{-2})
 q_{n-1} : Heat flux at $n-1$ th node (W m^{-2})
 q_s : Heat flux at the strand surface (W m^{-2})
 q_X, q_Y, q_Z : Heat fluxes along X , Y , and Z coordinates (W m^{-2})
 r_s : Pouring stream radius (m)
 S : The source term in the governing heat flow equation (W m^{-3})
 T_{liq} : Liquidus temperature ($^\circ\text{C}$)
 T_{sol} : Solidus temperature ($^\circ\text{C}$)
 U_o : Casting speed (or bulk motion) (m s^{-1})
 T : The temperature variable ($^\circ\text{C}$)
 T_{n-1} & T_{n-2} : Temperature at the $n-1$ and $n-2$ nodal points ($^\circ\text{C}$)
 T_o : The casting temperature ($^\circ\text{C}$)
 T_s : The cast surface temperature ($^\circ\text{C}$)
 T_w : Spray water temperature ($^\circ\text{C}$)
 X, Y, Z : Distances along X , Y , Z axes respectively (m)
 Greek letters
 ΔH_f : Latent heat of solidification (J kg^{-1})
 $\Delta X, \Delta Y, \Delta Z$: Lengths of control volume faces along X , Y , Z axes, respectively (m)
 α : Index of the coordinate system
 δ : Half of the control volume face thickness (m)
 $\delta X_e, \delta X_w$, etc.: Distances between the various nodes in three mutually perpendicular coordinate axes (m)
 ε : Emissivity of the oxidized iron surface ($=0.85$)
 γ_s : Solid fraction in the mushy zone
 λ : Index of coordinate dimension
 θ_a : Ambient temperature in absolute scale (K)
 θ_s : Surface temperature in absolute scale (K)
 ρ : Density of steel (kg m^{-3})
 σ : Stefan-Boltzmann constant

$$(\approx 5.67 \times 10^{-8}) \text{ (W m}^{-2} \text{ K}^{-4})$$

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