Application of Pinch Force to the Separation of Inclusion Particles from Liquid Steel

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The pinch force which is one of the most common electromagnetic forces in the metallurgical field can be generated by imposing an electrical current on a liquid metal. In this force field, electrically nonconductive particles suspended in a liquid metal experience a force in the opposite direction to the pinch force, and are squeezed out from the liquid metal.

In this study, the above principle was applied to the separation of nonmetallic inclusion particles from liquid steel. Firstly, the separation efficiency, η , of nonmetallic inclusion particles from the liquid steel flowing in a circular pipe was calculated by the trajectory method under laminar flow conditions. The value of η was found to be a function of V_R , $(C_1 D_R^2 / Re)Z$ and r_1 / δ , where r_1 is the radius of the pipe, δ is the skin depth and V_R , C_1 , D_R , Re and Z are nondimensional parameters. The estimated value of η in a continuous casting tundish equipped with a channel-induction furnace was greater than 95% for particles with a diameter over 60 μm . Secondly, the value of η for a square pipe was computed by the concentration method. In this case, the value of η was found to be a function of V_R and $(C_1 D_R^2 / Re)Z$ at $x_1 / \delta = 0$, where x_1 is the half width of the square pipe. However, with increasing x_1 / δ , secondary flows appeared in a cross section of the pipe because of the skin effect. These flows were found to increase the value of η by transporting the particles in the inner region toward the wall.

KEY WORDS: nonmetallic inclusion; inclusion separation; pinch force; electromagnetic force; tundish heater; pipe flow; clean steel.

1. Introduction

It is well known that nonmetallic inclusions cause various defects like slivers in steel products.¹⁾ As the demand for higher cleanliness becomes greater, separation of inclusion particles from liquid steel has been pursued in the continuous casting process by means such as floating up in the tundish^{2,3)} and mould,⁴⁾ adsorption on the surfaces of injected bubbles,⁵⁾ and separation by tundish filters.⁶⁾ However, for the next generation of clean steels other more positive methods for inclusion separation are currently needed.

In recent years, major advances have been made in the application of magnetohydrodynamics in metals and materials processing. The application of electromagnetic forces to inclusion separation has been proposed by Alemany *et al.*^{7,8)} Recently Asai *et al.*⁹⁾ have measured the migration velocity of polystyrene particles in a sodium chloride aqueous solution in which a DC current and DC magnetic field were simultaneously imposed. They found that the direction of migration is opposite to the electromagnetic force and the migration velocity agreed well with values calculated from the theory of Leenov and Kolin.¹⁰⁾

In this study, the application of the AC pinch force to inclusion separation is explored theoretically. The

pinch force, one of the most general forces in the metallurgical field, can be generated by the interaction between an imposed current and an induced magnetic field. For instance, a strong alternating current is usually applied in the channel-induction furnace or in the tundish heater. The applied current generates not only Joule heat but also a strong pinch force in the liquid metal. However, the role of this pinch force in inclusion removal has not been clarified yet.

2. Theory

2.1. Assumptions

In this study, infinitely long vertical pipes with circular and square cross section are considered. **Figure 1** shows the geometry of the pipes. In the axial direction of the pipe, an alternating current is imposed and a liquid metal containing suspended inclusion particles is passed through the pipe with a constant velocity, $\bar{w}$. The particles then will migrate laterally by the pinch force, attach to the wall of the pipe, and consequently be removed from the liquid metal. In order to obtain inclusion-removal efficiencies theoretically, the following assumptions were made:

(1) Variables such as electrical currents, magnetic flux density and liquid velocity do not change with the axial

Fig. 1. Metal container geometry analysed in this study.

distance, z .

(2) The electromagnetic field is not affected by the fluid flow.

(3) The fluid flow is laminar and in steady state.

(4) The fluid flow is not affected by suspended particles.

(5) The temperature is uniform.

(6) The inclusion particles are spherical with uniform diameter; and there is no interaction between particles.

(7) The particles are so small that their inertia is negligible.

(8) The particles respond to the time-averaged pinch force.

(9) The particles which arrive at the wall are fixed immediately.

2.2. Migration Velocity of Particles

Leenov and Kolin¹⁰⁾ derived a theoretical equation of the electromagnetic force on a spherical particle having an electrical conductivity different from that of the liquid metal.

$$F_p = -\frac{3}{2} \frac{\sigma_f - \sigma_p}{2\sigma_f + \sigma_p} \frac{\pi d_p^3}{6} F, \quad (1)$$

where F is the electromagnetic force acting on a unit volume of liquid metal; and σ_f and σ_p are the electrical conductivity of fluid and particle, respectively. In the case where the particle is non-conductive ($\sigma_p = 0$), the force, F_p , becomes

$$F_p = -\frac{3}{4} \frac{\pi d_p^3}{6} F. \quad (2)$$

It is seen from this equation that the migration force acts in opposition to the electromagnetic force. By equating F_p with the Stokes drag force, the migration velocity of the particles, v_{pr} , can be derived as follows:

$$v_{pr} = -d_p^2 F / 24\mu, \quad (3)$$

where μ is the viscosity of the liquid metal.

