Physical and Mathematical Modelling of Thermal Stratification Phenomena in Steel Ladles

Yuhua PAN and Bo BJÖRKMAN

A 1/4-scale hot-water model of industrial 107-tonne steel ladles was established in the laboratory. With this physical model, thermal stratification phenomena due to natural convection in steel ladles during the holding period prior to casting were investigated. By controlling the cooling intensity of the water model to correspond to the heat loss rate of steel ladles, which is governed by dimensionless numbers Fr and βAT, temperature distributions in the water model can simulate those in the steel ladles. Consequently, the temperature profile in the hot-water bath in the model can be used to deduce the thermal stratification phenomena in liquid steel bath in the ladles. In addition, mathematical simulations on fluid flow and heat transfer both in the water model and in the prototype steel ladle were performed using a computational fluid dynamics (CFD) numerical method. The CFD model was validated against temperatures measured in the water model. Comparisons between mathematically simulated temperature profiles in the prototype steel ladle and those physically simulated by scaling-up the measured temperature profiles in the water model showed a good agreement. Therefore, it can be concluded that, as long as accurate heat loss information is known, it is feasible to use a 1/4-scale water model to non-isothermally simulate fluid flow and heat transfer in steel ladles during the holding period before casting.

KEY WORDS: physical modelling; mathematical modelling; CFD; similarity; water model; steel ladle; natural convection; thermal stratification.

1. Introduction

Due to inevitable heat losses, natural convection is a common phenomenon occurring in steel ladles during the holding period prior to casting. A typical consequence of this phenomenon is thermal stratification of the liquid steel bath. The thermal stratification phenomenon in steel ladles and its potential influence on temperature control during continuous casting are important in steelmaking. This is because the temperature of liquid steel coming from a thermally stratified melt bath held in the ladles, i.e., teeming stream temperature, will have a direct impact on the temperature of steel melt held in the tundish. Therefore, investigations concerning this impact are obviously necessary, which, in turn, presupposes a good understanding of thermal stratification phenomena inside the steel ladles.

However, research work on this aspect still needs developing. From the limited extent of the literature it can be found that major efforts that have been made are mainly by means of laboratory and industrial measurements,1–5 mathematical modelling,6–10 and physical modelling.11

The measurements of thermal stratification were made mostly on pilot-scale 6–7.5 tonne steel ladles by using instrumented refractory rods on which thermocouples were mounted at different heights1,2) or by installing thermocouples (with some penetration into the steel bath) at different levels on the ladle wall.3,4) Grip6) directly measured the melt stratification in a 105-tonne production steel ladle also by inserting a refractory rod (equipped with three thermocouples at different levels) into the steel melt. The vertical temperature differences in steel ladles were successfully detected in these pilot-scale and industrial-scale experiments.

Based on the computational fluid dynamics (CFD) theory, several mathematical models have already been developed for simulating natural convection phenomena in steel ladles. The CFD models numerically solve the turbulent Navier–Stokes type partial differential equations describing the flow and heat transfer phenomena of interest, thus enabling the researchers to obtain flow patterns and temperature distributions inside the steel ladles. Ilegbusi and Szekely6) developed a two-dimensional CFD model to simulate the melt stratification phenomenon in a steel ladle equipped with the electromagnetic stirrer. Koo et al.7) also made a two-dimensional CFD model for the same purpose, but with argon agitation. Austin et al.8) established a two-dimensional CFD model to calculate thermal stratification in steel ladles of different tonnage (125, 200 and 275 tonnes). Based on the calculated results, the authors were able to correlate the development rate of thermal stratification to the bulk-cooling rate in a simple linear relationship that is common for ladles of all sizes investigated. Chakraborty and Sahai9) established a two-dimensional CFD model for simulations of thermal stratification phenomena in steel ladles with a special focus on the effect of top slag layer. Olika et al.10) set-up a two-dimensional CFD model to calculate the extent of thermal stratification in 105-tonne steel ladles. Influences of ladle thermal status and geometrical shape on the extent of thermal stratification were investigated. All the above-mentioned CFD simulations successfully revealed the thermal stratification phenomena in steel ladles.

An alternative approach that has the potential to achieve the same goal is physical modelling. Unlike the direct measurement of thermal stratification in steel ladles, physical models, established on the basis of the similitude theory, are normally easy to set up, economic and efficient in im-
plementation. In addition, the physical modelling results can also be used for verification of the mathematical modelling results.

