Validation of a Blast Furnace Solid Flow Model Using Reliable 3-D Experimental Results

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The finite element method (FEM) is used in conjunction with plasticity theory in granular materials to derive the stress field and velocity field inside a small experimental apparatus reproducing the blast furnace. The theory used, called hypo-plasticity, gave satisfactory agreement between numerical and experimental time lines, and was able to predict the shape of the stagnant region in the bottom part, the so called dead man, without any adjustable parameters. Specific numerical methods, like iterative remeshing, allowed it to reach steady flow conditions in an Eulerian frame. The stress field is characterized by a plastic active state in the upper part, and a plastic passive state in the lower part. The velocity field is characterized by a plug flow in the upper part, and a funnel flow in the lower part. This model can also simulate granular flows in all type of vessels, like silos. In modeling blast furnaces, its usefulness lies in its connection with a multi-phase total model.

KEY WORDS: blast furnace; finite element method; hypo-plasticity theory; granular material.

1. Introduction

The blast furnace is an industrial counter current metallurgical reactor whose aim is to produce molten iron (the pig iron) from ore (iron oxide). This reactor involves a high number of phases, species, chemical reactions, and physical phenomena. Among these phases, the solid phase requires special attention.

The combustion of coke at the lower part of the blast furnace creates a stagnant region in its center, the dead man. The dead man behaves like a quasi-static pyramid of granular material. However, its shape in real blast furnaces is not yet known with any accuracy. Industrial observation of real blast furnaces revealed that the granular diameter and void fraction inside the dead man are slightly different than in other parts of the burden. It is, therefore, believed that it plays an important role in the blast furnace process:

- to distribute gas produced in the raceway by offering different paths (blast furnace operators aim to concentrate gas flow in the central part);
- to change gas composition, because solid carbon species are involved in the solution loss equilibrium C + CO2 ⇌ 2CO and water gas reaction C + H2O ⇌ CO + H2, beside combustion itself;
- to solve carbon in pig iron, because it is crossed by a descending flow of liquid iron;
- finally, operation problems may arise if dead man empty spaces are gradually filled by powders.

The solid phase in the blast furnace is, therefore, a highly reactive phase, and permanently in motion, even if average residence time of the solid phase is much longer than for the other phases. The shape of the dead man, its void fraction distribution, its impact on internal chemical phenomena or fluid flow, as well as the melting of ore layers and its consequences on gas path, are some examples of phenomena that cannot be fully understood without a solid flow model.

Over the past few decades, steel companies recognized the need to have a blast furnace total model at their disposal, which enables the prediction of all phenomena that usually occur inside the apparatus. Unfortunately, the most difficult task was to achieve a complete solid flow sub-model, which describes its velocity field, stress field and void ratio field.

Indeed, knowledge of the velocity field leads to the derivation of solid trajectories and to the prediction of layers evolution with depth. On the other hand, knowledge of the stress field allows the granular deterioration with pressure to be predicted. This phenomenon has nasty effects on blast furnace efficiency: the powder produced by granular degradation, so called fines, usually block up the empty spaces, preventing gas circulation. Finally, knowledge of the void ratio field has an immediate impact not only on the gas path, but also on heat and chemical transfers and ultimately blast furnace efficiency.

The aforementioned difficulty is only connected to the lack of a well established granular flow theory, in opposition for instance to the description of viscous fluid flow by means of Navier Stokes equations. Thus, solid flow was often over-simplified. Most publications on this subject re-
flect the attempts of researchers to introduce several granular flow theories into blast furnace modeling. We shall review in the next section some of the major attempts made by these researchers, especially in Japan. One aspect of over-simplified theories was their inability to predict the dead man shape in experimental validation. To overcome this, a new solid flow model is needed for simulating the dead man shape accurately.

The purpose of this study is, therefore, to introduce an alternative solid flow model, based on hypo-plasticity theory, better suited to granular materials. The reliability of this proposed model will then be discussed by comparing numerically calculated results with experimental data, paying special attention to gas-solid mechanical interaction and dead man shape prediction.

2. Review of Solid Flow Models for Blast Furnace

2.1. Generalities

The simulation of solid flow in blast furnaces was performed using many different methods. Nevertheless, each one belongs to a specific class of numerical methods, according to the results it leads to. Five kinds of mechanical models were used to simulate solid flows. The distinction between them stands in the mathematical method they use:

1. Discrete element model: this is a discontinuous approach (in opposition to the next ones qualified as continuous approaches), which gives the position of individual particles.
2. Velocity model: includes the potential flow model and the kinematic model. Both models furnish velocity distribution without stress field.
3. Viscous flow model, which gives velocity field and viscous shear stresses.
4. Perfect plasticity, also known as limit state, which is derived from static soil mechanics theory.
5. Incremental plasticity, which is derived from dynamic soil mechanics theory.