The terminal velocity of the particle due to the buoyancy force can be expressed by Stokes Law,

$$v_{pt} = d_p^2 \Delta \rho g / 18\mu. \quad (4)$$

2.3. Pinch Force

2.3.1. Circular Pipe

The current density and the induced magnetic flux

density are expressed as a function of time, t , as follows:

$$J_t = \sqrt{2} J e^{j\omega t}, \quad B_{\theta t} = \sqrt{2} B_{\theta} e^{j\omega t}, \quad (5)$$

where $\omega = 2\pi f$; f is the frequency; and J and B_{θ} are complex numbers. The root-mean-square value of the fluctuating pinch force, F_r , can be obtained by

$$F_r = -\text{Re}\{J \cdot B_{\theta}^*\}, \quad (6)$$

where Re is the real part of the complex number and B_{θ}^* is the conjugate of the complex number, B_{θ} . Based on Eq. (6), F_r is obtained as follows:

$$F_r = \frac{\mu_e J_{rms}^2}{4\sqrt{2} \pi^2 r_1^3} f(\xi, R), \quad (7)$$

$$f(\xi, R) = \frac{\text{ber } \xi R (\text{ber}_1 \xi R - \text{bei}_1 \xi R) + \text{bei } \xi R (\text{ber}_1 \xi R + \text{bei}_1 \xi R)}{\text{ber}_1^2 \xi + \text{bei}_1^2 \xi},$$

where ber and bei represent the Kelvin functions; I_{rms} is the root-mean-square value of the imposed current; $\xi = \sqrt{2} r_1 / \delta$; and $R = r / r_1$. The skin depth, δ , is expressed by

$$\delta = (2 / \mu_e \sigma \omega)^{1/2}. \quad (8)$$

In the case of low frequency ($r_1 / \delta < 1$), Eq. (7) can be simplified as follows:

$$F_r = -\frac{\mu_e J_{rms}^2}{2\pi^2 r_1^3} R. \quad (9)$$

The migration velocity can be obtained by substituting Eq. (7) or (9) into Eq. (3).

2.3.2. Square Pipe

In the case of the square pipe, the electromagnetic equations must be solved numerically. The equations to be solved are as follows:

$$(1/\mu_e) \nabla \times (\nabla \times \mathbf{A}) = \sigma(-j\omega \mathbf{A} - \nabla \phi) + \mathbf{J}_{ext}, \quad (10)$$

$$\nabla \cdot [\sigma(-j\omega \mathbf{A} - \nabla \phi)] = 0, \quad (11)$$

where $\mathbf{A}$ is the vector potential of the magnetic flux density, $\mathbf{B}$; ϕ is the scalar potential; and $\mathbf{J}_{ext}$ is the imposed current density. Equation (10) can be modified into the following by applying the rectangular coordinates shown in Fig. 1(b).

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = \mu_e \sigma \left(j\omega A + \frac{d\phi}{dz} \right) - \mu_e J_{ext}, \quad (12)$$

where A and J_{ext} are the z components of $\mathbf{A}$ and $\mathbf{J}_{ext}$, respectively. In the case of infinitely long conductor of which the axis is z , A and J_{ext} do not have the x and y components.

For the second equation, Eq. (13) has been adopted instead of Eq. (11) in this study.

$$\frac{d\phi}{dz} = -\frac{j\omega}{S} \int_S A dS, \quad (13)$$

where S is the cross-sectional area of the pipe. Equation (13) can be derived by considering that the integrated value of the induced current density over the cross-

sectional area of the pipe should be zero.

The domain of calculation was restricted to a quarter cross section ($0 \leq x \leq nx_1$, $0 \leq y \leq ny_1$) in which ferromagnetic materials do not exist, i.e., $\mu_e = \mu_0$. Then the boundary conditions are as follows:

$$\begin{aligned} x=0, 0 < y < y_1 : \partial A / \partial x &= 0, \\ y=0, 0 < x < x_1 : \partial A / \partial y &= 0, \\ x=x_1, 0 < y < y_1 : \partial A / \partial x|_{x_1-} &= \partial A / \partial x|_{x_1+}, A|_{x_1-} = A|_{x_1+}, \\ y=y_1, 0 < x < x_1 : \partial A / \partial y|_{y_1-} &= \partial A / \partial y|_{y_1+}, A|_{y_1-} = A|_{y_1+}, \\ x=nx_1 \text{ or } y=ny_1 : A &= 0, \end{aligned} \quad \dots\dots\dots(14)$$

where $y_1 = x_1$. The value of n in the last condition was selected as 4 in this study.

Equations (12) to (14) were transformed into finite-difference equations by discretizing the quarter section into a uniformly divided mesh (in this study a 41×41 grid was adopted). The equations were solved by the successive over-relaxation method (SOR) for which the condition of convergence was $|\Delta A / A|_{\max} < 10^{-10}$. From the obtained values of A and $d\phi/dz$, J , B_x and B_y were calculated from the following equations.

$$J = -j\omega\sigma A - \sigma(d\phi/dz) + J_{\text{ext}}, \quad \dots\dots\dots(15)$$

$$B_x = \partial A / \partial y, \quad B_y = -\partial A / \partial x. \quad \dots\dots\dots(16)$$

Finally, the time-averaged pinch forces, F_x and F_y , were obtained from

$$F_x = -\text{Re}\{J \cdot B_y^*\}, \quad F_y = \text{Re}\{J \cdot B_x^*\}. \quad \dots\dots\dots(17)$$

In preliminary calculations, the above method was applied to a circular pipe problem; and the agreement between numerical and analytical solutions has been confirmed in the range of $r_1/\delta = 0-5$.