In steelmaking, water models have been widely used in simulating fluid flow in high temperature metallurgical vessels such as converters, electric arc furnaces, ladles, tundishes, and continuous casting moulds. So far, however, most of the established water models have been applied isothermally. In recent decades, a few non-isothermal applications of water models to study the thermal effects on fluid flow in tundishes have been reported. As for steel ladles, it seems that the only example that can be found in the literature is the work carried out by Hlinka and Miller, who used a hot water and acrylic plastic system to simulate liquid steel and refractory systems. Both fluid flow and heat transfer in such systems were studied and the results could be applied, by direct scale-up, to the ladle–tundish systems. This pioneering work points to the possibility of using water models to simulate fluid flow in steel ladles with thermal effects, typically the natural convection phenomena. Since then, little attention has been paid to this aspect. Therefore, the potential use of water model to simulate both fluid flow and heat transfer in steel ladles should be further explored.

The present study focuses on establishing a hot-water model to simulate natural convection phenomena in steel ladles during the holding period before casting for the following purposes:

- further evaluation of the key similarity criteria previously obtained for non-isothermal physical modelling of fluid flow and heat transfer in steel ladles, i.e., Fr and \( \beta A T \);
- validation of the CFD models developed for simulating natural convection in steel ladles; and,
- verification of the feasibility to simulate temperature stratification in steel ladles by scaling up the measured temperature stratification in the water model.

2. Experimental

For the purpose of simulating fluid flow and heat transfer in steel ladles by means of a water model, a systematic analysis on the similarity between natural convection phenomena in steel ladles and in hot-water models has been carried out by the authors. This similarity study suggests that water models with size scales in the range between 1/5 and 1/3 and using hot water of 45°C or higher could be appropriate for modelling natural convection phenomena in steel ladles. Accordingly, in the present work, a 1/4-scale hot-water model has been established in the laboratory. The model is based on the mid-aged 107-tonne steel ladles of SSAB Luleå Steelworks in Sweden. Figure 1 illustrates this physical model set-up with (a) showing a photo of the apparatus, (b) and (c) the sketches of internal arrangement.

The water model consists of two cooling chambers: the one is a cylindrical cooling chamber for simulating ladle wall, and the other is a flat cooling chamber for simulating ladle bottom. The cooling chambers are made of 2-mm thick stainless steel sheet (stainless steel type: ASTM 304). Hot water is used for simulating liquid steel held in ladles, while cold water with controllable temperatures is tangentially introduced into the cooling chambers in directions shown in Figs. 1(b) and 1(c). The diameter of the hot-water bath (i.e., the inner diameter of the side-cooling chamber) is 0.682 m and the height of the hot-water bath is 0.657 m. The gaps, through which the cooling water passes, are...
0.018 m for both the side and bottom cooling chambers. T-type (copper–constantan) thermocouples (TCs) were employed to get the temperature information from the water model. Figures 1(b) and 1(c) also schematically illustrate the thermocouple positions in the water model (cf. Fig. 1). 21 TCs were used for measuring the temperature profile in the water bath on a vertical plane bounded by sidewall and centre axis. 7 TCs were used for measuring the temperature distribution in the side-cooling chamber under the level of the hot-water bath. 4 TCs were used for measuring temperatures of water inflows and outflows of the cooling chambers. In addition, for the purpose to check the symmetry of cooling intensity, 3 more TCs (No. 8, No. 16 and No. 24) were used, together with TC No. 21, to measure temperatures along the periphery of the hot-water bath, as shown in Fig. 1(c). All the temperature signals were recorded into an HP34970A data logger for post processing. To prevent heat loss from the top free surface of the water bath, the free surface was covered with a light porous plastic plate that can float on the surface. In order to homogenise the hot-water bath, if needed, pressurised air can be blown into the water bath via the tuyere located at the centre of the bottom-cooling chamber.

A series of tests using this hot-water model was carried out in the following general procedure:
1) supply cold water to cooling chambers at certain temperatures and flowrates;
2) fill the model with hot water (around 50°C) to a certain bath level;
3) stir the hot-water bath by blowing pressurised air until the average temperature of the water bath decreases to about 45°C;
4) stop stirring the hot-water bath for a certain time lapse (i.e., the holding period for the development of thermal stratification); and, then,
5) stir the hot-water bath again or drain the hot water out.