In the last theoretical model, we have included both the older elasto-plastic theory and the newer hypo-plastic theory, introduced in the next section, since both theories require the use of the finite element method. This numerical method aims to solve incrementally the equilibrium equations. In this specific case, the plastic state, characterized by steady flow conditions, is reached step by step. On the other hand, in the so called perfect plasticity model, steady flow is already assumed in all parts of the granular bed.

The differences between the elasto-plastic and the hypo-plastic theories stand in the definition of the constitutive equation, although both models can be used within the finite element method, and their ability to take into account most aspects of granular behaviour. On one hand, the elasto-plastic model assumes an elastic, i.e. reversible, state before the limit state. On the other hand, the hypo-plastic model assumes a plastic, i.e. irreversible, state even before the limit state. Therefore, the limit state is also called perfect plastic state to distinguish it from intermediate states. Nevertheless, the latter model is more appropriate to granular materials than the former, because granular materials have irreversible behaviour.

In most cases, each method was compared to a three dimensional small scale blast furnace cold model, where time lines and dead man profile were the two main phenomena observed. The first two mechanical models were based on empirical formulations, where numerical constants required adjustments according to experiments. Besides, none of them use momentum equations, hence they are not able to consider mechanical effects, like interaction between phases. In the following, we shall review in more details the last three cases, which overcome this limitation.

2.2. Viscous Flow Model

A major improvement was indeed brought by the viscous fluid model in the frame of a new total model project called “Four Fluid Flow”, where highly coupled momentum equations are considered for liquid (metal and slag), gas, solid and fines (powder). For each phase, the momentum equation reads:

\[ \nabla \cdot \vec{\sigma} = \rho \nabla \cdot \vec{\dot{u}} \]

where \( \vec{\sigma} \) represents the stress field, \( \rho \) the density, \( \vec{\dot{u}} \) the velocity field, and \( \vec{F} \) the interaction forces with the other phases (like Ergun’s solid–gas drag force). The stress field is decomposed into a hydrostatic part and a deviatoric part which is related to the shear strain rate, \( \dot{\varepsilon} \), equal to the deviatoric part of the velocity gradient tensor:

\[ \vec{\sigma} = -p \vec{I} + 2 \mu \dot{\varepsilon} \]

Here, \( \mu \) is the viscosity coefficient. After some calculations, the final equation is a general transport equation, with a convective part and a diffusive part on the left hand side, and a source term on the right hand side (which includes all terms that originate the motion, like gravity or pressure gradients):

\[ \nabla \cdot \left( \rho \mu \frac{\partial \vec{u}}{\partial t} - \mu \nabla \cdot \vec{u} \right) = S \]

Here is one of the components of the velocity vector field. The advantages of this method is that the same equation is used to describe the motion of all four phases, hence simplifying the calculations (use of the finite volume technique and SIMPLE algorithm). Note that a similar approach is under development in Australia, and takes into account gas–solid interactions. By adjusting the so called source term \( S \), many kinds of interactions can be considered. For instance, special slip boundary conditions were also assumed for the wall and the dead man boundary. A small scale physical model was used for model validation, where numerical and experimental time lines were compared. The fluid melt approach gave better agreement with experiments than the kinematic or the potential flow method. Indeed, the solid viscous coefficient, \( \mu \), was empirically determined according to experimental simulations.

Assuming the solid phase motion to behave as a fluid has a major drawback though: the dead man shape cannot be obtained, therefore its shape is prescribed before calculation, giving birth to complex mesh structure. In the case of, the cell in the calculation domain that belong to the supposed dead man are removed from the calculation. More
generally speaking, the use of a viscous flow theory for describing granular material behavior is questionable.

2.3. Perfect Plasticity

In this case, the stress field is obtained by assuming an additional relationship between stress components, usually the Mohr Coulomb criterion:

\[ \left(\sigma_{11} - \sigma_{22}\right)^2 + \left(\sigma_{12}\right)^2 = \left(\frac{\sigma_{11} + \sigma_{22}}{2}\right)^2 \sin \phi \]......(4)

This choice is justified by the fact that the three unknown stress components \(\sigma_{11}, \sigma_{12}\) and \(\sigma_{22}\) need three equations to be determined.

