A Mathematical Model for Prediction of Thickness of Mould Flux Film in Continuous Casting Mould

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A mathematical model of mould flux infiltration and heat transfer through the flux film has been developed. The model considers the effect of static pressure of molten steel, temperature dependency of flux viscosity and determination of liquid flux thickness with pressure gradient. Present model gives the results that agree well with actual plant data such as mould flux consumption, mould friction and the heat transfer. The results of the model give remarkable effect of static pressure of molten steel as follows: decrease of the static pressure causes increase of liquid and total flux film thickness, reduction of heat flux and decrease of friction force on solidified shell.

KEY WORDS: continuous casting; mould flux; heat transfer; solidification; lubrication; model; viscosity; static pressure; infiltration; fluid dynamics; shear stress.

1. Introduction

The lubrication and the heat transfer in the mould are very important factors to maintain stable operation of casting and slab quality.1–4 It is necessary to know conditions of mould flux infiltration between the mould and solidifying shell during continuous casting of steel in order to determine lubrication and heat transfer conditions. Several numerical calculations of the flux infiltration have been reported since 1980's.5–7 These reports are successful to describe the conditions of near meniscus region. However, thickness of liquid flux film, the most important factor for lubrication and heat transfer, was given as a linear function of the distance from meniscus without examination in the models of all the studies. So the models are applied only for the region near meniscus. Because the behaviour of the mould flux film near meniscus may not be independent from the behaviour of the flux film in lower part of the mould, it is necessary to describe the whole part of mould in order to determine the lubrication and heat transfer conditions. Moreover, the effect of static pressure of molten steel has been neglected in the models of flux infiltration. Extreme care should be exercised in the treatment of the static pressure of molten steel because it depends upon the shell strength. Since the static pressure influences the infiltration of the mould flux at broad face in the slab casting mould except near corner, where the solidifying shell is strong. However, the steel pressure should be considered in the slab casting.

The way of approaching lubrication study can be categorized into two types. One is to describe infiltration of mould flux, which is already mentioned, and the other is to measure load of actual casting mould.8–10 However, besides an empirical method11 no one reported theoretical prediction of the mould flux thickness distribution. The studies of heat transfer through mould flux film have progressed especially during the past 20 years though they are all in static conditions.12–19

In the present study, a trial of combining heat transfer model and infiltration model has been performed to predict mould flux thickness. The empirical factors have been avoided as much as possible in the model. Flux consumption rate and friction in the mould are also discussed from the view point of the result of the numerical calculations.

2. Mathematical Model

In order to determine distribution of mould flux thickness, present integration model consists of two models. The first represents the flux infiltration between the mould and the strand. The second heat transfer model involves solidification sub-model by Matsuno et al.20 and an original sub-model for heat transfer through flux film. Surface temperature of the strand is calculated with the result of infiltration calculation. The surface temperature is used to determine the temperature of the mould flux film and liquid flux viscosity. The schematic illustration for the infiltration model is shown in Fig. 1. Both the heat transfer and the infiltration models are linked and the results are exchanged as shown in Fig. 2.

2.1. Model of Flux Infiltration

As shown in Fig. 1, the level of meniscus on the mould wall is set as the origin of the coordination. The x-axis of the system is in the direction along the length of the strand. The y-axis is drawn in parallel to the level of the liquid steel. The lubrication with the mould flux film in the area between the mould and the strand is represented in the form of a hydrodynamic lubrication with a viscous fluid between two non-parallel surfaces. The temperature distribution in
the liquid flux film is not taken into consideration in the model. Also, the shell strength is assumed to be very small and the static pressure of molten steel is transferred immediately to the flux film. The infiltration of the mould flux occurs mainly in the direction of the strand, and for this, Navier–Stokes equation for continuity becomes

\[ \frac{\partial^2 u}{\partial y^2} = \frac{\partial P}{\partial x} - (\rho_t - \rho_m)g \]  \hspace{1cm} (1)

\[ u = V_c \quad \text{at} \quad y = d_1, \quad \text{equation (2)} \]

\[ u = V_m = 2 \pi f A \cos(2 \pi f t) \quad \text{at} \quad y = 0, \quad \text{equation (3)} \]

Because the solidified shell except in corner is not strong enough to support the static pressure of molten steel inside the shell, the influence of the static pressure is considered explicitly in this model.