2.4. Removal Efficiency of Particles

2.4.1. Circular Pipe (Trajectory Method)

The velocity distribution of the liquid metal can be expressed by

$$w = w_m[1 - (r/r_1)^2], \quad \dots\dots\dots(18)$$

where w_m is the maximum axial velocity. A small buoyant particle, for which the terminal velocity is v_{pt} , has the following velocity in the liquid metal flow.

$$v_{pz} = w_m[1 - (r/r_1)^2] + v_{pt} = dz/dt, \quad \dots\dots\dots(19)$$

$$v_{pr} = -\frac{\mu_e I_{\text{rms}}^2 d_p^2}{96\sqrt{2} \pi^2 \mu r_1^3} f(\xi, R) = dr/dt. \quad \dots\dots\dots(20)$$

Dividing Eq. (19) by Eq. (20), the equation of the particle trajectory can be obtained from

$$dz/dr = [w_m\{1 - (r/r_1)^2\} + v_{pt}]/v_{pr}. \quad \dots\dots\dots(21)$$

If a backward integration of Eq. (21) is made from the surface of the wall ($r=r_1, z=0$) to an arbitrary radial position, the axial position of the particle ($-z$) can be obtained. At this axial position, all the particles which exist in the outer radial position must be collected on the wall at $z=0$. As a result, the removal efficiency of particles can be expressed by the following equation:

$$\begin{aligned} \eta &= \int_r^{r_1} v_{pz} 2\pi r dr / [\pi r_1^2 (w_m/2 + v_{pt})] \\ &= \frac{(w_m + v_{pt})(r_1^2 - r^2) - (w_m/2r_1^2)(r_1^4 - r^4)}{r_1^2 (w_m/2 + v_{pt})}. \quad \dots\dots\dots(22) \end{aligned}$$

Equations (21) and (22) can be transformed into Eqs. (23) and (24) by introducing the dimensionless parameters shown in Eq. (25).

$$\frac{dZ}{dR} = -24\sqrt{2} \pi^2 \frac{1 + V_R - R^2}{(C_I D_R^2 / Re) f(\xi, R)}, \quad \dots\dots\dots(23)$$

$$\eta = \frac{2(1 + V_R)(1 - R^2) - (1 - R^4)}{1 + 2V_R}, \quad \dots\dots\dots(24)$$

$$\begin{aligned} Z &= z/r_1, \quad V_R = v_{pt}/w_m, \quad C_I = \mu_e I_{\text{rms}}^2 / \rho v^2, \\ Re &= 2r_1 \bar{w}/v, \quad D_R = d_p/2r_1. \end{aligned} \quad \dots\dots\dots(25)$$

From the above equations, the particle-removal efficiency, η , is found to be correlated by the following equation.

$$\eta = f[V_R, (C_I D_R^2 / Re)Z, r_1/\delta]. \quad \dots\dots\dots(26)$$

The numerical integration of Eq. (23) has been carried out by the Runge-Kutta-Gill method in which the increment ΔR was fixed at -0.01 . In the case of low frequency, Eq. (9) can be used as the pinch force expression and an analytical solution for η can be derived. From a preliminary calculation, the agreement between the analytical and numerical result of η has been confirmed at $r_1/\delta = 0$.

2.4.2. Square Pipe (Concentration Method)

In the case of the square pipe, secondary flows may appear in the x - y cross sectional plane because the pinch force will be concentrated near the corner of the pipe at high frequencies. These flows, coupled with the axial flow, may affect the particle transport phenomena in a complicated way, making it difficult to adopt the particle-trajectory method. Therefore the concentration method has been used to obtain η after solving the following flow equations.

The nondimensional flow equations to be solved are as follows:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0, \quad \dots\dots\dots(27)$$

$$U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = -\frac{\partial P}{\partial X} + \frac{\partial^2 U}{\partial X^2} + \frac{\partial^2 U}{\partial Y^2} + F_x, \quad \dots\dots\dots(28)$$

$$U \frac{\partial V}{\partial X} + V \frac{\partial V}{\partial Y} = -\frac{\partial P}{\partial Y} + \frac{\partial^2 V}{\partial X^2} + \frac{\partial^2 V}{\partial Y^2} + F_y, \quad \dots\dots\dots(29)$$

$$U \frac{\partial W}{\partial X} + V \frac{\partial W}{\partial Y} = -\frac{\partial P}{\partial Z} + \frac{\partial^2 W}{\partial X^2} + \frac{\partial^2 W}{\partial Y^2}. \quad \dots\dots\dots(30)$$

The dimensionless variables in the above equations are as follows:

$$\left. \begin{aligned} X &= x/x_1, \quad Y = y/x_1, \quad Z = z/x_1, \\ U &= x_1 u/v, \quad V = x_1 v/v, \quad W = x_1 w/v, \\ P &= x_1^3 p/\rho v^2, \quad F_x = x_1^3 F_x/\rho v^2, \quad F_y = x_1^3 F_y/\rho v^2, \end{aligned} \right\} \quad \dots\dots\dots(31)$$

where U , V and W are the fluid velocities in the X , Y and Z direction respectively, and P is the pressure. According to the first assumption stated earlier, the Z derivatives, with the exception of the pressure term, can be omitted.

