Relationship between Primary- and Secondary-dendrite Arm Spacing of C–Mn Steel Uni-directionally Solidified in Steady State

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The relationship between primary dendrite arm spacing and secondary one was investigated in experiment, and was found that the latter is roughly one half of the former. Theoretically, the primary arm spacing is interpreted by thermodynamic perturbations ahead of the primary tip. On the contrary growth mechanism of secondary arm is ascribed to Ostwald-ripening and/or sintering/coalescence mechanism. The experimental result is analysed by the mechanisms, and is shown the 'one half relationship' is a good approximation.

KEY WORDS: dendrite arm spacing; dendrite morphology; thermodynamic perturbation; Ostwald-ripening.

1. Introduction

In the previous publication¹⁾ by the author, dendrite arm spacings of C–Mn-steel in steady state uni-directional solidification are investigated. The physical variables characterizing solidification conditions, *e.g.*, growth rate of primary dendrite tip: R and temperature gradient around solid/liquid zone: G are the most significant ones, and the results in the publication¹⁾ are correlated to the following relationships;

$$\lambda_1 [\mu\text{m}] = 1750R^{-0.14}G^{-0.50} \quad [R: \mu\text{m/s}, G: ^\circ\text{C/mm}] \quad \dots\dots\dots(1)$$

$$\lambda_2^f \sim 0.5\lambda_1 \quad \dots\dots\dots(2)$$

where λ_1 is primary arm spacing and λ_2^f is secondary arm spacing at completely solidified zone. The experimental results are examined by the theories based mainly on Kurz and Fisher,^{2,3)} who evolved it from a thermodynamic consideration which is first proposed by Mullins and Sekerka,⁴⁾ and is developed by Langer and Müller-Krumbhaar^{5–8)} into perturbation around primary dendrite tips. For the secondary dendrite growth, Ostwald-ripening mechanism⁹⁾ and/or sintering/coalescence behaviour¹⁰⁾ are applied.

2. Theoretical Bases

Mullins *et al.*⁴⁾ explained the statistical fluctuation of solute concentration and temperature around a solid/liquid interface which moves at a constant velocity R , applying three equations of continuity in terms of a co-ordinate whose origin is set at the operating point moving in the z -direction at the same velocity R ;

$$\left. \begin{aligned} \nabla^2 C + (R/D)\partial C/\partial z &= 0 \\ \nabla^2 T + (R/D_{th})\partial T/\partial z &= 0 \\ \nabla^2 T' + (R/D'_{th})\partial T'/\partial z &= 0 \end{aligned} \right\} \quad \dots\dots\dots(3)$$

where C , T and T' represent solute concentration in liquid, temperature in liquid and temperature in solid, respectively. Each solution for C , T and T' is analysed with infinitesimal sinusoidal Fourier components which simulate sinusoidal ripple of the tip interface expressed as

$$z = \delta(t) \sin \omega x \quad \dots\dots\dots(4)$$

where $\delta(t)$ is amplitude of perturbation, ω corresponds to the ripple frequency, and x is radial co-ordinate. Mullins *et al.* put the following four conditions;

(i) perturbed C , T and T' around the tip-interface should agree with the unperturbed field at several wavelengths from the interface,

(ii) at the tip interface, temperature: T_I and solute concentration: C_I at the operating tip and temperature of supercooled melt ahead of the tip: T_N are related to the solute concentration gradient M , as $T_I = MC_I + T_N$, and

(iii) the classical law for critical nucleation radius at the tip: $T_N = T_E + 2\sigma T_E/\Delta r$ is applied, where T_E is equilibrium melting temperature, σ is the specific solid/liquid interface free energy, and Δ is latent heat of melt per unit volume,