From the water model tests, temperature profiles in the hot-water bath were measured. The measured temperatures can be used to compare the predictions of the CFD model developed for the water model as an indirect verification of the CFD model developed for steel ladles. Furthermore, by controlling the cooling intensity to correspond to the heat loss rate of steel ladles, obeying the similarity criteria, the temperature profiles (exhibiting thermal stratification) obtained from the water model may be used to deduce the extent of thermal stratification in the prototype steel ladles.

3. Mathematical Simulation
3.1. Governing Equations
In order to use the water modelling results to validate the CFD model developed for simulating natural convection phenomena in steel ladles, the same CFD model should be applied to the water model as well. In fact, the natural convection flows in liquid steel and in hot water are governed by the same set of turbulent Navier–Stokes type partial differential equations, except for the differences in thermal-physical properties of the fluids and initial and boundary conditions. In the Cartesian tensor notation, the governing equations can be expressed, for both liquid steel and hot water, as follows:

- **The Continuity Equation**

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0 \quad \text{..................................................................(1)}
\]

- **The Momentum Equations**

\[
\frac{\partial (\rho u_i)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = -\frac{\partial p}{\partial x_j} + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) - \rho \beta \frac{\partial T}{\partial x_j} \quad \text{..............................................(2)}
\]

where the Boussinesq approximation of the buoyancy force is adopted.

- **The Energy Equation**

\[
\frac{\partial (\rho C_T)}{\partial t} + \frac{\partial (\rho u_i C_T)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \lambda + \frac{\partial T}{\partial x_j} \right) \quad \text{..............................................(3)}
\]

- **The Turbulence Equations**

In the present work the standard high- Reynolds number k–ε two-equation turbulence model was employed to account for the turbulence effect involved in natural convection phenomena in liquid steel and in hot water, which reads:

i) The equation of turbulence kinetic energy:

\[
\frac{\partial (\rho k)}{\partial t} + \frac{\partial (\rho u_j k)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu + \frac{\partial k}{\partial x_j} \right) + \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} - \rho \varepsilon \quad \text{..................................................................(4)}
\]

ii) The equation of dissipation rate of turbulence energy:

\[
\frac{\partial (\rho \varepsilon)}{\partial t} + \frac{\partial (\rho u_j \varepsilon)}{\partial x_j} = \frac{\partial}{\partial x_j} \left( \mu_\varepsilon + \frac{\partial \varepsilon}{\partial x_j} \right) + \mu_\varepsilon \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} - C_1 \mu_\varepsilon \frac{\varepsilon^2}{k} \quad \text{..................................................................(5)}
\]

In addition, the effective viscosity \( \mu_\text{eff} \) is defined as

\[
\mu_\text{eff} = \mu + \mu_\varepsilon
\]
where \( \mu_l \) and \( \mu_t \) are, respectively, laminar (molecular) and turbulent viscosities. In the same way, \( \lambda_{\text{eff}} \) is also defined as the sum of laminar and turbulent heat conductivities, i.e.,

\[
\lambda_{\text{eff}} = \lambda_l + \lambda_t
\]

Furthermore, using the turbulence model described above, \( \mu_l \) and \( \lambda_l \) can be determined through

\[
\mu_l = C_{\mu} \frac{k^2}{\varepsilon} \quad \text{..................................(6)}
\]

and

\[
\lambda_l = \frac{\mu_l C_{\mu}}{Pr_t} \quad \text{................................(7)}
\]

where \( Pr_t \) is the turbulent Prandtl number, which can be taken as 0.9 for fully turbulent flows.\(^{18} \)

According to the recommendation of Launder and Spalding,\(^{19} \) the constants involved in the \( k-\varepsilon \) turbulence model can be assigned with the following values:

\[
C_{\mu} = 0.09, \quad C_1 = 1.44, \quad C_2 = 1.92, \quad \sigma_\varepsilon = 1.0, \quad \sigma_k = 1.3.
\]

Due to the use of high-Reynolds number turbulence model, logarithmic wall functions were adopted to supply boundary conditions along solid walls to the governing equations.