The following may be applied as the boundary conditions:

$$\left. \begin{aligned} X=0, \quad 0 < Y < 1 : U = \partial V / \partial X = \partial W / \partial X = 0, \\ Y=0, \quad 0 < X < 1 : \partial U / \partial Y = V = \partial W / \partial Y = 0, \\ X=1, \quad 0 < Y < 1 : U = V = W = 0, \\ Y=1, \quad 0 < X < 1 : U = V = W = 0. \end{aligned} \right\} \dots (32)$$

Equations (27) to (29) have been solved first, followed by Eq. (30), because the former equations are independent of the latter. The stream function and vorticity were used to transform Eqs. (27) to (29). Subsequently, these equations were transformed into finite-difference equations by the upwind-difference scheme (UDS) and solved by the SOR method.¹¹⁾ Equation (30) was then transformed by UDS and solved by the SOR method at various values of $(-\partial P / \partial Z)$. A uniformly divided mesh (11×11 grid points in the x - y plane) has been adopted for the numerical calculations. For conditions of convergence in the computation, $|\Delta \zeta / \zeta|_{\max} < 10^{-5}$ and $|\Delta W / W|_{\max} < 10^{-5}$ were employed, where ζ is vorticity.

As mentioned above, the particle-trajectory method has not been used to obtain η because of the complexity of the flow field. The particle-concentration method used for the square tube is based on the following equation.

$$\frac{\partial U_P C}{\partial X} + \frac{\partial V_P C}{\partial Y} + \frac{\partial W_P C}{\partial Z} = 0. \dots (33)$$

The dimensionless variables in the above equation are as follows:

$$U_P = x_1 u_P / v, \quad V_P = x_1 v_P / v, \quad W_P = x_1 w_P / v, \quad C = c / c_0, \dots (34)$$

where c_0 is the particle concentration at the inlet of pipe, and u_P , v_P and w_P are the particle velocities as given in the following equations.

$$\begin{aligned} u_P &= u - (d_p^2 / 24 \mu) F_x, \quad v_P = v - (d_p^2 / 24 \mu) F_y, \\ w_P &= w + v_{pi}. \end{aligned} \dots (35)$$

The boundary conditions are expressed by

$$\left. \begin{aligned} Z=0 : C=1, \\ X=0 \text{ or } 1, \quad 0 < Y < 1 : \partial C / \partial X = 0, \\ Y=0 \text{ or } 1, \quad 0 < X < 1 : \partial C / \partial Y = 0. \end{aligned} \right\} \dots (36)$$

The X and Y -direction derivatives in Eq. (33) have been transformed into finite differences by UDS, and the equation has been integrated along the Z axis by the Crank-Nicolson method for each step of ΔZ . The SOR method has been used to obtain convergent predictions of C ; and $|\Delta C / C|_{\max} < 10^{-4}$ was applied for the condition of convergence.

Finally, the value of η has been calculated by the following equation by using computed values of C and W_P .

$$\eta = 1 - \int_0^1 \int_0^1 C W_P / \{ \bar{W} (1 + V_R') \} dX dY. \dots (37)$$

The particle-concentration method mentioned above has been adopted for the circular pipe, and the results for η agree well with the predictions from the particle-trajectory method.

The following six nondimensional parameters may affect the value of η in the case of the square pipe.

$$\begin{aligned} Z, \quad C_I (= \mu_e I_{rms}^2 / \rho v^2), \quad (-\partial P / \partial Z), \\ V_R' (= v_{pi} / \bar{w}), \quad D_R (= d_p / 2x_1), \quad x_1 / \delta. \end{aligned} \dots (38)$$

Among these parameters, $(-\partial P / \partial Z)$ can be replaced by the pipe Reynolds number, Re .

$$Re = 2x_1 \bar{w} / v. \dots (39)$$

As in the case of the circular pipe, η can be correlated by the following equation while secondary flows are negligibly weak.

$$\eta = f[V_R', (C_I D_R^2 / Re) Z, x_1 / \delta]. \dots (40)$$

On the other hand, Eq. (41) may be applicable when the secondary flows become strong, because the parameters which directly affect the secondary flows should be taken into account.

$$\eta = f[V_R', (C_I D_R^2 / Re) Z, x_1 / \delta, C_I, D_R, Re]. \dots (41)$$

3. Results and Discussion

3.1. Circular Pipe

3.1.1. Effects of Various Parameters on η

Figure 2 shows the effect of $V_R (= v_{pi} / w_m)$ on the relation between η and $(C_I D_R^2 / Re) Z$ at $r_1 / \delta = 0$. The value of η increases rapidly with increasing $(C_I D_R^2 / Re) Z$ in the initial stage, but the rate of increase becomes small as η approaches unity. This indicates that the flux of inclusion particles to the wall is high at the inlet of the pipe and it becomes lower as Z increases. As seen from the figure, η decreases with increasing V_R , because the residence time of particles in the pipe becomes short. However, the effect of V_R on η is not so strong in a practical range of V_R which may be less than 0.1.

Figure 3 shows the effect of r_1 / δ on the relation between

Fig. 2. Effect of V_R on the relationship between η and $(C_I D_R^2 / Re) Z$ at $r_1 / \delta = 0$.

Fig. 3. Effect of r_1/δ on the relationship between η and $(C_I D_R^2 / Re) Z$ at $V_R = 0$.