(iv) at any point of tip interface, the velocity $R(x)$ must satisfies the following equation of solutal diffusion,

$$\begin{aligned} R(x) &= \{K_S(\partial T'/\partial z)_I - K_L(\partial T/\partial z)_I\}/\Delta \\ &= D(\partial T/\partial z)_I/\{C_I(k-1)\} \end{aligned}$$

and obtained the relative differential rate of the amplitude;

$$\begin{aligned} \dot{\delta}/\delta &= R\omega\{-2T_E\Gamma\omega^2[\omega^* - (R/D)p] \\ &\quad - (g' + g)[\omega^* - (R/D)p] + 2MG_C[\omega^* - (R/D)]\}/ \\ &\quad \{(g' - g)[\omega^* - (R/D)p] + 2\omega MG_C\} \quad \dots\dots\dots(5) \end{aligned}$$

which signifies stability/unstability of the ripple, or perturbation. Very clearly

$$\dot{\delta}/\delta = 0 \quad \dots\dots\dots(6)$$

means that the perturbation ahead of the tip interface continues to propagate with a ripple shape unchanged, i.e., perturbed tip proceeds forward with stable morphology.

Langer *et al.*⁵⁻⁸⁾ applied the central result of Mullins *et al.*,⁴⁾

$$\lambda_s = 2\pi(2Dd_0/R)^{1/2} \quad (7)$$

to side-branching behavior around primary dendrite tip, using a parabolic co-ordinate (cartesian co-ordinate), where λ_s is minimum wavelength of perturbation, d_0 is capillarity length of primary arm-buds, or height of ripple at the tip.

According to their calculation,⁶⁻⁸⁾ displacement of interface from unperturbed paraboloid, or perturbation, is given with $F(\xi, t)$ as shown in Fig. 1, where $F(\xi, t)$ is expressed as $\exp[\Omega(t-\xi)]J_0(2\Omega\sqrt{\xi})$ with ξ representing distance from primary tip center in parabolic co-ordinates, Ω : supersaturation and J_0 : ordinary Bessel function, and the relationship between tip radius: r and dimensionless growth rate, or Péclet number: $Rd_0/2D$ is given as shown in Fig. 2. From their computer calculation, the dimensionless parameter: $(2D/Rd_0)/(r/d_0)^2$ was given as Eq. (8),

$$(2D/Rd_0)/(r/d_0)^2 = 2Dd_0/Rr^2 \sim 0.025 \quad (8)$$

at the dynamically stable operating point of a dendrite surface where the secondary capillarity length $d_0 = F(\xi, t)$ is calculated for a supercooling of 0.05 (A/Cp) for which experimental data was available. The dimensionless parameter originates from a stability criterion which is ordinarily referred to Ivantsov isothermal mode, *etc.*, expressed as $Rr^2 = \text{constant}$. Equation (8) is inserted into Eq. (7), then Eq. (9) is obtained.

$$\lambda_s \sim r \quad (9)$$

Kurz *et al.*^{2,3)} simplified Eq. (5) with the following delicate approximations, (i) localized tip is deemed as flat plane, (ii) $\Lambda \sim 0$, (iii) $D_{th} \sim D'_{th}$, (iv) $k \sim 0$, and introduced

$$\lambda_s = 2\pi[\Gamma/(MG_C - G)]^{1/2} \quad (10)$$

from which a relationship between R and r is obtained as Eq. (11). According to Kurz *et al.*, Eq. (11) is further simplified to Eq. (12), provided R is large enough comparing to dendrite/cellular criterion; $R_{tr} = GD/\Delta T_0 k$,

$$R = 2D(Gr^2 + 4\pi^2\Gamma)/(r^3pG - 2r^2pC_0M + 4\pi^2\Gamma rp) \quad (11)$$

$$R \sim 4\pi^2D\Gamma/r^2\Delta T_0k \quad (12)$$

Kurz *et al.* further postulated an axial-symmetric ellipsoid shape for an overall primary dendrite arm, and devised a relationship between λ_1 and solidification variables, as follows,

$$\lambda_1 = 4.3(\Delta T_m)^{1/2}(D\Gamma/\Delta T_0k)^{1/4}R^{-1/4}G^{-1/2} \quad (13)$$