If Eq. (1) is integrated twice with boundary conditions given in Eqs. (2) and (3), one obtains

\[ u = \frac{1}{2 \eta} \left[ \frac{\partial P}{\partial x} (\rho_t - \rho_m)g \right] \left( y^2 - d_1 y \right) + \frac{1}{d_i} \left( V_c - V_m \right) y + d_i V_m \]

\[ \text{equation (4)} \]

Usually, the hydrodynamic lubrication with a viscous fluid is described with the fundamental equation by Reynolds that is valid for a two-dimensional flow rate of the fluid. The flow rate, \( Q \), is derived by the integration of Eq. (4) in the \( y \)-axis direction:

\[ Q = \int_0^{d_1} u \, dy \]

\[ \text{equation (5)} \]

That is,

\[ Q = \left( V_c - V_m \right) d_i \cdot \frac{d_1}{2} \left[ \frac{\partial P}{\partial x} (\rho_t - \rho_m)g \right] \]

\[ \text{equation (6)} \]

or alternatively,

\[ \frac{\partial P}{\partial x} = \frac{\delta P(V_c - V_m)}{d_i^2} + \frac{12 \eta Q}{d_i^3} (\rho_t - \rho_m)g \]

\[ \text{equation (7)} \]

The thickness of the liquid flux film has been examined in two regions, 0 ≤ \( x \) ≤ \( L_1 \) and \( L_1 < x \leq L_2 \), that are shown in Fig. 1.

1. Region 0 ≤ \( x \) ≤ \( L_1 \)
   According to Jimbo et al., \( ^{22} \) static meniscus shape in the mould is expressed by

\[ d_l = -\sqrt{2a^2 - x^2} + \sqrt{2a^2 - x^2} \]

\[ \left[ 1 - \frac{1}{\sqrt{2}} \log \left( \sqrt{2} + 1 \right) a \right] \]

\[ \text{equation (8)} \]

where

\[ a^2 = \frac{2 \sigma_{mf}}{(\rho_m - \rho_f)g} \]

\[ \text{equation (9)} \]

When typical values are given for physical properties of molten flux and steel as shown in Table 1, the meniscus shape is calculated as shown in Fig. 3. However, Eq. (8) is too complicated to solve analytically. Fortunately, the meniscus shape in this region can be approximated by Eq. (10) when appropriate parameters are given.

\[ d_l = d_l e^{\log(d_l)/L_1} \]

\[ d_l e^{\log(d_l)/L_1} = d_l e^{\log(d_l)/L_1} \]

\[ \text{equation (10)} \]

where

\[ m = \frac{1}{L_1} \log(d_i/d_l) \]

\[ \text{equation (11)} \]

Here, an example of the case, \( L_1 = 0.007 \) m, \( d_i = 0.015 \) m and

\[ \begin{array}{|c|c|}
\hline
\text{Steel} & \rho_v & 7200 \text{ kgm}^{-3} \\
\hline
\text{Thermal conductivity} & \kappa & 45.8 \text{ Wm}^{-1} \text{K}^{-1} \\
\hline
\text{Mould flux} & \rho_f & 2500 \text{ kgm}^{-3} \\
\hline
\text{Interfacial tension of molten steel and flux} & \sigma_{sf} & 1.00 \text{ Nm}^{-1} \\
\text{Viscosity \(^{(*)}\)} & \eta_{flux} & 1763 \text{ Pa.s} \\
\text{Initial Temperature} & \theta_0 & \text{K} \\
\hline
\text{Solidus temperature} & \lambda_1 & 0.60 \text{ Wm}^{-1} \text{K}^{-1} \\
\text{Thermal conductivity of liquid phase} & \lambda_2 & 1.14 \text{ Wm}^{-1} \text{K}^{-1} \\
\text{Thermal conductivity of crystalline phase} & \lambda_3 & \frac{1}{2} \left( 2.10 - \frac{2}{3} \right) \\
\text{Emissivity} & \alpha & 2.0 \quad - \\
\text{Refraction coefficient} & \gamma & 700 \text{ m}^{-1} \\
\hline
\end{array} 
\]

\( (*) \) Measured by Sakai Chemical Co.

\[ \begin{array}{|c|c|c|}
\hline
\text{Dimensions and casting condition} & d_1 & 0.015 \text{ m} \\
\hline
& L_1 & 0.007 \text{ m} \\
& L_2 & 0.800 \text{ m} \\
& L_3 & 0.020 \text{ m} \\
\hline
\text{Casting speed} & V_c & \text{m min}^{-1} \\
& 1.0, 1.5, 2.0, 3.0, 5.0 & \text{m min}^{-1} \\
& 0.0167, 0.0200, 0.0333, 0.0500, 0.0833 & \text{m min}^{-1} \\
\hline
\text{Oscillation Stroke} & S & 0.008 \text{ m} \\
\hline
\text{Negative strip time ratio} & f & \text{Hz} \\
& 1.05Vc/25 & \text{Hz} \\
\hline
\text{Thickness of strand (mould)} & D & 0.260 \text{ m} \\
\hline
\text{Thermal resistance of mould} & R_{\text{mould}} & 0.076 \text{ m}^2 \text{K} / \text{W} \\
\hline
\text{Heat transfer coefficient of water cooling} & h_w & 28.0 \text{ kWm}^{-1} \text{K}^{-1} \\
\hline
\end{array} 
\]

Table 1. Physical properties and casting conditions for calculations.\(^5,13,24,25,27\)
\[ d_l = 0.0002 \text{ m}, \] is shown in Fig. 3 with the results of theoretical calculation. Approximation by Eq. (10) agrees well with the theoretical one in, and hence Eq. (10) is used in the following calculations.