3.2. Simulation Tools and Assumptions

Equations (1) through (5) were solved numerically using a personal computer and a commercial CFD simulation software package, PHOENICS. The solutions were made, in a two-dimensional cylindrical-polar coordinate system, both on the water model and on the steel ladle under the corresponding initial and boundary conditions. In the present CFD simulations, the following general assumptions were made:

1) the steel ladle and the model ladle are simplified as cylinders with constant radii;
2) the flow and heat transfer phenomena are axis-symmetrical;
3) non-slip condition is applied to all boundary walls;
4) zero-flux condition is considered along the centre axis;
5) no heat loss is considered at the top free surface both in the water model and in the prototype steel ladle (assuming the steel melt is covered with a thick layer of slag).

3.3. Simulation Conditions and Numerical Models

As mentioned before, the major purposes of the present work are to check the validity of the similarity criteria derived previously, to verify the feasibility to simulate natural convection phenomena in steel melt held in the prototype ladle by using a non-isothermal water model, and to validate the CFD models developed for simulating natural convection phenomena in steel ladles. To realise these purposes, two important conditions should be established. One is to get the temperature distribution in steel melt in the prototype ladle for use to verify the water model predictions; the other is to establish an appropriate relationship between the heat loss flux in the steel ladle and that in the water model. For the first condition, it is obviously difficult or impossible to measure the temperature distribution in the real plant steel ladles. For this reason, the authors turned to the use of a CFD numerical model to predict the temperature distribution in the steel ladle as the standard for comparison with the temperature distribution in the same steel ladle but simulated by the water model. As for the second condition, logi-
transfer CFD model), were directly applied to the boundaries of the liquid baths. In this simplified CFD model, a two-dimensional computation domain was defined as a slice of water or steel volume bounded by sidewall, bottom, centre axis and top free surface; and a cylindrical-polar grid with 20 (radial) × 25 (axial) cells was applied to this domain as shown in Fig. 2(b). In the present work, the simplified CFD model was used to predict temperature distributions in the prototype steel ladle for use to compare those simulated by the water model, as a means of verifying the feasibility of using the water model (by scaling up the measured water temperatures) to predict the extent of thermal stratification in steel ladles.

Table 2 gives the thermal-physical properties of solid steel (model shell material), liquid steel and water used in the present CFD simulations. Table 3 lists the descriptions of various regions in the computation domains defined in Fig. 2.

4. Results and Discussion

4.1. Comparison between Water Model and CFD Model Simulation Results

Figure 3 shows the comparisons between the measured water temperatures and those predicted using the conjugate-heat-transfer CFD model for a water model experimental case. In this experimental case, initially, after stop of gas bubbling, the hot-water bath was nearly homogeneous and

Table 2. Thermal-physical properties of solid steel, liquid steel and water.

<table>
<thead>
<tr>
<th>Property</th>
<th>Unit</th>
<th>Solid steel*</th>
<th>Liquid steel</th>
<th>Water</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature</td>
<td>°C</td>
<td>&lt; 400</td>
<td>1580</td>
<td>45</td>
</tr>
<tr>
<td>Density</td>
<td>kg/m³</td>
<td>7000</td>
<td>6962.8</td>
<td>990.22</td>
</tr>
<tr>
<td>Thermal expansivity</td>
<td>°C⁻¹</td>
<td>0.00015</td>
<td>0.0004</td>
<td></td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>W/m°C</td>
<td>15</td>
<td>27.9</td>
<td>0.147</td>
</tr>
<tr>
<td>Thermal capacity</td>
<td>J/kg°C</td>
<td>500</td>
<td>787</td>
<td>4182</td>
</tr>
<tr>
<td>Viscosity</td>
<td>Pa·s</td>
<td>-</td>
<td>0.006</td>
<td>0.00689</td>
</tr>
<tr>
<td>Source of references</td>
<td></td>
<td>(20)</td>
<td>(21)</td>
<td>(22)</td>
</tr>
</tbody>
</table>

* Model shell material: ASTM 304 stainless steel

Table 3. Description of computation domains in Fig. 2.