The material parameter is the internal angle of friction \(\phi\). In this case, the characteristic lines method is used to derive the stress field inside the packed bed.\(^{11, 12}\) Takahashi et al.\(^{12}\) proposed an interesting and accurate description of the lower part of the blast furnace: according to them, there is a dead zone, where no motion is noticeable, bounded by a stress characteristic curve; a slow moving zone, where the particle motion is erratic, bounded by a velocity characteristic curve; and above them a normal moving zone with distinguishable stream lines. The velocity line is also a discontinuity line since flow direction and velocity magnitude both change through this line. This line was used in the fluid flow method\(^{12}\) to determine the prescribed dead man boundary, but its shape is not related to the overall blast furnace geometry.

The derivation of characteristic lines assumed by Takahashi et al.\(^{9, 12}\) is controversial: the authors assume constant mean stress and linear variation of shear stress with distance from axis. Besides, the characteristic line method is valid only for two dimensional cases. Fur-thermore, assuming perfect plasticity at every point of the calculation domain is not reasonable and does not allow switches from active state to passive state, nor the existence of regions where perfect plasticity is not yet reached. Incidentally, this mechanical method could not be used to derive the solid velocity field inside the blast furnace.

2.4. Incremental Plasticity

Incremental plasticity overcomes these limitations, by making use of the finite element method (FEM) to solve the weak form of the momentum Eq. (1):

\[ \int_{\Omega} \mathbf{d} \mathbf{e} : \mathbf{H} \mathbf{d} \mathbf{v} = \int_{\Omega} \mathbf{d} \mathbf{u} : \mathbf{f} \mathbf{d} \mathbf{v} \]......(5)

Here, \(\mathbf{d} \mathbf{e}\) and \(\mathbf{d} \mathbf{u}\) are virtual strain and virtual displacement quantities, respectively. Surfacic forces on the boundary of the arbitrary control volume \(\Omega\), wherever displacement is not prescribed, were omitted in Eq. (5). External volumetric forces, like gravity, and drag forces due to the interaction with other phases, were grouped together in \(\mathbf{f}\). The physical meaning of Eq. (5) is that the virtual power of external forces, which is the scalar product of a virtual velocity field by the external forces field, is equal to the virtual power of internal forces, which is the double scalar product of a virtual strain field, deduced from the previous virtual velocity field, by the internal forces field represented by the stress tensor field.

The Eq. (5) is incrementally solved by assuming that stress increments are function of strain increments:

\[ \Delta \mathbf{\sigma} = \mathbf{H} : \Delta \mathbf{\varepsilon} \]......(6)

The fourth order operator \(\mathbf{H}\) is called the Jacobian of the constitutive equation of the granular material. The Eq. (5) is, therefore, re-written in the following manner:

\[ \int_{\Omega} \mathbf{d} \mathbf{e} : \mathbf{H} \mathbf{d} \mathbf{v} = \int_{\Omega} \mathbf{d} \mathbf{u} : \mathbf{f} \mathbf{d} \mathbf{v} - \int_{\Omega} \mathbf{d} \mathbf{\varepsilon} : \mathbf{\sigma} \mathbf{d} \mathbf{v} \]......(7)

This formulation leads afterward to the discretized form:

\[ [K][\Delta \mathbf{U}] = [F_i] - [F_e] \]......(8)

where \([K]\) is the overall rigidity matrix, deduced from the Jacobian \(\mathbf{H}\), \([\Delta \mathbf{U}]\) is the displacement increment array at mesh nodes. Continuous strain and velocity fields are deduced by appropriate interpolation functions. \([F_i]\) is the external forces array at mesh nodes deduced from the external forces field \(\mathbf{f}\), and \([F_e]\) is the internal forces array at mesh nodes deduced from the stress field \(\mathbf{\sigma}\).

Efficient numerical methods, like Newton Raphson, are used to solve Eq. (8) iteratively. Convergence is judged likely to occur when the stress field equilibrates the volumetric forces, that is when the second hand residual becomes smaller than some fixed value, or when the displacement increment found after inverting Eq. (8) is small enough. In either case, the sum of all displacement increments constitutes the total displacement associated to the stress field solution of the momentum equation. This approach is qualified as Lagrangian since the total nodal displacement implies the permanent deformation of the mesh: the mesh points always stay connected to material points and deform with the solid body. Several remeshings are in fact required in order to reach a steady stress field and velocity field in an Eulerian frame.