Additional boundary conditions are:
\[ Q_{x=0} = \frac{3(V_c - V_m)\eta}{md_1^2} e^{-2mL_1} + \frac{4Q_{x=0}}{md_1^2} (\rho_l - \rho_m)gx + C \]
\[ \frac{\partial P}{\partial x} \Big|_{x=0} \]

When \( x = L_1 \), pressure, \( P \), is derived from integration of Eq. (7) as
\[ P = \frac{3(V_c - V_m)\eta}{md_1^2} e^{-2mL_1} + \frac{4Q_{x=0}}{md_1^2} (\rho_l - \rho_m)gx + C \]
\[ \frac{\partial P}{\partial x} \]

where \( C \) is integration constant and is derived by Eq. (14) and the boundary conditions (13) as
\[ C = \rho_l gL_1 - \frac{3(V_c - V_m)\eta}{md_1^2} + \frac{4Q_{x=0}}{md_1^2} (\rho_l - \rho_m)gL_1 + \rho_l gL_2 \ldots \]

At the same position, another expression for the flux flow rate \( Q \) is derived from Eq. (7) as
\[ Q_{x=L_1} = \frac{(V_c - V_m)}{2} d_2 - \frac{1}{12\eta} \left[ \frac{P_{x=L_1} - P_{x=0}}{L_1} + (\rho_l - \rho_m)gL_1 \right] \]
\[ \frac{\partial P}{\partial x} \]

Substituting Eqs. (13) and (16) into Eq. (17), the flux flow rate is expressed as
\[ Q = \frac{3(V_c - V_m)d_l}{4} \beta \left\{ \frac{2 \log \beta + (1 - \beta^2)}{3 \log \beta + (1 - \beta^2)} \right\} \]

where \( \beta = d_l/d_1 \).

An additional assumption here is that the film thickness does not change during oscillation cycle, i.e., the mould velocity does not affect the average thickness of liquid flux film in this model. For the calculation of average flux thickness, the value of \( V_m \) is assumed zero. Of course, the evaluation of friction in the mould should take into account the influence of \( V_m \) that will be discussed in the following section.

Flux flow rate for one oscillation cycle, \( Q_{osc} \), is thus derived by integrating of Eq. (18) as
\[ Q_{osc} = \frac{3V_c d_l \beta}{4f} \left\{ \frac{2 \log \beta + (1 - \beta^2)}{3 \log \beta + (1 - \beta^2)} \right\} \]

The mould oscillation term disappears in Eq. (19), since this model assumes steady state flow of liquid flux and no change of the meniscus shape by the mould oscillation. Namely, the effect of the meniscus shape change during oscillation is not considered in this model for simplicity.

2 Region 2 \((L_1 < x < L_2)\)

The purpose of this study is to define the thickness of mould flux film. From Eq. (7), a cubic equation for the liquid flux thickness, \( d_l \), is derived as
\[ \frac{\partial P}{\partial x} = (\rho_l - \rho_m)g \]
\[ \frac{\partial Q}{\partial x} = 12\eta Q \]
\[ \frac{\partial Q}{\partial x} = \frac{12\eta Q}{2} \]

Substituting Eqs. (21) and (22) into Eq. (20), Cardano’s solution is derived as
\[ \varphi = \frac{2\eta(V_c - V_m)}{\frac{\partial P}{\partial x} - (\rho_l - \rho_m)g} \]
\[ \frac{\partial P}{\partial x} = 12\eta Q \]
\[ \frac{\partial Q}{\partial x} = \frac{12\eta Q}{2} \]

Solutions of Eq. (23) are:
\[ d_l = \kappa + \lambda, \quad \kappa \omega^2 + \lambda \omega, \quad \text{or} \quad \kappa \omega + \lambda \omega^2 \]

where
\[ \kappa = \frac{-\xi + \sqrt{\xi^2 + 4\phi^3}}{2} \]
\[ \lambda = \frac{-\xi - \sqrt{\xi^2 + 4\phi^3}}{2} \]
\[ \omega = -1 + \frac{3i}{2} \]

However, the thickness of the liquid flux film should take real value. Therefore, the solution is only one, \( d_l = \kappa + \lambda \), when \( \xi^2 + 4\phi^3 < 0 \).

\[ d_l = \frac{1}{2} \left\{ -\xi + \sqrt{\xi^2 + 4\phi^3} \right\} + \frac{1}{2} \left\{ -\xi - \sqrt{\xi^2 + 4\phi^3} \right\} \]

Since the flow rate of mould flux, \( Q \), is not a function of position in the mould at steady state, \( d_l \) can be obtained by Eqs. (21), (22) and (28) when \( \partial P/\partial x \) and \( \eta \) are known.