η and $(C_I D_R^2 / Re) Z$ at $V_R = 0$. It is seen from the figure that η does not change with r_1/δ in the range of $r_1/\delta < 1$, but it decreases gradually as r_1/δ increases above unity. This is because of the skin effect on the pinch force. While $r_1/\delta < 1$, the pinch force does not change with r_1/δ as shown in Eq. (9), but it becomes concentrated near the wall in the case of $r_1/\delta > 1$. Although the pinch force increases near the wall, the force in the inner region becomes so weak that the particles there migrate only slowly toward the wall.

The operating condition to achieve a desired value of η for a certain inclusion size can be determined from Fig. 3. The value of $(C_I D_R^2 / Re) Z$ which corresponds to the desired η includes C_I , Re and Z as variables at fixed r_1/δ and D_R ; thus any combination of these variables is permitted in a mathematical sense.

3.1.2. Inclusion Removal in the Tundish Heater

Recently, several tundishes in steel continuous casting machines have been equipped with heating systems in order to improve the cleanliness of the steel.¹²⁻¹⁵⁾ Figure 4 shows a schematic view of a tundish heater which is based on the channel-induction furnace.¹²⁾ This tundish is composed of two chambers, two pipes which connect each chamber, and an iron core wrapped with a coil. These pipes and chambers form a one-turn coil when the tundish is filled with liquid steel. A strong alternating current can be induced in these pipes and the poured steel is heated by the Joule effect. By using this tundish, considerable reduction of nonmetallic inclusions has been achieved, but the reason for this effect has not been elucidated yet.

The operating conditions for this tundish heater are shown in Table 1. Assuming an electric power efficiency of 80%, the maximum current in the heater is estimated to be 44 kA. From this value and the geometrical conditions shown in the table, values of η have been calculated based on the result in Fig. 3. In the calculation, the end effects at the inlet and outlet of the pipe were neglected.

Figure 5 shows the relationships between η and z for various sizes of inclusion particles. Thus it is seen that most of the large particles ($d_p > 60 \mu\text{m}$) are removed by the time the liquid steel is passed through the pipe.

Fig. 4. Schematic diagram of tundish heater¹²⁾ for continuous casting of steel.

Table 1. Operating conditions of the tundish heater.¹²⁾

Diameter of pipe ($2r_1$)	0.1 m
Length of pipe (z_1)	1.15 m
Flow rate of steel	1.5 t/min
Maximum power:	1 MW
Power efficiency	80 %
Maximum current (I_{rms})	44 200 A
Frequency	60 Hz
C_I	6.9×10^{11}
r_1/δ	0.65
Re	31 800

$$(\sigma_{Fe} = 7.2 \times 10^5 \text{ S m}^{-1}, \rho_{Fe} = 7000 \text{ kg/m}^3, \mu_{Fe} = 0.005 \text{ Pa} \cdot \text{s})$$

Fig. 5. Plot of η against z on η_{exit} against electric power ratio in the tundish heater for different particle diameters.

On the other hand, the small particles ($d_p < 20 \mu\text{m}$) are difficult to separate effectively. However, the value of η for the small particles can be increased by decreasing the flow rate of the liquid steel and the diameter of the pipe, or by increasing the length of the pipe. Collision and coagulation may also be expected between small particles

in the oscillatory pinch-force field, which has not been considered in this study.

Although Fig. 5 was obtained at the maximum electric power (1 MW), the effect of a decrease in the electric power on η can also be read from the same plots. The electric power, as well as C_I , is proportional to I_{rms}^2 , so the change in the electric power corresponds to the change in C_I . As seen from Fig. 3, each of C_I and Z has exactly the same effect on η . Consequently, Fig. 5 shows the relation between η at the exit of the pipe and the specific power ratio as well as the relation between η and z . As seen from Fig. 5, η_{exit} for large particles shows a small change with decreasing specific power. These results support the effectiveness of the pinch-force separation of inclusion particles.

Laminar flow is assumed in the present analysis, but the flow of liquid steel in the pipe may be turbulent at the high Reynolds number shown in Table 1 ($Re=31\,800$). If the turbulent flow does not retard the inclusion-removal rate in the vicinity of the wall, its sole effect may be to make the particle concentration uniform in the cross section of the pipe. In this case, the value of η may increase because the particles in the inner region of the liquid steel can be transported to the wall easily by turbulent diffusion. However, further investigations are needed to clarify the effect of turbulence on the particle removal as well as the end effects of the pipe.

3.2. Square Pipe

3.2.1. Velocity Profile

Figure 6 shows velocity distributions in the quarter cross section of the square pipe for four values of x_1/δ . There are two recirculating flows in the cross section and the velocity increases quite rapidly with increasing x_1/δ . These secondary flows are generated due to the pinch force being concentrated in the vicinity of the pipe wall especially near the corners of the pipe. These flows are expected to affect η when their velocities are comparable to the axial flow velocity.

Fig. 6. Velocity profiles in the x - y plane of a quarter cross-section of a square pipe. ($C_I=10^8$)

The maximum velocity, U_m , is plotted, together with the maximum pinch force, F_m , against x_1/δ in Fig. 7. The value of U_m exhibits a sharp increase with x_1/δ , and it tends to saturate in the vicinity of $x_1/\delta=5$. On the other hand, F_m remains constant in the range of $x_1/\delta < 1$ and increases beyond x_1/δ of unity.