3. Experiments

Sample steels containing approximately 0.6%C-0.6%Mn in mass% were solidified under steady state uni-directional solidification conditions. Steel a; 0.62%C-0.12%Si-0.58%Mn-0.006%P-0.073%S-0.049%Al-0.0011%O, Steel b; 0.52%C-0.10%Si-0.56%Mn-0.003%P-0.046%S-0.037%Al-0.0011%O. The solidification variables in the experiments ranged for R , from 4.1 to 56 $\mu\text{m/s}$, and for G , from 1.8 to 16°C/mm. During the sample was solidifying at the aimed steady state

Fig. 1. Perturbation ahead of primary dendrite tip illustrating ripple of thermodynamic fluctuation of solute-concentration/temperature, after Langer *et al.*⁵⁻⁸⁾

Fig. 2. Dimension-less growth rate, or Péclet number: $Rd_0/2D$ and dimension-less tip radius: r/d_0 , after Langer *et al.*⁵⁻⁸⁾

conditions, it was quenched, and dendrite arm spacings of primary arms: λ_1 and secondary ones: λ_2 were measured around the solid/liquid zone. Details of the experiments were explained in the previous publication.¹⁾

4. Analysis

4.1. Primary Arm Spacing

All the experimental data, the physical data and solidification variables applied are listed in **Table 1**. λ_1 -values are calculated by Eq. (13) and Eq. (1) and are shown in **Table 2**. The correlation between experiments and theoretical calculations is good as shown in Table

2. Also in **Fig. 3**, a calculation result of Eq. (11) is shown, taking a typical solidification conditions; $G=2^\circ\text{C}/\text{mm}$. It is understood Eq. (11) gives a good evaluation of λ_1 for wide range of R -value, although the exponent of R in the experiment was -0.14 as mentioned before.¹⁾ The exponent of G in the experiment coincides with the theory. This means the perturbation ahead of primary tip in the experiment can be explained with the theories by Mullins *et al.*, i.e., Eqs. (6) and (7), Langer *et al.*, i.e., Eqs. (8) and (9), and finally theoretical approximations by Kurz *et al.*, i.e., Eqs. (10)–(12). Only the discrepancy about R -exponent remains unsolved. For further theoretical approach it must be necessary to continue additional

Table 1. Observed data of the present study¹⁾ and some physical values used in the present discussion.

Test No.	#9	#5	#3	#11	#12	#13	#17
Steel	a	a	a	a	b	b	b
[mass% C]	0.62	0.62	0.62	0.62	0.52	0.52	0.52
T_L [$^\circ\text{C}$]	1490	1490	1490	1490	1496	1496	1496
T_S [$^\circ\text{C}$]	1423	1423	1423	1423	1438	1438	1438
T_e [$^\circ\text{C}$]	1320	<1320	<1320	<1320	1350	1350	1350
ΔT_0 [$^\circ\text{C}$]	67	67	67	67	58	58	58
ΔT_m [$^\circ\text{C}$]	103	(>103)	(>103)	(>103)	88	88	88
$D^{15)}$ [cm^2/s]	2.74×10^{-5}	2.74×10^{-5}	2.74×10^{-5}	2.74×10^{-5}	2.79×10^{-5}	2.79×10^{-5}	2.79×10^{-5}
$\Gamma^{16)}$ [$^\circ\text{C cm}$]	1×10^{-5}	1×10^{-5}	1×10^{-5}	1×10^{-5}	1×10^{-5}	1×10^{-5}	1×10^{-5}
k	0.34	0.34	0.34	0.34	0.34	0.34	0.34
R [$\mu\text{m/s}$]	28.7	56.2	28.8	21.8	5.28	4.97	4.12
G [$^\circ\text{C}/\text{mm}$]	1.99	1.81	4.81	16.3	2.04	4.83	10.9
λ_1 [μm]	862	717	474	272	910	626	462
λ_2^f [μm]	316	301	257	172	434	362	228
$\lambda_2(T_S)$ [μm]	175	175	128	73	284	210	140

Table 2. Comparison between observed λ_1 and calculated λ_1 with Eq. (13).