The viscosity, \( \eta \) has been measured and expressed as a function of temperature, Eq. (29), in order to define the viscosity of liquid flux in the mould along withdrawal direction:
\[ \eta = \eta_0 \exp \left\{ -\frac{B}{T_{\text{ln}}} \right\} \]

For the calculation of \( d_l \), Eq. (7) is digitized as
Therefore, the thickness of the liquid mould is expressed as
\[ \beta \equiv \frac{d_2}{d_1} \] ..........................(33)

The temperature, \( T_0 \), for the viscosity is calculated by heat transfer model.

(3) Determination of \( d_2 \)

The only parameter involved in this model is the thickness, \( d_2 \), of liquid flux film at the point where meniscus touches with mould wall. This can vary with casting speed, mould velocity and mould flux viscosity as shown in Eq. (16) that replaces Eq. (7).

\[
\left( \frac{\Delta P}{\Delta x} \right)_{x=d_1} = \frac{2 \eta \left( V_e - V_m \right)}{\beta^2 d_2^3} 
\]

\[
\eta = \frac{12 \eta Q}{(\rho_l - \rho_m)g} \] ..........................(35)

\[
\xi = \frac{12 \eta Q}{(\rho_l - \rho_m)g} \] ..........................(36)

The temperature, \( T_0 \), for the viscosity is calculated by heat transfer model.

The viscosity, \( \eta \), is approximated by such a function as
\[ \eta = \eta_0 \exp \left( -\frac{B}{\theta_{th}} \right) \] ..........................(31)

Initial conditions are
\[ \left( \frac{\Delta P}{\Delta x} \right)_{x=0} = P_{x=0} - P_{x=0} \] ..........................(32)

and
\[ d_0 = d_1 = \beta d_1 \] ..........................(33)

Therefore, the thickness of the liquid flux film is expressed as
\[ d_2 = \beta d_1 \] ..........................(34)

where
\[
\varphi = \frac{2 \eta \left( V_e - V_m \right)}{\beta^2 d_2^3} 
\]

\[
\xi = \frac{12 \eta Q}{(\rho_l - \rho_m)g} \] ..........................(35)

\[
\beta = \frac{\left( \frac{\partial P}{\partial x} \right)_{x=d_1} - (\rho_l - \rho_m)g d_2^2}{\frac{1}{\beta^2} \left( \frac{1}{3 \log \beta + (1 - \beta^3)} \right) + (\rho_l - \rho_m)g} \] ..........................(36)

The accuracy of the approximation, Eq. (39), is shown in Fig. 5. The approximation agrees within a few percent difference with the \( \beta \) function, Eq. (38), in the range of \( \beta \) between 0.003 and 0.02. Equation (39) gives us an idea that \( \beta \) is proportional to \( V_e \) when \( V_m \) is zero and \( \eta \) is constant. Therefore, \( d_2 \) can be expressed as:
\[ d_2 = \sqrt[4]{A V_e^{-0.6280}} \] ..........................(40)

where \( d_2^* \) is the thickness of liquid flux film at reference casting speed (1 m/min in present study). This assumption involves some difficulty because the pressure gradient is not a constant value but a function of \( \beta \). Hiraki \(^{11}\) reported that the thickness of mold flux film is proportional to \( V_e^{-0.6280} \). This agrees well with the present approximation.
However, theoretical meaning of this relationship requires future clarification.

The value of $d^*_f$ is estimated to be 0.25 mm according to a heat transfer model that is described in Sec. 2.2.2. An example of slag rim with flux film is taken in the present investigation at the meniscus of a plant mould. The sample gives 0.15 mm for solid flux film as shown in Fig. 6. The average casting speed was 1.5 m/min before taking the sample. Simultaneous heat flux measurements give an estimation that the thermal resistance of liquid flux film and film thickness. The temperature gradient is derived from solidus temperature, i.e. 1373 K, and surface temperature of the shell, which is approximately 1873 K. Assuming that effective thermal conductivity of the liquid is 1.1 W/mK and heat flux at the meniscus region is 2000 W/m², the liquid flux thickness and the value of $d_f$ is 0.2 mm and 0.252 mm accordingly.