This method is very general, and is used for all plasticity problems. What makes the difference between one case and another is the constitutive equation coded in the components of the fourth order Jacobian tensor \(\mathbf{H}\).\(^{13}\)

This method was used to simulate three dimensional small scale blast furnaces,\(^{14}\) using a very common constitutive equation for granular materials, called Drucker Prager.\(^{15}\) The resulting stress field on the side wall and furnace bottom was compared with experimental measurements. Active state and passive state regions in the blast furnace\(^{16, 17}\) as well as yielding regions were also redefined according to numerical simulations. The dead man is defined as an iso-velocity curve, and was compared with experimental observations.\(^{18}\)

However, the choice of the Drucker Prager constitutive law is not the best one for three dimensional calculations. In fact, this constitutive law is only a poor transposition of metal elasto-plasticity into granular elasto-plasticity, generated by adding pressure dependence to the Von Mises criterion.

More generally speaking, elasto-plastic constitutive equations contain an elastic component before yielding, which would imply linear behavior for very small cycle, which is a seldom observed phenomenon for granular mate-
3. Hypo-plasticity Theory for Granular Materials

This theory was developed during the last few years at the University of Karlsruhe, in Germany. It was proved that this constitutive equation makes use of a small set of constants, among them the internal angle of friction, often written \( \phi \).

3.1. General Expression of the Jacobian

The Jacobian that appears in Eq. (6) is a function of stress, and incremental strain direction in the following way:

\[
H(\bar{\sigma}, \Delta \bar{\epsilon}) = f_s \left( L(\bar{\sigma}) + N(\bar{\sigma}) \otimes \frac{\Delta \bar{\epsilon}}{|| \Delta \bar{\epsilon} ||} \right).............(9)
\]

where \( f_s \) is the stiffness coefficient. The necessary non linearity (or irreversibility) towards strain increment is expressed by the fact that for any strain increment \( \Delta \bar{\epsilon} \) one has:

\[
H(\bar{\sigma}, -\Delta \bar{\epsilon}) \neq -H(\bar{\sigma}, \Delta \bar{\epsilon})....................(10)
\]

With this approach, no switch function is needed, since loading and unloading are obtained with the same Jacobian.

In Eq. (9), \( L \) is a fourth order tensor and \( N \) is a second order tensor. Their expressions are function of stress state and state variables only, and provide all the information concerning yielding. Indeed, yielding is characterized by the fact that non-zero strain increment entails zero stiffness or zero stress increment, with no volumetric change. Therefore, \( L \) and \( N \) must fulfill the following statements at yield:

\[
\text{tr}(\Delta \bar{\epsilon}) = 0............................(11)
\]

\[
L: \Delta \bar{\epsilon} + N || \Delta \bar{\epsilon} || ^{-1} = 0...................(12)
\]

or:

\[
\text{tr}(L^{-1}: N) = 0............................(13)
\]

\[
| L^{-1}: N |...............................(14)
\]

3.2. Yielding or Failure Dependence

The simplest expressions for \( L \) and \( N \) used in the previous couple of equations are functions of the dimensionless stress ratio \( \bar{\sigma}^* \) (ratio of the stress tensor over its trace): \( ^{24} \)

\[
L(\bar{\sigma}^*) = I + \frac{1}{f_y} \bar{\sigma}^* \otimes \bar{\sigma}^*...........................(15)
\]

\[
N(\bar{\sigma}^*) = \frac{1}{f_y} (\bar{\sigma}^* + 3 \bar{s}^*)...........................(16)
\]

where \( \bar{s}^* \) is the dimensionless deviatoric stress ratio, \( I \) is the fourth order identity tensor, and \( f_y \) is a yield factor that is a function of the failure criterion and the invariants of the deviatoric stress ratio. A failure criterion is expressed in terms of stress invariants, not individual components, because it is independent of the choice of a referential.

The expression (obtained after combining the previous equations):

\[
| \bar{s}^* | = f_y(\Theta).............................(17)
\]

is simply the equation in the polar coordinates \( (| \bar{s}^* |, \Theta) \) of the failure criterion in the deviatoric plane. \( ^{25} \) Note that these polar coordinates are also stress invariants defined by:

\[
| \bar{s}^* | = \frac{1}{p} (\bar{s}^* : \bar{s}^*).....................(18)
\]

\[
\cos(\Theta) = \frac{r}{q}.................................(19)
\]

with:

\[
p = -\text{tr}(\bar{\sigma}).............................(20)
\]

\[
q = 2 \sqrt{3} \left( \frac{3}{2} \bar{s}^* : \bar{s}^* \right)...........................(21)
\]

\[
r = \left[ \frac{3}{2} (\bar{\Sigma} : \bar{\Sigma}) : \bar{s}^* \right] .........................(22)
\]

The ratio

\[
\mathcal{F} = \frac{| \bar{s}^* |}{f_y(\Theta)}...........................(23)
\]

is used to measure the plasticity index. The value ranges from 0 (hydrostatic state) to 1 (fully plastified state). Intermediate states are qualified as partially plastified states. Note that in hypo-plasticity theory, absence of full plasticity is not equivalent to elasticity: there is no elastic state in hypo-plasticity theory. Elasticity concept is related to linearity: this behavior is missing in real granular material behavior.