Test No.	#9	#5	#3	#11	#12	#13	#17
Steel	a	a	a	a	b	b	b
Obs. λ_1 [μm]	862	717	474	272	910	626	462
Cal. λ_1 [μm] by Eq. (13)	787	698	506	295	1140	754	526
Cal. λ_1 [μm] by Eq. (1)	775	740	499	282	970	636	435

$$\text{Eq. (13)} \quad \lambda_1 = 4.3(\Delta T_m)^{1/2}(D\Gamma/\Delta T_0 k)^{1/4}R^{-1/4}G^{-1/2}$$

$$[\text{for steel a}] \quad \lambda_1 [\mu\text{m}] = 2570R^{-0.25}G^{-0.50} [R: \mu\text{m/s}, G: ^\circ\text{C}/\text{mm}]$$

$$[\text{for steel b}] \quad \lambda_1 [\mu\text{m}] = 2470R^{-0.25}G^{-0.50} [R: \mu\text{m/s}, G: ^\circ\text{C}/\text{mm}]$$

$$\text{Eq. (1)} \quad \lambda_1 [\mu\text{m}] = 1750R^{-0.14}G^{-0.50} [R: \mu\text{m/s}, G: ^\circ\text{C}/\text{mm}]$$

Fig. 3.

Relationship between velocity of primary dendrite tip: R and tip radius: r for 0.6% C–0.6% Mn-steel, temperature gradient along solid/liquid: $G \sim 2^\circ\text{C}/\text{s}$. Solid circle: Steel a, open circle: Steel b in Table 1.

c.f., Open/solid circles indicate each calculated λ_1 -value.

observations on the actual shape of primary dendrite arm, although the present work involves no additional experiment about it.

4.2. Secondary Dendrite Arm Spacing

The incipient state of nucleation of secondary arms is obviously interpreted by the idea proposed by Langer and Müller-Krumbhaar as explained above. Then the growth, or exactly, roughening of minute secondary arms, is thought to be explained by Ostwald-ripening mechanism⁹⁾ and sintering/coalescence.¹⁰⁾ The result of the present author¹⁾ as well as other published data¹¹⁻¹³⁾ shows secondary arm spacing along solid/liquid zone: λ_2 depends on the time duration after primary tips intrude into the concerned location, τ , as expressed with Eq. (14). Dann *et al.*¹⁴⁾ thought that Ostwald-ripening mechanism prevails in relatively

$$\lambda_2 = (\text{constant}) \cdot \tau^m \quad \text{.....(14)}$$

early stage of secondary arm roughening, where the exponent m falls on around 0.25, on the contrary the later stage is explained by sintering/coalescence, where $m \sim 0.4$. On the other hand, many reports give the time-exponent m of λ_2 as $m \sim 1/3$. To give an answer to the question whether m is constant, *e.g.*, $1/3$, or it changes along solid/liquid zone, further experiment is required. In the following discussion, $m = 1/3$ is used. The data of the present author fluctuate around these figures as shown in another publication (refer to Fig. 3 in the publication¹⁾). The transition of the kinetics is not necessarily clear.