2.2. Model for Heat Transfer and Solidification of Steel

2.2.1. Solidification Model

Following assumptions are considered for the heat transfer and solidification of steel in the mould:

1) No effect of convection at broadface
2) Only one-dimensional heat transfer in y-axis-direction, across mould flux film
3) Constant density of molten steel
4) Steady state heat transfer

Since flow of the molten steel does not reach directly from submerged entry nozzle to the broadfaces, the flow of the molten steel does not reach directly from submerged entry nozzle to the broadfaces, the flow of the molten steel does not reach directly from submerged entry nozzle to the broadfaces. The solution model. The only difference is that the origin is not set on the surface of the mould but on the surface of the shell.

In order to proceed with the numerical calculation conveniently, Eq. (41) is rearranged with converted temperature $\phi$ defined in the following and enthalpy $H$:

$$\phi = \int_0^\theta \frac{K_m}{K_d} d\theta$$

(45)

From which one gets

$$\frac{d\phi}{dy} = \frac{K_m}{K_d} \frac{d\theta}{dy}$$

(46)

$$\frac{d^2\phi}{dy^2} = \frac{1}{K_d} \left( \frac{d\theta}{dy} \right)^2$$

(47)

and hence

$$\frac{d\phi}{d\theta} = \frac{K_m}{K_d} \frac{d\theta}{d\theta}$$

(48)

$$\frac{dH}{d\theta} = \frac{dH}{d\phi} \frac{d\phi}{d\theta} = \frac{K_m}{K_d} \frac{dH}{d\phi}$$

(49)

From the definition of $H$, one obtains

$$\frac{dH}{dx} = \frac{K_d}{\rho_m c_\text{m} V} \left( \frac{d^2\phi}{dy^2} \right)$$

(50)

Actual values for converted temperature $\phi$ and enthalpy $H$ are given in Matuno’s study.20)

When $i$ and $j$ are the integers of indicating coordination for $x$-axis and $y$-axis, differentials of $H$ and $\phi$ are expressed as:

$$\frac{dH}{dx} = \frac{1}{\Delta x} \left( H_{i,j+1} - H_{i,j} \right)$$

(51)

$$\frac{d^2\phi}{dy^2} = \frac{1}{(\Delta y)^2} \left( \phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j} \right)$$

(52)

Equation (50) is digitized as

$$H_{i,j+1} = H_{i,j} + \frac{K_d}{\rho_m c_\text{m} V} \frac{\Delta x}{(\Delta y)^2} \left( \phi_{i+1,j} - 2\phi_{i,j} + \phi_{i-1,j} \right)$$

(53)

The boundary conditions for Eqs. (42), (43) and (44) are also modified as

$$y = 0 (i=k); \quad q_y = \frac{K_d}{2\Delta y} \left( \phi_{i+1,j} - \phi_{i-1,j} \right)$$

(54)

$$y = \frac{D}{2} (i=l); \quad \phi_{i+1,j} - \phi_{i-1,j} = 0$$

(55)

When $q_y$ is given, $H$ is obtained by substituting Eqs. (54) and (55) into Eq. (53) iteratively with calculations of $\theta$ and $\phi$.

2.2.2. Model for Heat Transfer through Mould Flux Film

Steady state heat flux, $q_y$, at the surface of the shell is defined with overall thermal resistance, $R_i$, temperatures of the shell ($\theta_{sh}$) and of cooling water ($\theta_w$), as

$$q_y = \frac{1}{R_i} (\theta_{sh} - \theta_w)$$

(56)

where

$$R_i = R_{c} + R_{f} + R_{\text{int}} + R_{\text{mold}} + \frac{1}{h_w}$$

(57)

Alternatively, $q_y$ can be derived from Eq. (58)24) with the value of $\theta_{sh}$, $d_i$ and physical properties of mold flux for heat transfer as

$$q_y = \frac{\lambda_{c}}{d_i} (\theta_{sh} - \theta_{sol}) + \alpha (\theta_{sh} - \theta_{sol}^4)$$

(58)

where

$$\alpha = \frac{n^2 \sigma}{0.75d_i + e_{s}^{2} + e_{m}^{2} - 1}$$

(59)
In this case, the most important value is the solidus of mould flux (\(\theta_{\text{sol}}\)) because it is assumed that the thickness of liquid film (\(d_l\)) is approximately same as the distance between shell surface and the position of the solidus. Thus, surface temperature of the mold and average temperature of solid flux film facing to the mould are derived, respectively, from Eqs. (60) and (61) as

\[
\theta_{\text{mould}} = \theta_w + q_i \left( \frac{1}{h_w} + R_{\text{mould}} \right) \quad \text{(60)}
\]

\[
\theta_{\text{lm}} = \theta_w + q_i \left( \frac{1}{h_w} + R_{\text{mould}} + R_{\text{int}} \right) \quad \text{(61)}
\]

where

\[
h_w = 0.023 \frac{\lambda_w}{d_w} Re^{0.8} Pr^{0.33} \quad \text{(62)}
\]

\[
Re = \frac{D_w u_w d_w}{\eta_w} \quad \text{(63)}
\]

\[
Pr = \frac{c_w \eta_w}{\lambda_w} \quad \text{(64)}
\]

\[
R_{\text{int}} = \left( 9.6014 \theta_{\text{sol}} - 9604.3 \right)^2 \times 10^{-11} \quad \text{(65)}
\]