Figure 8 shows axial velocity profiles in the quarter cross section for various values of x_1/δ at $-\partial P/\partial Z=10^3$. Although a parabolic profile of W can be seen at $x_1/\delta=0$, this profile is distorted at $x_1/\delta=0.5$ and becomes flat at larger values of x_1/δ . The axial velocity is found to decrease with increasing x_1/δ because of the increase in flow resistance due to the development of secondary flows.

Figure 9 shows the effect of C_I on the relationship between $Re (=2\bar{W})$ and x_1/δ at $-\partial P/\partial Z=10^3$. In the region of small x_1/δ , the value of Re agrees with that of $C_I=0$, because the secondary flows are so weak that they never affect the axial flow in this region. But the Re begins to decrease with increasing x_1/δ beyond a critical value until near $x_1/\delta=5$ whereupon the decrease is arrested. This corresponds to the relation between U_m and x_1/δ shown in Fig. 8. The critical x_1/δ value where

Fig. 7. Maximum pinch force and velocity as a function of x_1/δ . ($C_I=10^8$)

Fig. 8. Axial velocity maps in the x - y plane of a quarter cross-section of a square pipe. ($C_I=10^8$, $-\partial P/\partial Z=10^3$)

Fig. 9. Relation between Re and x_1/δ for different values of C_f at $-\partial P/\partial Z=10^3$.

Fig. 10. Relation between Re and $-\partial P/\partial Z$ for different values of C_f .

Re commences its decrease rises with decreasing C_f ; and the extent of the decline in Re increases with increasing C_f . This means that the secondary flows become stronger with increasing C_f .

The relation between Re and $(-\partial P/\partial Z)$ at different values of C_f is shown in Fig. 10. For the case of $C_f=0$ in which secondary flows do not exist, the following empirical correlation for the friction factor in a square pipe has been reported¹⁶⁾ as follows:

$$\Delta p = 4C_f(\rho \bar{w}^2/2)(z/2x_1), \quad C_f = 14.21/Re. \quad \dots(42)$$

As shown in the figure, this correlation agrees well with the present result. In the case of $C_f=10^8$ ($x_1/\delta=2$), the Re values are smaller than for $C_f=0$, but the linear relationship between Re and $(-\partial P/\partial Z)$ does not change.

3.2.2. Effects of Various Parameters on η

The value of η in a square pipe is compared with that in a circular pipe for the condition of $x_1/\delta=r_1/\delta=0$ in Fig. 11. Thus it is seen that the value of η for the square pipe is smaller than that for the circular pipe. The value of η in the circular pipe with the same cross-sectional area and flow rate as the square pipe is also shown in the same figure. In this case, the value of η becomes closer to that of the square pipe. This indicates that the shape of the pipe has only a small effect on η in the case without the secondary flows.

Figure 12 shows the effect of x_1/δ on the relationship between η and $(C_f D_R^2/Re)Z$ at $C_f=10^8$, $D_R=10^{-3}$,

Fig. 11. Comparison of η between circular and square pipes. ($x_1/\delta=r_1/\delta=0$)

Fig. 12. Effect of x_1/δ on the relationship between η and $(C_f D_R^2/Re)Z$. ($C_f=10^8$, $D_R=10^{-3}$, $Re=100$)

$Re=100$ and $V'_R=0$. It is found from the figure that the value of η increases with increasing x_1/δ . This result is completely opposite to that of the circular pipe shown in Fig. 3. This is because secondary flows transport particles from the inner region of the pipe to the vicinity of the wall where the pinch force is strong. In this case, the skin effect also accelerates the particle removal rate, because the force near the wall becomes stronger as x_1/δ increases. As shown in Fig. 7, the intensity of secondary flows is considerably strong even in the region $x_1/\delta < 1$, and hence η starts to increase at $x_1/\delta=0.2$.

The effects of Re , C_f and D_R on the relationship between η and $(C_f D_R^2/Re)Z$ have been examined at $V'_R=0$ and $x_1/\delta=5$. Firstly, no effect of Re on the relation between η and $(C_f D_R^2/Re)Z$ was detected throughout the range of Re from 10 to 1000 under the condition of $C_f=10^8$ and $D_R=10^{-3}$. Therefore, Re can be eliminated from Eq. (41). Secondly, effects of C_f and D_R have been computed; and the results are shown in Figs. 13(a) and 13(b) where the value of η at constant $(C_f D_R^2/Re)Z$ is seen to decrease gradually as C_f and D_R increase. However, the extent of decrease in η is less than 3% in the range $C_f=10^7-10^9$ and $D_R=0.5 \times 10^{-3}-2 \times 10^{-3}$.

Finally, Fig. 14 shows particle-concentration profiles in the quarter cross section of the square pipe for various values of x_1/δ at $\eta=0.5$. In the case of $x_1/\delta=0$, that is, without secondary flows, the particle concentration

Fig. 13. Effects of C_I and D_R on the relationship between η and $(C_I D_R^2 / Re) Z$ at $x_1/\delta = 5$.

Fig. 14. Concentration maps of inclusion particles at $\eta = 0.5$ in the x - y plane of a quarter cross-section of a square pipe. ($C_I = 10^8$, $D_R = 10^{-3}$, $Re = 100$)

shows a maximum at the axis of the pipe and decreases smoothly toward the wall. As x_1/δ increases, the profile becomes flat except at the axis. At $x_1/\delta = 5$, the concentration has a peak near the corner of the pipe as well as at the axis, because particles accumulate there due to highly developed secondary flows.