Based on the Langers' theory, let the roughening mechanism be brought in effect at a location behind the primary tip by a distance of the radius of primary tip along the curvature, and at this location the wave length of perturbation is λ_s , which corresponds to λ_2 there, as shown in Fig. 1. Here the primary axial-distance between the tip-top and the starting point of roughening is taken as *ca.* $(1/8)r$ as shown in Fig. 1, where r is primary tip radius and is regarded $\sim \lambda_s$, then Eq. (14) becomes Eq. (15).

$$\lambda_2 = \lambda_s^{1-m} (8R)^m \cdot \tau^m \quad \text{.....(15)}$$

Using Kurz's calculation result of Eq. (12), $r(\sim \lambda_s) = 2\pi(D\Gamma/R\Delta T_0 k)^{1/2}$ is obtained, then Eq. (15) is expressed as

$$\lambda_2 = (2\pi)^{1-m} (D\Gamma/R\Delta T_0 k)^{(1-m)/2} (8R)^m \cdot \tau^m \quad \text{.....(16)}$$

Putting $m = 1/3$ as reported in the publications elsewhere, the secondary arm spacing: λ_2^e at $\tau = \Theta_f = \Delta T_0/RG$ is calculated, and Eq. (17) is obtained.

$$\lambda_2^e = 6.8(D\Gamma/RGk)^{1/3} \quad \text{.....(17)}$$

The calculated λ_2^e by Eq. (17) evaluates the experimental data of secondary arm spacing at T_s with satisfactory coincidence as shown in Table 3.

Obviously λ_2^e is not the final secondary arm spacing: λ_2^f which is the spacing at locations of fully solidified zone. There is no method calculating λ_2^f from any of the above mentioned theoretical approaches, since the inter-dendritic enriched zone becomes an independent system from the thermodynamic point of view. The ratio: λ_2^e/λ_2^f , where λ_2^e is secondary arm spacing at T_s calculated by Eq. (17) and λ_2^f is observed at quenched specimen, is obtained and shown in Table 4. The ratio λ_2^e/λ_2^f ranges 0.47–0.52 for steel a, and 0.61–0.78 for steel b. That means the final secondary arm spacing approximately doubles the theoretical value of λ_2^e .

4.3. Ratio of λ_2^f/λ_1

Using the theoretical values for λ_1 by Eq. (13) and λ_2^e by Eq. (17), ratio of λ_2^e/λ_1 is given by Eq. (18). The data of experiments are calculated with the Eq. (18) and listed

$$\lambda_2^e/\lambda_1 = 1.6[(\Delta T_0)^{1/4}/(\Delta T_m)^{1/2}](D\Gamma/k)^{1/12} R^{-1/12} G^{1/6} \quad \text{.....(18)}$$

in Table 4. In Table 4 the semi-theoretical results of λ_2^e/λ_2^f are calculated, and are listed together. From the results in Table 4, it is thought the one half relationship: Eq. (2),

$$\lambda_2^f \sim 0.5 \lambda_1 \quad \text{.....(2)}$$

Table 3. Comparison of observed $\lambda_2(T_s)$ with calculated λ_2^e with Eq. (17).

Test No. Steel	#9 a	#5 a	#3 a	#11 a	#12 b	#13 b	#17 b
$\lambda_2(T_s)$ [μm]	175	175	128	73	284	210	140
Cal. λ_2^e [μm] by Eq. (17)	164	135	122	89	288	220	179

Eq. (17) $\lambda_2^e = 6.8(D\Gamma/RGk)^{1/3}$

[for steel a] λ_2^e [μm] = $63.3(RG)^{-1/3}$ [R : cm/s, G : °C/cm]

[for steel b] λ_2^e [μm] = $63.6(RG)^{-1/3}$ [R : cm/s, G : °C/cm]

Table 4. λ_2^e/λ_2^f and calculated λ_2^e/λ_1 by Eq. (18).