Here, the Matsuoka–Nakai \( ^{26} \) criterion is used:
because it provides a very good approximation of the Mohr–Coulomb criterion for tridimensional calculations.

The failure criterion (24) depends on the following constants:

\[ k = 9 + 8 \tan^2 \phi \] ..........(25)

\[ \lambda = \frac{9 - k}{3 - k} \] ..........(26)

\[ \mu = \frac{\lambda}{2(3 - \lambda^2)} \] ..........(27)

where the only independent parameter is the internal angle of friction \( \phi \). A triaxial test is needed to obtain the value of \( \phi \).

### 3.3. Stiffness or Pressure Dependence

The stiffness factor \( f_s \), in Eq. (9) is the only function of mean pressure (another stress invariant):

\[ f_s = \frac{1 + e}{e} h_s \left( \frac{3P}{h_s} \right)^{1-n} \] ..........(30)

where \( h_s \) is called granulate hardness. With the exponent \( n \), and the internal angle of friction \( \phi \), granulate hardness is another independent material parameter. An oedometric test is needed to obtain the values of both \( h_s \) and \( n \). The coefficient \( h_s \) expresses the slope of the compression curve given by Eq. (30), whereas \( n \) reflects its curvature.

The stiffness coefficient is:

\[ f_s = \frac{1 + e}{e} h_s \left( \frac{3P}{h_s} \right)^{1-n} \] ..........(31)

with:

\[ h_s = 1 + a_e \frac{a_e}{3} - \frac{a_e}{\sqrt{3}} \] ..........(32)

\[ a_e = \frac{3 - \sin \phi}{\sin \phi} \] ..........(33)

A more detailed derivation of Eqs. (24) and (31) can be found in Ref. 25) Note that in Eq. (9), \( L \) and \( N \) carry the yield or failure information through \( f_y \) (pressure independent information), while \( f_s \) carries the material compressibility (pressure dependent information).

### 4. Numerical Simulations

A small scale blast furnace cold model was used in the past to validate the fluid flow model.3) In the present paper, the same experimental apparatus is used to validate the solid flow model (see next section). The dimensions of the model are gathered in Table 1. One characteristic of this model is the absence of bosh.

The constants used in the solid flow model are summarized in Table 2. They include both material properties, through the set of constants \( \rho, \Phi, h_s, n, e_0 \) introduced in the previous section, and boundary conditions through the wall roughness \( \Phi_w \).

#### 4.1. Mesh Distortion

During the solid flow simulation, the mesh is strongly distorted (Fig. 1). Therefore, in order to reach an Eulerian steady flow, remeshing is necessary. The extracted elements are removed from calculation, and new layers are numerically added at the top of the burden. The solution is then transferred from the nodes of the deformed mesh to the
nodes of the undeformed mesh, by means of proper interpolation functions. Afterwards, the extraction is resumed.

During the remeshing procedure, the velocity field is computed. Up to nine cycles were performed, but a steady velocity field appeared since the fifth cycle. The upper limit of the remeshing counter is a compromise that takes into account computation time and storage capacity. The stability of both stress field and velocity field was already studied by the first author for other shapes of blast furnaces, and similar remarks were given.

The remeshing procedure is necessary for two reasons: first it ensures that the blast furnace is always filled up with granular material. Indeed, during discharge the top surface is lowering, which changes the height of burden, hence flow conditions. Second, calculations do not converge if the mesh is too distorted, hence the need for remeshing. A criterion used to decide remeshing is the shear of the elements located at the boundary of the extraction zone.

Even if remeshing is performed, the blast furnace solid flow model can be considered to be in steady flow conditions since perfect plasticity is finally reached in most parts of the blast furnace (see right side of Fig. 2). Once perfect plasticity occurs, stresses do not change anymore, nor velocities.

4.2. Stress Field

Figure 2 represents the steady stress field on the left and the plasticity index on the right. The plasticity index, \( \varphi \), ranges from 0 (hydrostatic state) to 1 (fully plastified state).