Although the heat generation due to the Joule effect has not been considered in the present study, this effect is very important for designing the inclusion separator. This effect should be investigated in the future study.

4. Conclusions

The application of a pinch force to the separation of nonmetallic inclusion particles from liquid steel has been discussed theoretically in this study. Inclusion-removal efficiencies, η , from liquid metal to the wall of the pipe with circular and square-cross section, in which an alternating current is imposed, have been calculated for various physical conditions. In the case of the circular pipe, η computed by a trajectory method is found to be a function of V_R , $(C_I D_R^2 / Re) Z$ and r_1/δ .

The value of η rises with increasing $(C_I D_R^2 / Re) Z$ and

decreases gradually with increasing V_R and r_1/δ . The value of η in a continuous casting tundish equipped with a channel-inclusion furnace has been estimated. The value of η reaches over 95 % for particles with a diameter greater than $60 \mu\text{m}$ at the maximum electric power.

In the case of the square tube, the generation of secondary flows is found in the cross section of the pipe at high frequencies because the pinch force is concentrated near the corner of the pipe. Due to these secondary flows, the axial flow velocity decreases with increasing x_1/δ and C_I at constant $(-\partial P/\partial Z)$. In such cases, the relation between Re and $(-\partial P/\partial Z)$ is linear. The value of η calculated by a concentration method is smaller than that for the circular pipe. Contrary to the results for the circular pipe, η increases as x_1/δ increases, because secondary flows transport the particles from the inner region to the vicinity of the wall where strong forces exists. Parameters which affect η are V_R , $(C_I D_R^2 / Re) Z$ and x_1/δ while secondary flows are negligibly small, but C_I and D_R should be added when secondary flows develop.

Nomenclature

- A : root-mean-squared vector potential ($\text{T} \cdot \text{m}$)
- B : root-mean-squared magnetic flux density (T)
- C : nondimensional particle concentration ($= c/c_0$)
- C_I : nondimensional parameter including current ($= \mu_e I_{\text{rms}}^2 / \rho v^2$)
- c : particle concentration ($1/\text{m}^3$)
- D_R : nondimensional particle diameter ($= d_p/2r_1$ or $d_p/2x_1$)
- d_p : particle diameter (m)
- F : pinch force (N/m^3)
- F_p : electromagnetic force on a particle (N)
- F_r : root-mean-squared pinch force (N/m^3)
- $F_{x,y}$: nondimensional pinch force ($= x_1^3 F_{x,y} / \rho v^2$)
- f : function in Eqs. (26), (40) and (41)
- I_{rms} : root-mean-square value of imposed alternating current (A)
- J : root-mean-squared current density (A/m^2)
- J_{ext} : imposed current density ($= I_{\text{rms}}/S$) (A/m^2)
- P : nondimensional pressure ($= x_1^2 p / \rho v^2$)
- p : pressure (Pa)
- R : nondimensional radial distance ($= r/r_1$)
- Re : pipe-Reynolds number ($= 2r_1 \bar{w}/v$ or $2x_1 \bar{w}/v$)
- r : radial distance (m)
- S : cross-sectional area of the square pipe (m^2)
- t : time (s)
- U : nondimensional velocity in the x -direction ($= x_1 u/v$)
- u : velocity in the x direction (m/s)
- V : nondimensional velocity in the y direction ($= x_1 v/v$)
- V_R : nondimensional terminal velocity of particle in the circular pipe ($= v_{pt}/w_m$)
- V'_R : nondimensional terminal velocity of particle in the square pipe ($= v_{pt}/\bar{w}$)
- v : velocity in the y direction (m/s)
- v_{pt} : radial velocity of particle due to pinch force (m/s)
- v_{pt} : terminal velocity of particle (m/s)

W : nondimensional velocity in the z direction
 ($=r_1 w/v$ or $x_1 w/v$)
 w : velocity in the z direction (m/s)
 X : nondimensional distance ($=x/x_1$)
 x : distance shown in Fig. 1b (m)
 Y : nondimensional distance ($=y/x_1$)
 y : distance shown in Fig. 1b (m)
 Z : nondimensional axial distance ($=z/r_1$ or z/x_1)
 z : axial distance shown in Fig. 1b (m)
 δ : skin depth ($=(2/\mu_e \sigma \omega)^{1/2}$) (m)
 ζ : vorticity ($=\partial U/\partial Y - \partial V/\partial X$)
 η : inclusion-removal efficiency
 μ : viscosity (Pa·s)
 μ_0 : magnetic permeability in vacuum (H/m)
 μ_e : magnetic permeability ($=\mu_0$) (H/m)
 ν : kinematic viscosity (m^2/s)
 ξ : nondimensional value ($=\sqrt{2} r_1/\delta$)
 ρ : density (kg/m^3)
 σ : electrical conductivity (S/m^{-1})
 ϕ : scalar potential ($\text{T} \cdot \text{m}^2/\text{s}$)
 ω : angular frequency (rad/s)

Subscripts

0: initial value
 1: wall
 m : maximum value
 P : particle
 x : x direction
 y : y direction
 θ : azimuthal direction

Superscript

($\bar{}$): mean value

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Appendix

A-1. Inclusion Separation by a Solenoid

Korovin^{17,18)} proposed a method for inclusion separation based on a pinch force in a solenoid. As shown in Fig. A-1, a pinch force can be induced in a liquid-metal cylinder by applying an AC current in a solenoid, and this force is also applicable to inclusion separation. The effectiveness of this method will be discussed in comparison with the present method.