<table>
<thead>
<tr>
<th>Regions</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABCD</td>
<td>Hex-water bath</td>
</tr>
<tr>
<td>EFGH</td>
<td>Bottom-cooling chamber</td>
</tr>
<tr>
<td>HGIA</td>
<td>Bottom-cooling water</td>
</tr>
<tr>
<td>HJIK</td>
<td>Inner shell of bottom-cooling chamber*</td>
</tr>
<tr>
<td>UKL</td>
<td>Side-cooling chamber</td>
</tr>
<tr>
<td>BILC</td>
<td>Side-cooling water</td>
</tr>
<tr>
<td>FG</td>
<td>Inlet of bottom-cooling chamber</td>
</tr>
<tr>
<td>EM</td>
<td>Outlet of bottom-cooling chamber</td>
</tr>
<tr>
<td>U</td>
<td>Inlet of side-cooling chamber</td>
</tr>
<tr>
<td>LK</td>
<td>Outlet of side-cooling chamber</td>
</tr>
<tr>
<td>ED</td>
<td>Centre axis</td>
</tr>
<tr>
<td>MF</td>
<td>Interface between cooling water and outer shell of bottom-cooling chamber</td>
</tr>
<tr>
<td>JK</td>
<td>Interface between cooling water and outer shell of side-cooling chamber</td>
</tr>
<tr>
<td>ab</td>
<td>Interface between liquid bath (steel melt or hot water) and bottom wall</td>
</tr>
<tr>
<td>bc</td>
<td>Interface between liquid bath (steel melt or hot water) and side wall</td>
</tr>
<tr>
<td>cd</td>
<td>Free surface of liquid bath</td>
</tr>
<tr>
<td>ad</td>
<td>Centre axis</td>
</tr>
</tbody>
</table>

* 3 cells are assigned for this region in z direction.
** 3 cells are assigned for this region in r direction.

Fig. 2. Computation domains and grids used for CFD simulations of water model and steel ladle.

Fig. 3. Comparison between temperatures calculated (lines) and measured (symbols) at different positions in the water model (cf. Fig. 1(b)).
had an average temperature of 44.8°C. It was cooled by introducing 10°C cold water into the side and bottom cooling chambers at flowrates of 9 and 10 l per min, respectively. The cooling lasted 6 min to allow the development of thermal stratification in the hot-water bath. It can be seen from this figure that a quite satisfactory agreement between calculated temperatures and measured ones was obtained, showing that the conjugate-heat-transfer CFD model developed in the present study is feasible for use to obtain the information about the heat loss fluxes from the hot-water bath to the cooling chambers and their distributions in the water model. This heat loss information will be applied to the simplified CFD model to simulate the similar flow and heat transfer phenomena in steel ladles.

4.2. Flow and Temperature Fields in Hot-water Model

Figure 4 illustrates the flow and temperature fields in the hot-water model after 6 min of cooling, predicted by the conjugate-heat-transfer CFD model, for the above-mentioned experimental case. Due to axis-symmetry, only a half of the flow field and a half of the temperature field are shown in the figure. It is seen that the CFD model well revealed the natural convection flow pattern and thermal stratification in the hot-water bath. Comparisons of the temperatures in the hot-water bath with those in the cooling chambers show that the temperature difference between the hot-water bath and the bottom-cooling chamber is almost uniform, implying a nearly uniform heat flux from the former to the latter, while the temperature difference between the hot-water bath and the side-cooling chamber is the smallest at the level of top free surface but increases downwards, reflecting that a gradient distribution of the heat flux from the former to the latter in the vertical direction can be expected in the water model. This feature of heat transfer confirms the fact that thermal stratification is dependent not only on the average heat loss flux but also, more crucially, on the distribution of the heat flux along the sidewall, and this should also hold true for the thermal stratification phenomena in steel ladles. Therefore, one of the advantages of the conjugate-heat-transfer CFD model developed in this work is that it can provide the details of heat loss information, which can be used to estimate the heat loss flux distributions in the prototype steel ladles.

4.3. Relationship between Heat Loss Fluxes of Water Model and Steel Ladle

One of the major aims of the present study is to explore the possibility of simulating thermal stratification phenomena in steel ladles by using the temperature information obtained from the hot-water model. This can be realised by scaling up, via certain similarity criteria, the water temperature distribution in the model into the steel temperature distribution in the prototype ladle. The criteria for such a scale-up can be derived from the similarity analysis on the flow phenomena concerned. According to the previous study on the similarity between natural convection phenomena in water models and in steel ladles, the key similarity criteria were found to be the following:

\[ Fr_m = Fr_p \] ..........................(8)

and

\[ (\beta \Delta T)_m = (\beta \Delta T)_p \] ..........................(9)

where the subscripts m and p stand for the water model and the prototype steel ladle, respectively.