In order to define \(R_{\text{int}}\), Yamauchi et al.’s relationship between heat flux and solidus has been modified in Fig. 7. The plots of \((R_{\text{int}})^{1/2}\) vs. solidus show excellent linearity in the Fig. 7.

Equation (66) represents the heat transfer across the solid flux film.

\[
q_i = \frac{\lambda_s}{d_s} (\theta_{\text{sol}} - \theta_{\text{lm}}) \quad \text{(66)}
\]

Since \(\theta_{\text{sh}}\), \(\theta_{\text{sol}}\) and \(\theta_{\text{lm}}\) are already known, the thickness of solid flux film is derived as Eq. (67) from the ratio of liquid to solid flux thickness defined by the temperatures and given \(d_l\).

\[
d_i = d_l \frac{\theta_{\text{sh}} - \theta_{\text{lm}}}{\theta_{\text{sh}} - \theta_{\text{sol}}} = d_l \frac{\theta_{\text{sol}} - \theta_{\text{lm}}}{\theta_{\text{sh}} - \theta_{\text{sol}}} \quad \text{(67)}
\]

2.3. Model for Calculation of Liquid Friction on Solidified Shell

Model for lubrication is necessary to evaluate the infiltration model mentioned above. In the continuous casting process, lubrication has been evaluated by measuring friction force on solidified shell. The friction force can be calculated theoretically with given liquid flux thickness, \(d_l\), viscosity of mould flux as a function of temperature, and velocity of the mould \((V_m)\) and the strand \((V_c)\). Figure 8 represents the model for calculation of the friction.

Fundamental equation for the velocity distribution in the liquid flux film in transient state is

\[
\frac{\partial v_x}{\partial t} = \frac{\eta}{\rho_l} \frac{\partial^2 v_x}{\partial y^2} \quad \text{...............(68)}
\]

The distribution of velocity in the liquid flux film can be calculated with boundary condition, Eqs. (69) and (70), analytically.

\[
v_0 = V_c - V_m = V_c - \pi f \cos(2\pi ft) \quad \text{(69)}
\]

\[
v_{y,0} = 0 \quad \text{...............(70)}
\]

In this case, the origin of the coordination system is assumed to move with the oscillation of the mould.

Equation (71) gives shear stress with velocity gradient in the liquid film obtained by the solution of Eq. (68):

\[
\tau_x = \eta \left( \frac{\partial v_y}{\partial y} \right) \quad \text{...............(71)}
\]

The friction force acting on the solidified shell is given by an integration of the shear stress at \(y=0\) toward casting direction, \(x\), as

\[
F_i = \int_0^{l_i} \tau_x(x)dx \quad \text{...............(72)}
\]

However, the viscosity is not constant in the liquid flux film, and hence Eq. (71) is digitized and solved numerically.

\[
\tau_i^k = \eta_i \left( \frac{v_i^k - v_{i-1}^k}{\Delta y} \right) \quad \text{...............(73)}
\]

where \(i\) and \(k\) represent the number of element for \(y\)-axis and time, and \(\eta_i\) is defined by Eq. (31). Equation (73) can be expressed as

\[
\eta_i (v_i^k - v_{i-1}^k) = \eta_i (v_{i-1}^{k-1} - v_i^{k-2}) \quad \text{...............(74)}
\]

Thus, one gets
3.1. Validity of the Model

Some of the result can be confirmed with either plant data or experimental results in order to show the validity of the present model. Figure 9 shows the relationship between casting speed and the mould flux consumption, $Q_c$ [kg/m² slab], calculated by the present model. Since $Q_{osc}$ in Eq. (19) represents the volume of mould flux flows during one oscillation cycle in a unit width, $Q_c$ can be calculated by

$$Q_c = \rho f Q_{osc} \frac{3 \rho f \beta}{4} \left[ \frac{2 \log \beta + (1 - \beta^2)}{3 \log \beta + (1 - \beta^2)} \right] \cdots (80)$$