Assuming an infinitely long melt and coil, the root-mean-square value of the fluctuating pinch force is expressed by

$$F_r = -\frac{\mu_e (NI_{rms}/L)^2}{\sqrt{2} r_1} f^*(\xi, R),$$

$$f^*(\xi, R) = \xi \frac{\text{ber } \xi R (\text{ber}_1 \xi R + \text{bei}_1 \xi R) - \text{bei } \xi R (\text{ber}_1 \xi R - \text{bei}_1 \xi R)}{\text{ber}_1^2 \xi + \text{bei}_1^2 \xi} \quad \text{.....(A-1)}$$

Substituting Eq. (A-1) into Eq. (3) and dividing by Eq. (4), the following equation can be derived.

$$\frac{v_{Pr}}{v_{Pt}} = \frac{3}{4\sqrt{2}} \frac{C_I^*}{Gr} f^*(\xi, R), \quad \text{.....(A-2)}$$

where $Gr = r_1^3 \Delta \rho g / (\rho \nu^2)$ and $C_I^* = r_1^2 \mu_e (NI_{rms}/L)^2 / (\rho \nu^2)$.

On the other hand, v_{Pr}/v_{Pt} for the present method is expressed as follows:

$$\frac{v_{Pr}}{v_{Pt}} = -\frac{3}{16\sqrt{2} \pi^2} \frac{C_I}{Gr} f(\xi, R) \quad \text{.....(A-3)}$$

The calculated values of v_{Pr}/v_{Pt} by Eqs. (A-2) and (A-3) are given in Figs. A-2 and A-3 respectively. In the figures, contour lines of v_{Pr}/v_{Pt} are plotted with r_1/δ as ordinate and $R (=r/r_1)$ as abscissa. There can be seen a distinct difference between these results especially in the lower range of r_1/δ . In the case for the solenoid (Fig. A-2), the value of v_{Pr}/v_{Pt} decreases suddenly after r_1/δ becomes less

Fig. A-1. Method for applying current in a liquid metal cylinder by a solenoid.

Fig. A-2. Contour lines of v_{Pr}/v_{Pt} for the system shown in Fig. A-1.

Fig. A-3. Contour lines of v_{Pr}/v_{Pt} for the system shown in Fig. 1.

than 2, because the pinch force acts only in a skin layer. On the other hand, v_{Pr}/v_{Pt} keeps considerable strength even in the lower range of r_1/δ for the present method (Fig. A-3). Taking into account the difficulty of imposing the pinch force in a long cylinder because of the limitation in coil length, it may be concluded that effective separation of inclusion particles by using the solenoid is not expected.

A-2. Pinch Effect in a Circular Pipe

In the case of low static pressure, the liquid metal cylinder may be squeezed and broken because of the pinch force, which is called the pinch effect.¹⁹⁾ The limiting condition for the pinch effect will be discussed based on the present theory. It is postulated that a

Table A-1. Values of β in Eq. (A-7).

r_1/δ	β	r_1/δ	β
0.1	1.0000	3	0.7718
0.5	0.9996	4	0.6782
1	0.9932	5	0.6300
2	0.9141	6	0.6027

horizontal pipe is connected with a metal reservoir containing a liquid metal of which the free surface is kept at the height, h , from the axis of the pipe. Neglecting the pressure losses due to the friction and contraction of the metal stream, Bernoulli's theorem can be applied between the free surface and the cross section of the pipe.

$$p - p_a = \rho gh - \rho \bar{w}^2/2, \quad \text{.....(A-4)}$$

where p is the liquid pressure in the pipe; p_a is the atmospheric pressure. If the value of p is less than the pressure due to the pinch force, the pinch effect will appear. The pressure due to the pinch force is expressed as follows:

$$dp/dr = F_r, \quad \text{.....(A-5)}$$

Substituting Eq. (7) into the above equation, the following equation may be obtained by integrating between the limits of 0 to r .

$$p - p_a = -\frac{\mu e I_{rms}^2}{4\sqrt{2}\pi^2 r_1^2} \int_0^1 f(\xi, R) dR, \quad \text{.....(A-6)}$$

Equating the maximum pressure obtained by Eq. (A-6) with the pressure in Eq. (A-4), the limiting height of the free surface, h_c , below which the pinch effect appears can be given by

$$H_c = (Re^2/2 + \beta C_I/\pi^2)/4Ga,$$

$$\beta = -\frac{1}{\sqrt{2}} \int_0^1 f(\xi, R) dR, \quad \text{.....(A-7)}$$

where $H_c = h_c/r_1$ and $Ga = r_1^3 g/\nu^2$. The values of β in Eq. (A-7) at various values of r_1/δ are given in Table A-1. As seen in the table, β is equal to 1 in the range $r_1/\delta < 1$ while it decreases with increasing r_1/δ in the range $r_1/\delta > 1$.

The value of H_c for the operating condition of the tundish heater (Table 1) was estimated to be 6, so the pinch effect may not appear in the range of $h > 0.3m$.

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