The above similarity criteria lead to a relationship between the heat loss flux in the water model and that in the prototype steel ladle as:

\[ q_m = q_p \rho_m C_m \beta_p \] ..........................(10)

with

\[ f_i = \frac{f_m}{f_p} = \sqrt{\frac{T_{1005}}{T_{1002}}} \] ..........................(11)

where, for the present hot-water model, the geometry scale factor, \( f_G \), is 1/4 and, accordingly, the time scale factor, \( f_t \), is 0.5.

Introducing the thermal-physical properties of liquid steel and water given in Table 2 into Eq. (10), after rearrangement, results in:

\[ q_p = 7.0574 q_m \] ..........................(12)

Equations (11) and (12) are used in this work for scaling up the transient heat loss flux for the water model to that for the steel ladle.

4.4. Prediction of Steel Temperatures in Ladle by Using Water Temperatures in Model

The present work focuses on using the temperature distribution in the water model to predict the temperature distribution in steel ladles. The relation between the temperatures in the water model and the temperatures in the steel ladle is governed by Eq. (9), which can be further expressed as:

\[ T_p = T_{0,p} - \frac{\beta_m}{\beta_p} (T_{0,m} - T_m) \] ..........................(13)

where \( T_{0,p} \) and \( T_{0,m} \) are the initial temperatures of liquid steel and hot water, respectively. Equation (13) is actually adopted in the present study for scaling up the water temperatures in the model to the steel temperatures in ladles.

A method of verifying the feasibility of using the measured water temperatures in the model to predict the steel temperatures in ladles has been developed in the present work, as shown in Fig. 5. This figure depicts how to define the heat loss flux and its distribution in the steel ladle based on those obtained from the water model by using for example the conjugate-heat-transfer CFD model. Once the heat loss flux and its distribution in the steel ladle are defined, the simplified CFD model can then be used to predict the steel temperatures as standards to compare the steel temper-
atures simulated by the water model by scaling up, via Eq. (13), the measured water temperatures. This comparison will indicate the feasibility of using the water model to deduce the extent of thermal stratification in steel ladles.

Following the flow chart in Fig. 5, the same water model experimental case as described in Sec. 4.1 was further analysed. In the first step, the conjugate-heat-transfer CFD model was applied to this water model experimental case so as to predict the heat loss flux and its distribution in the water model. Figure 6 illustrates the local heat fluxes from hot-water bath to cooling chambers at different times during cooling, predicted by the conjugate-heat-transfer CFD model. It is seen from Fig. 6(a) that the distribution of the heat flux along the height of the side-cooling chamber looks non-linear and rather complicated. However, Fig. 6(b) shows that during cooling the heat flux from the hot-water bath to the bottom-cooling chamber appears to be nearly uniformly distributed along the radius of the chamber, except for the region close to the lower corner.

In the second step, in order that the heat fluxes predicted by the conjugate-heat-transfer CFD model, as shown in Fig. 6, can be applied as thermal boundary conditions to the simplified CFD model, empirical equations were used to approximate these heat fluxes and their distributions on the sidewall and bottom of the water model. In the present work, the empirical equations were obtained by using regression analysis and curve fitting treatment on the data shown in Fig. 6.

For the heat loss flux to the sidewall, according to Fig. 6(a), it is assumed that the heat flux has a linear distribution along the height of sidewall but with the slope and intercept as functions of time. Thus, through linear regression analysis (with respect to the distance from the bottom of the hot-water bath) and exponential curve fitting treatment (with respect to time), the heat loss flux and its distribution on the sidewall can be approximately represented by the following empirical equation:

\[ q_{s,m} = 5.572.0 + 2.897.9e^{-3.9858 \times 10^{-3} t_m} \]

\( t_m \) is the time from the start of heating; \( h_m \) is the distance from the bottom of the hot-water bath (0 ≤ \( h_m \) ≤ \( H_m \)).

Equation (14) is illustrated in Fig. 7(a) and Fig. 8 shows the results of this CFD simulation in comparison with measured temperatures in the water model. As seen, a good agreement is obtained between the predictions and the measurements. This result verifies that the heat loss fluxes from the hot-water bath to the cooling chambers predicted by the conjugate-heat-transfer CFD model are generally accurate and Eqs. (14) and (15) give a satisfactory approximation of the heat flux distributions on the sidewall and bottom of the water model. On the other hand, Fig. 8 also proves that the simplified CFD model, previously developed for simulating natural convection and resulting thermal stratification phenomena in steel ladles, can give reliable simulations on the similar flow and heat transfer phenomena in the water model. Therefore, in the present work this CFD model was used to predict the temperature distribution in the prototype steel
ladle for use to compare the temperature distribution in the same ladle but simulated by the water model by scaling up the measured water temperatures.