The reason for the lack of oscillation term in Eq. (80) is that $\beta$ was assumed to be constant during one oscillation period. $\beta$ should reflect the pressure change at around meniscus caused by the oscillation. Despite the absence of such term, Fig. 9 shows that the values of measured flux consumption in commercial plants and the result of the calculation agree well. An empirical equation, Eq. (81) proposed by Hiraki et al.\textsuperscript{11} indicates that the contributions of mould oscillation such as stroke, $S$, frequency, $f$, and positive strip time, $t_p$ are small.

$$d_f = 79.1512 \cdot V_c^{0.628} \cdot (\theta_{\text{mold}} [K] - 273.15)^{-0.866} \cdot S^{0.341} \cdot f^{0.076} \cdot t_p^{0.116} \cdots (81)$$

Other results of calculation for the friction force acting on the solidified shell in the mould can be presented in the following. Figure 10 represents the relationship between the friction forces acting on solidified shell and lubrication index, $\eta \cdot V_c$. Measurements of the friction force at commercial plant by Ohmiya et al.\textsuperscript{12} and Mizukami et al.\textsuperscript{4} are also plotted in Fig. 10. The range of the calculated friction force is in the same order of magnitude with the plant measurements by Ohmiya. Ohmiya mentioned that when he measured the friction force, Al₂O₃ pick up of mould flux in the mould reaches about 10% or more. He confirmed that the effect of Al₂O₃ pick up caused increase of viscosity and melting point of the molten flux in the mould, i.e., initial viscosity of about 0.1 Pa·s increased to 0.5 to 1.0 Pa·s. On the other hand, the calculated friction force gives good agreement with data by Mizukami.\textsuperscript{4} Figures 9 and 10 show that the model gives reasonable results for lubrication, though the model has a tendency to give slightly thinner flux film thickness. The heat flux calculated with the model and given in Fig. 11 shows excellent agreement with that observed by Nakato et al.\textsuperscript{26}

Observed local heat flux\textsuperscript{1,27} just below the meniscus is also plotted against casting speed in Fig. 12 with that calculated by the present model. Heat flux observed by Yamauchi et al.\textsuperscript{27} agrees very well in the lower casting speed range. Hiraki et al.\textsuperscript{11} reported heat flux value for high speed casting of 90 to 120 mm thickness slab. The data for low casting speeds (such as less than 3 m/min, 0.05 m/s in the Fig. 12) do not agree with other data by Yamauchi et al., nor the result of calculation. The reason for the disagreement with the data of Hiraki et al. may be due to the difference in mould material or dimensions. Another reason may be that the model gives thinner thickness of liquid flux.
film at meniscus region.

3.2. Effect of Casting Speed

Figure 13 shows the effect of casting speed on the pressure, $P$, and gradient of pressure $dP/dx$ at meniscus region ($x=\L_1$). Position of the mould during one oscillation cycle is also shown in Fig. 13. When time is zero, upward mould velocity is at its maximum and $P$ and $dP/dx$ are at their minimum. Flux flow rate, $Q$, becomes minimum at this time of the oscillation cycle. In this particular case, $Q$ has negative sign as shown in the Fig. 13. This means that the mould flux flows upward. Moreover, from Fig. 13, $P$, $dP/dx$ and $Q$ increase with casting speed until $t=2/4f$ when downward velocity of the mould is maximized.

The thickness of liquid flux film, $d_\ell$, does not behave simply in this model as shown in Fig. 14. According to Eq. (6), balance of $dP/dx$ and $Q$ determines $d_\ell$ in this model. At higher casting speeds, the contribution of $Q$ exceeds that of $dP/dx$. This is the reason for the behaviour of $d_\ell$ mentioned above. On the other hand, total thickness of flux film decreases with casting speed and increases with distance from meniscus as shown in Fig. 15. Surface temperature of the shell contributes the total flux thickness, i.e., the thickness decreases with the surface temperature.

3.3. Effect of Viscosity

Assuming that the viscosity of mould flux at the meniscus region is 0.053 to 0.318 Pa·s, $dP/dx$ behaves, similarly
to Fig. 13, as shown in Fig. 16. The results of calculation for \( d_l \) are shown in Fig. 17. The liquid film thickness \( d_l \) increases with the viscosity. However, \( d_l \) becomes only about twice as large when the viscosity increases ten times. As already mentioned, the viscosity of mould flux is calculated by Eq. (31). Certainly, the viscosity of the mould flux should be measured when this model is applied to another system under discussion, since temperature dependent term, \( B \), which represents activation energy of viscosity, will be very different for various flux compositions.

### 3.4. Effect of Solidus Temperature

Solidus temperature contributes to \( d_l \) only by controlling heat transfer that determines surface temperature of the shell. Increase of the solidus temperature causes decrease of heat transfer through flux film, decrease of shell thickness, increase of the surface temperature and decrease of the flux viscosity. Therefore, \( d_l \) decreases with the solidus temperature as shown in Fig. 18. However, since the solidus temperature determines the boundary of liquid and solid flux film, total flux film increases with the solidus temperature as shown in Fig. 19.