The stress field in the burden is characterized by an active state in the upper part, where the major pressure is vertical, and by a passive state in the lower part, where the major pressure is rather horizontal. The active state is due to the uniform flow submitted only to gravity. The expression active state means that the granular bed acts on the wall. The passive state is due to the convergent flow towards the extraction zone. Indeed, in the lower part, the flow is bounded by a vertical wall and the dead man (see its definition below). The expression passive state means that the granular bed undergoes the action of these boundaries.

The switch between active state and passive state occurs at the junction of the shaft with the belly. This region is also crossed by an arc of major pressures, that spans from the bottom plane to the belly’s wall. The contact point of this arch with the belly’s wall is characterized by a wall pressure jump. This contributes to the erosion of walls in real blast furnaces.

When the flow initiates, the passive state region appears just above the extraction zone. Then this region gradually expands as the flow continues. The point where wall pressure jump occurs and the point where stress state switch occurs both gradually move upward. It seems that this displacement is stopped when the shaft is encountered, and especially when wall slope changes.

The dead man, characterized by very low velocities, undergoes highest pressures (see left side of Fig. 2). But high pressures do not necessarily mean fully plastified state. Indeed, the plasticity index is rather low in this part (see right side of Fig. 2), as well as inside the arc of major pressures. Therefore, the plasticity criterion can be used to describe the dead man shape. But the next section will introduce another method based on velocities.

4.3. Velocity Field

Figure 3 describes the steady velocity field, by means of trajectories (left side) and iso-velocity curves (right side). Markers were placed every 4 min along trajectories. The iso-velocity values represent a fraction of the maximum velocity that occurs above the extraction zone.

The shaded region represents the dead-man boundary. Its definition is part of the calculation, and is based on the steady velocity field. The dead-man is bounded by an iso-velocity curve, in this case at more or less 5% of maximum velocity. The starting point of this curve on the central axis is the location of an inflexion point of the velocity profile.
Indeed, the burden’s velocity on the central axis is a monotonously decreasing function of height. But there is a point where this decrease is maximum. This point was then attributed to the start of the dead-man profile.

The velocity field is characterized by a plug flow in the upper part, where trajectories are equally spaced, and by a funnel flow in the lower part where trajectories converge towards the extraction zone.

The region located at belly level is characterized by uniform velocity. Above, in the shaft, the velocity is gradually decreasing with height, and the space between markers is also gradually decreasing. This is due to the increase of radius, which increases the volume to be filled by granular materials. Note that for tridimensional axisymmetric cases, the increase of volume in the shaft is proportional to the square of the increase of the radius.

There is a strong imbalance of velocities in the lower part, between central axis side where velocities can be lower than 5% of the maximum velocity, and the wall side where velocities can be higher than 95% of maximum velocity. Note that high velocities occur where pressures are low (raceway and top of blast furnace), and conversely low velocities occur where pressures are high (inside the dead man).

According to Takahashi et al., the dead man is a region where particles do not enter, and hence was called the non moving zone. The region above is called the slow moving zone, and was characterized by erratic trajectories. In our numerical simulations this region spans from iso-velocity 5% to iso-velocity 15% of maximum velocity. Above, the region is called the fast moving zone.

These definitions agree with our numerical results, except for the dead zone. In fact, if one waits enough, the so called dead-zone is penetrated by trajectories, but at a much slower speed than the surroundings. The calculated residence time inside this dead zone is approximately equal to four hours. This long period, which is definitely lower than infinity, confuses experimentators who try to define the dead-man’s exact shape. The boundary of the dead man is rather a layer than a curve.

5. Experimental Validation

5.1. General Remarks

5.1.1. Definition of Time Lines

The previous numerical simulations are validated by comparing experimental and numerical time lines. Experimental time lines are obtained by using colored markers distributed at fixed time intervals at the top of the physical model. As soon as the first colored marker appears in the lower part, the flow is stopped. Then each layer is extracted from the physical model, and the positions of the markers are indexed.

Numerical time lines are obtained by following equal residence time along solid trajectories. These time lines correspond to well established steady flow, not to transient flow. The time line denoted 0° in subsequent figures (see Figs. 4 to 6) represents an arbitrary starting time for trajectories, not the starting time of the experiment. Note that the experiment was conducted in transient conditions. In other words, the position of the markers do not correspond to the ones of a well established steady flow.

5.1.2. Parameter Evaluation

In the previous validation, the solid viscosity, \( \mu \), was adjusted to match numerical time lines with experimental ones. This value was obtained by trial and error. This is an empirical procedure that validates the solid flow model only for the type of material considered (sand), and the type of geometry (small scale models with artificially prescribed dead men).