Test No. Steel	#9 a	#5 a	#3 a	#11 a	#12 b	#13 b	#17 b
① λ_2^e/λ_2^f	0.52	0.45	0.47	0.52	0.66	0.61	0.78
② λ_2^e/λ_1 by Eq. (18)	0.21	0.20	0.25	0.31	0.26	0.30	0.34
② ÷ ① $\sim \lambda_2^f/\lambda_1$	0.40	0.44	0.53	0.60	0.39	0.49	0.44

Eq. (18) $\lambda_2^e/\lambda_1 = 1.6[(\Delta T_0)^{1/4}/(\Delta T_m)^{1/2}](D\Gamma/k)^{1/12} R^{-1/12} G^{1/6}$

[for steel a] $\lambda_2^e/\lambda_1 = 0.079R^{-0.0833}G^{0.167}$ [R : cm/s, G : °C/cm]

[for steel b] $\lambda_2^e/\lambda_1 = 0.082R^{-0.0833}G^{0.167}$ [R : cm/s, G : °C/cm]

is supported by the semi-theoretical considerations.

5. Discussion

In introducing Eq. (13) in Kurz's theory, shape of the primary dendrite is postulated as axial-symmetric ellipsoid, which leads to the R -exponent: $m = -0.25$. There is another theoretical explanation¹⁷⁾ giving the same figure for R -exponent, and some experiments, *e.g.*, a very careful work done by McCartney *et al.*¹⁸⁾ using Al-Mg-Si alloys, support these theoretical results.

Nevertheless, many experimental works using a variety of materials ranging from metallic alloys^{11-13,19-21)} and transparent organic compounds²²⁾ do not converge into the theoretical figure: $m = -0.25$. Furthermore, Mason *et al.*²⁰⁾ proved that neither R -exponent: m nor G -exponent: n is constant for steady dendritic solidification, analysing their own work and those of Klaren *et al.*¹⁹⁾ and Spittle *et al.*²¹⁾ any of which is done with also very careful experiments using Pb-Sn alloys. Mason *et al.*²⁰⁾ even suggested that the relationship: $\lambda_1 = (\text{const.}) R^m G^n$ is 'merely a convenient form for representing experimental data taken over a limited range of experimental variables'. Indeed, another observation done by Somboonsuk *et al.*²²⁾ with succinonitrile-5.5mol% acetone system, in which the present author see the primary significance to verification of the premises for the above theories, gives quite different m -value, notwithstanding the tip morphology of the primary dendrite coincides quantitatively with the theories which sustained the above mentioned theoretical treatments of primary dendrite arm spacing.

As far as C-Mn steel is concerned, only a few works have been opened. And there seems to be no adequate background concluding what is the reliable m -, n -value. Therefore, it is the best numerical treatment to take a regression adopting $n = -0.5$, which many experimental works as well as the theories converge into. Eq. (1) in the present work unexpectedly coincides with theories in this respect, although the regression¹⁾ did not suppose any *a priori* n -value, and it should be reminded the experimental regression, *i.e.*, Eq. (1) gives nearly the same λ_1 -value which is calculated with the theoretical Eq. (13) as shown in Table 2. This assures the present discussion about λ_2^f vs λ_1 relationship based on Eq. (13) does not lead to wrong result practically, although it might not be selfconsistent logically.

The starting point of secondary arm roughening is arbitrarily designated as the point withdrawn in the axial direction by $(1/8)r$ from the primary tip as shown in Fig. 1. It is based on the following approximation. If the first one wavelength of perturbation along the curvature: $\lambda_S^{(1)}$ is equal to the radius of curvature: r , it is calculated mathematically that the first secondary arm spacing: $\lambda_2^{(1)}$ is slightly smaller than one tenth of r in axial direction, or $\lambda_2^{(1)} \sim 0.1r$, which is again adjusted to the easiness of numerical treatment; $\sim 2^{-3}r$. This introduces Eq. (15), and Eq. (17). Applying Eq. (17) to steel a and steel b, the numerical formulae,

$$\text{for steel a: } \lambda_2^e [\mu\text{m}] = 63.3(RG)^{-1/3} \\ [R: \text{cm/s, } G: ^\circ\text{C/cm}]$$

$$\text{for steel b: } \lambda_2^e [\mu\text{m}] = 63.6(RG)^{-1/3} \\ [R: \text{cm/s, } G: ^\circ\text{C/cm}]$$

are obtained. The calculated λ_2^e s coincide with the experimental $\lambda_2(T_S)$ as shown in Table 3.