### 3.5. Effect of Metal Density

Contribution of metal density, \( \rho_m \), is considered in the infiltration model, as already mentioned. Density, \( \rho_m \) represents not only the density itself, but also the pressure change in the mould or shell strength. Considering slab casting, shell strength is higher in the near corner and the shell does not transmit the static pressure of molten steel to flux film. It is different from the centre of the mould. We can introduce the pressure term from the solidified shell metal density in Eq. (1). However, it is convenient to add static pressure term of molten steel for understanding the effect of pressure. Therefore, the metal density should be taken as a measure of distance from the corner of the mould when examining the following figures. Figure 20 shows the effect of the metal density on \( \frac{dP}{dx} \) in meniscus region. The shape of the curve does not change when \( \rho_m \) changes. It only shifts downward with \( \rho_m \). The liquid film thickness increases remarkably with the decrease of \( \rho_m \) as shown in Fig. 21. This change will cause:

1. Increase of total flux film thickness (Fig. 22)
2. Reduction of heat flux (Fig. 23)
3. Increase of shell surface temperature
4. Decrease of friction (Fig. 24)

These are very important when discussing uniform cooling or lubrication in the mould. Because these suggest that not only the deformation of the shell but also the pressure change from the shell causes the change of local heat transfer and lubrication condition. According to Fig. 23, the distribution of pressure in slab width direction may be one of the reasons for longitudinal surface cracks to occur.

### 4. Conclusion

A mathematical model of flux infiltration and heat transfer through the flux film has been developed. The model gives results that agree well with actual plant data such as mould flux consumption, mould friction and heat transfer.
The introduction of metal density and the method of determining the liquid flux thickness seem to be successful in the model. The most important result of the calculation is the contribution of the pressure by molten steel to the local heat transfer across and lubrication by the flux film. The pressure distribution will also cause corresponding flux flow distribution in the direction of slab width. The motion of solid flux film will be also influenced by the pressure because if the static pressure is very small, solid flux film does not stay on the mould wall any more. In order to improve the model, following issues should be considered in the future:

1) Determination of $d_2$ in dynamic way
2) Prediction of shell strength and introduction of shell deformation
3) Three-dimensional infiltration
4) Introduction of solid flux film motion

**Nomenclature**

- $a$: Absorption coefficient (m$^{-1}$)
- $c$: Specific heat capacity (J kg$^{-1}$ K$^{-1}$)
- $D$: Thickness of strand (m)
- $d$: Thickness of flux film (m)
- $f$: Frequency of mould oscillation (Hz)
- $F$: Friction force (N)
- $g$: Gravity constant ($g=9.80 \text{ m s}^{-2}$)
- $h$: Heat transfer coefficient (W m$^{-2}$ K$^{-1}$)
- $H$: Enthalpy of steel (J kg$^{-1}$ K$^{-1}$)
- $K$: Thermal conductivity of steel (W m$^{-1}$ K$^{-1}$)
- $L$: Length of mould flux film along casting direction (m)
- $n$: Refraction index
- $P$: Pressure in liquid flux (Pa)
- $q$: Heat flux (W m$^{-2}$)
- $Q$: Flow rate of liquid flux (m$^3$ s$^{-1}$)
- $R$: Thermal resistance (m$^2$ K W$^{-1}$)
- $u$: Velocity of liquid flux (m s$^{-1}$)
- $V_c$, $V_m$: Velocity of strand and mould (m s$^{-1}$)
- $\eta$: Viscosity of liquid flux (Pa s)
- $\vartheta$: Temperature (K)
- $\alpha$: Radiation heat transfer coefficient (W m$^{-2}$ K$^{-1}$)
- $\varepsilon$: Emissivity
- $\phi$: Converted temperature (K)
- $\lambda$: Thermal conductivity of mould flux (W m$^{-1}$ K$^{-1}$)
- $\rho$: Density (kg m$^{-3}$)
- $\sigma$: Stefan-Boltzmann constant ($\sigma=5.669\times10^{-8}$ W m$^{-2}$ K$^{-4}$)
- $\sigma_{m}$: Interfacial tension of molten steel and flux
- $\tau$: Shear stress (Pa)

**Subscripts**

- $c$: Conduction heat transfer
- $int$: Interface between mold and solid flux
- $l$: Liquid phase of mold flux
- $mold$: Surface of mould
- $p$: Mould flux or surface of mould flux
- $r$: Radiation heat transfer
- $s$: Solid phase of mold flux
- $sh$: Steel shell
- $sol$: Solids
- $w$: Cooling water

**REFERENCES**