No parametrical study is performed in the present case, because there was no need to adjust any empirical constant, like viscosity, to experiments. The material constants \( \phi \), \( h \), and \( n \) are determined independently by very common soil mechanics tests, like the triaxial test and the oedometric test.

Such tests can be performed for all types of granular materials, from sand to gravel, from ore to coke. Beside, these constants are independent of the geometry of the vessel that will contain them. The constants used in the hypo-plastic theory do not represent numerical parameters, but real physical constants of the granular material considered.

5.1.3. Description of Operating Conditions

Three different conditions were considered (see Table 3). The case denoted I (see Fig. 4), is characterized by a solid top velocity of 1.1 cm·s\(^{-1}\). The case denoted II (see Fig. 5), is characterized by a solid top velocity of 2.2 cm·s\(^{-1}\). In case III (see Fig. 6), a counter current gas flow was added, characterized top velocity equal to 0.7 cm·s\(^{-1}\). The figures show a comparison between experimental and numerical

<table>
<thead>
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<th>Table 3. Cases characteristics.</th>
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<tbody>
<tr>
<td>Case I</td>
</tr>
<tr>
<td>Solid flow rate (10(^{-5}) x kg·s(^{-1}))</td>
</tr>
<tr>
<td>Solid inlet velocity (cm·min(^{-1}))</td>
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<tr>
<td>Gas flow rate (10(^{-9}) x kg·s(^{-1}))</td>
</tr>
<tr>
<td>Gas outlet velocity (m·s(^{-1}))</td>
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time lines. Elapsed time is measured in minutes.

The comparison between case I and case II emphasizes the influence of the solid descending velocity, whereas the comparison between case II and case III emphasizes the influence of an additional gas flow.

5.2. Influence of Solid Descending Velocity

The time lines are flat in the upper part of the experimental model (corresponding to the shaft of the blast furnace), where plug flow occurs, and much more curved in the lower part (corresponding to the bosh), where funnel flow occurs. The shape of time lines in both regions is not influenced by the solid descending velocity.

The last time line before the first marker was extracted represents what was believed to be the boundary of the dead man. The height on the central axis of this supposed dead man is influenced by the speed of solid flow: a higher descending velocity implies a higher position of the top of the supposed dead man on the central axis. Note that if the speed is multiplied by two, the indicated residence time is divided by two.

The scatter between the experimental and numerical time line in case II at central axis (see Fig. 5) is due to the fact that experimental steady flow was not yet reached when the first marker was extracted. However, the experimental and numerical time line in case I (see Fig. 4) match perfectly for slower descending velocity, because the experimental steady flow was reached. Therefore, reaching steady flow conditions during experimental observations should be considered in validation procedures.

In both cases I and II there is a quite good agreement between calculated and observed time lines, especially in the plug flow region. There is, however, a little scatter above the extraction zone, near the wall. This is due to a limitation in the present finite element model to simulate very rough boundary conditions in a region where stress levels are low and velocities are high.

Note that in Ref. 5) a different Reynolds number was used and adjusted as a boundary condition within the fluid flow model to match this scatter. There is no theoretical justification behind this procedure. Unfortunately, there is also no connection between Re_w, the Reynolds number at the wall used in calculations, and \( \phi_w \), the measured friction angle at the wall.

5.3. Influence of a Counter Current Gas Flow

The influence of gas flow was considered by adding the Ergun gas–solid drag force to the external loads (gravity) that is acting on the burden. The drag force:

\[
F_g^s = \left[ 1.75 \rho_g \left( V_g^s + 150 \mu_g \frac{e_g}{(1-e_g)d_s \phi_s} \right) \right] \\
\times \left( \frac{e_s}{(1-e_s)d_s \phi_s} \right)^2 
\]

is a function of gas velocity \( V_g \), gas density \( \rho_g \), gas viscosity \( \mu_g \), and also solid volume fraction \( e_s \), grain diameter \( d_s \), particle shape \( \phi_s \).

The solid volume fraction, \( e_s \), is a function of solid mean pressure \( P \):

\[
e_s = \frac{1}{1 + \left[ \frac{1}{e_s^0} - 1 \right] f(P)} 
\]

where:

\[
f(P) = \exp \left[ -\left( \frac{-\text{tr} (\tilde{f})}{h_s} \right)^\alpha \right]
\]

is the pressure coefficient, which depends on the stress.
field, and $e^s$ is the solid volume fraction at rest. Note that in the previous validation, the solid volume fraction was kept constant and equal to its initial value.