6. Conclusion

The primary dendrite arm spacing: λ_1 of C-Mn steel solidified uni-directionally in steady state is well described by Kurz's theoretical analysis which is based on thermodynamic perturbation theory, or marginal stability criterion. Their approximation with axial symmetric ellipsoid morphology for dendrite shape gives a fairly good evaluation for the dependency of λ_1 -value upon primary tip velocity, however experimental results indicating some deviations from the theoretical value should be also acceptable.

Growth or roughening behavior of secondary arm spacing: λ_2 is also numerically described, depending on a model proposed by the present work. It is composed of two mechanisms, one is thermodynamic perturbation around primary dendrite tip, and the other is Ostwald ripening or sintering/coalescence, both of which require time dependency of roughening kinetic as time-exponent with 0.25–0.4. The present author regarded the exponent simply as 1/3, and obtained an equation with which the ratio λ_2^f/λ_1 is estimated to be nearly one half, with aid of a complementary experimental result of λ_2^f -relation to the calculated λ_2^e .

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Nomenclature

- λ_1 : Primary dendrite arm spacing
- λ_2 : Secondary dendrite arm spacing
- λ_2^f : Secondary dendrite arm spacing when overall system completes solidification
- λ_2^e : Secondary dendrite arm spacing at the location where temperature is T_S
- R : Growth rate of primary dendrite tip
- G : Temperature gradient ahead of primary dendrite tip, or approximately along solid/liquid zone
- C : Concentration of solute (=carbon, in the present report)
- D : Diffusion coefficient of carbon in melt steel ahead of dendrite tip
- D_{th}, D'_{th} : Thermal diffusivity in steel melt, and that in dendrite body around primary dendrite tip
- T : Temperature at arbitrary point in steel melt around primary dendrite tip
- T' : Temperature at arbitrary point in dendrite body
- ω : Frequency of thermodynamic perturbation along radial co-ordinate of primary dendrite axis, or along parabolic co-ordinate on primary dendrite tip

- σ : Free energy of solid/liquid interface per unit volume
 Λ : Latent heat of steel melt per unit volume
 K_S, K_L : Average curvature at a point on solid-side, and that on liquid-side of interface
 T_E : Equilibrium melting temperature
 Γ', Γ : $\Gamma' = \sigma/\Lambda$, typically given as $\sim 10^{-8}$ cm according to Mullins *et al.*¹⁶⁾ $\Gamma = T_E \Gamma' = \sigma T_E/\Lambda$ is called as Gibbs-Thompson-parameter, given in the present discussion as $\sim 10^{-5}$ °C cm
 ω^* : $(R/2D) + \{(R/2D)^2 + \omega^2\}^{1/2}$
 k : Partition coefficient of solute(=carbon), $p = 1 - k$
 M : Slope of liquidus line in equilibrium phase diagram; °C/unit-concentration
 g, g' : $g = 2\kappa_L G_L/(\kappa_L + \kappa_S)$, $g' = 2\kappa_S G_S/(\kappa_L + \kappa_S)$, where κ and G mean thermal conductivity and temperature gradient with suffix L for liquid-side and S for solid-side of the unperturbed interface
 G_C : Concentration gradient in the liquid at unperturbed interface
 δ : Amplitude of perturbation
 d_0 : Perturbed capillarity length at primary dendrite tip
 r : Radius of curvature at primary dendrite tip
 λ_S : Wave length of perturbation along unperturbed dendrite surface
 $\Delta T_0, \Delta T_m$: $\Delta T_0 = T_L - T_S$, $\Delta T_m = T_L - T_e$, where T_L : liquidus, T_S : solidus and T_e : eutectic solidification temperature in residual melt

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