In this way, gas flow and solid flow are fully coupled: on one hand, the solid flow is determined by the stress field which equilibrates the external forces, including the gas drag force. On the other hand, the gas drag force depends on the solid volume fraction, which is affected by the stress field.

By comparing Figs. 5 and 6, it can be seen that in the upper part of the model scale, the gas flow has little influence on the solid flow, since the time lines are located approximately at the same height.

On the other hand, in the lower part near the central axis, the time lines become narrower in the case of an existing gas flow. It can also be seen that the starting point on the central axis of the 23’ time line is located at a higher position in case III than in case II. Therefore, gas flow contributes to the increase in height of the boundary of the slow moving inside the blast furnace. This assertion also proves that solid flow and gas flow mechanical interaction should not be miscalculated or simply neglected in blast furnace modeling.

5.4. Validation of Dead Man Shape

In the previous validation, an artificial dead man shape was prescribed prior to calculations. This artifact was justified by a weakness in fluid flow theory, which is unable to take into account stagnant zones. If this prescribed dead man shape was missing, then the flow would occur in all parts of the model, even at the bottom and central part.

The dead man shape was theoretically bounded by a characteristic curve. Prescribing dead man shapes in this way for 3D experiments is problematic, since the only validations of such shapes were conducted with 2D experiments, which did not even take into account the strong influence of front and rear observation glass sheets.

Unlike in the previous studies, there is no need to impose a shape for the dead man prior to calculations in the current study. The actual solid flow model can take into account stagnant regions, and there is no need to suppress from the calculation domain those regions where stagnant flow may occur. The calculation domain spans from the bottom up to the top of the model. One consequence is that the mesh was considerably simplified. The shape of the dead man came out naturally as a result of the calculations.

The dead man influence on the flow is evident in the figures. Unfortunately, the experiments conducted by Ref. 5) do not show the exact shape of the dead man, but rather the position of the last time line before one marker came out through the extraction slot. In the current study, it was possible to reproduce all time lines, including the last observed one. What was defined as the dead man in Ref. 5) was in fact the boundary of a slow moving zone according to the definition of Refs. 9), 12) Obviously, the dead man shape does not coincide with this time line.

The numerical simulation described in the previous section gave another definition of the dead man, as the iso-velocity curve where the velocity profile is decreasing the most. Beneath this curve, residence time can reach 4 hr.

Other researchers have also described the dead man shape as an iso-velocity curve, but the value of the bounding velocity was arbitrarily chosen, whereas it is precisely defined in the current study. Beside, according to the elasto-plastic constitutive equation used, since the dead man is merely flowing, it would be in elastic state. The hypo-plastic constitutive equation predicts a partially, although not perfectly, plastified state. Therefore, it is assumed that the current study allows an improved prediction of the dead man state than other studies.

6. Conclusion

A solid flow model derived from soil mechanics and granular materials theories was studied to describe solid motion in the blast furnace burden. The superiority of this model over other solid flow models stands in the following points:

• it is based on a more recent and yet very efficient constitutive equation, known as hypo-plastic,
• the numerical scheme used to solve the momentum equations, based on the finite element method, introduces remeshing which allows to reach steady flow after few iterations,
• it has proved to be more suitable for simulating both the stress field and the velocity field, and in particular the stagnant zone without using any arbitrary parameters,
• the parameters needed for the calculation are a small set of constants, calibrated using common soil mechanics devices, and are independent of the vessel geometry.

The steady flow obtained was clearly characterized by plug flow in the upper part and funnel flow in the lower part. A reasonable dead-man shape was also observed from the velocity field solution, and was defined by an iso-velocity curve. Finally, good agreement between numerical and experimental time lines was obtained. Scatters are located in regions too small to be significant.

These results suggest a new possibility that the present total mathematical model of a blast furnace can be improved by incorporating this solid flow model into it.

Nomenclature

- $\rho$: density (kg · m$^{-3}$)
- $\mu$: viscosity (Pa · s)
- $\phi$: granular internal angle of friction ($^\circ$)
- $e$: granular void ratio (—)
- $e^s$: porosity (—)
- $P$: solid pressure (Pa)
- $\sigma^p$: plasticity index (—)
- $V$: velocity (m · s$^{-1}$)
- $g$: gravity (m · s$^{-2}$)
- $\sigma$, $s$: stress tensor and its deviatoric part (Pa)
- $\varepsilon$, $\varepsilon^s$: strain tensor and its deviatoric part (—)
- $H$: constitutive equation’s Jacobian (Pa)

REFERENCES