For this purpose, in the fourth step, Eqs. (14) and (15) were scaled up, via Eqs. (11) and (12), so that similar expressions (empirical equations) of heat loss fluxes to the sidewall and bottom of the prototype steel ladle were derived as

\[ q_{s,p} = 39323.8 + 20451.6e^{-1.9929 \times 10^{-8} h_p} - (6673.5 + 30298.8e^{-3.0928 \times 10^{-10} H_p}) \frac{h_p}{H_p} \]  

and

\[ q_{b,p} = 13433.8 + 14020.2e^{-2.8953 \times 10^{-8} h_p} \]  

where \( q_{s,p} \) and \( q_{b,p} \) are, respectively, the local heat fluxes to the sidewall and bottom of the prototype steel ladle; \( H_p \) is the height of the liquid steel bath and \( h_p \) is the distance from the bottom of the liquid steel bath (0 ≤ \( h_p ≤ H_p \)).

In the final step, Eqs. (16) and (17) were applied as thermal boundary conditions to the simplified CFD model to simulate the natural convection flows in the prototype steel ladle that have dynamic and thermal similarity to those in the water model. Figure 9 shows comparisons between the steel temperatures in the prototype ladle predicted by the simplified CFD model, under the thermal boundary conditions given by Eqs. (16) and (17), and those simulated by the water model by scaling up, via Eq. (13), the measured water temperatures. In these comparisons, the steel temperatures calculated by the simplified CFD model are treated as standards to verify the accuracy in using the hot-water model to simulate thermal stratification phenomena in steel ladles. It can be seen that Fig. 9 gives a generally good agreement between the temperatures in the prototype steel ladle calculated by the mathematical CFD model and those simulated by the physical hot-water model, except for the region very close to the ladle bottom. This result proves that it is feasible to use the hot-water model established in the present work to simulate thermal stratification phenomena in steel ladles, as long as the heat loss fluxes and their distributions in steel ladles are known.

4.5. Influence of Heat Loss Rate on Thermal Stratification in Water Model and Steel Ladle

With feasible use of the hot-water model to simulate the thermal stratification phenomena in steel ladles, further water model tests were performed to investigate the influ-
ence of heat loss rate on the development rate of thermal stratification in steel ladles. Table 4 gives the conditions for these tests. In the water model tests the initial hot-water temperature was controlled around 45°C and the cooling time lasted about 6 min. In each test, the average heat loss rate of the hot-water bath was obtained in terms of bulk-cooling rate, which was calculated as the average drop rate (in °C/min) of the temperatures measured in the hot-water bath.

According to the similarity criteria, i.e., Eqs. (8) and (9), the relation between the bulk-cooling rate of the water model and that of the prototype steel ladle can be derived. The bulk-cooling rate is defined as

\[ \dot{\phi} = \frac{\Delta T}{\Delta t} \] ..........................(18)

According to this definition, there is

\[ \frac{\dot{\phi}_p}{\dot{\phi}_m} = \frac{\Delta T_p/\Delta t_p}{\Delta T_m/\Delta t_m} \] ..........................(19)

where \( \dot{\phi}_p \) and \( \dot{\phi}_m \) are the bulk-cooling rates of steel ladle and water model, respectively.

Considering \( f = \Delta t_p/\Delta t_m \) and introducing Eqs. (9) and (11) into Eq. (19), the relation between the bulk-cooling rate of the prototype steel ladle and that of the water model becomes

\[ \phi_p = \phi_m \frac{\beta_m}{\beta_p} \sqrt{f_G} \] ..........................(20)

Equation (20) makes it possible to calculate the bulk-cooling rates of steel ladles from those measured from the water model, and vice versa.

The bulk-cooling rates measured from the water model and those calculated for steel ladles, via Eq. (20), are also listed in Table 4 for different tests. The results show that the bulk-cooling rates of steel ladles in the range between 0.3 and 0.9°C/min can well be simulated by the water model by manipulating the flowrates and temperatures of cooling water. For example, a steel ladle with a bulk-cooling rate of 0.5°C/min may be simulated with the water model by introducing about 20°C water into the side-cooling chamber at a flowrate of 18 l/min and into the bottom-cooling chamber at a flowrate of 10 l/min, e.g., test No. 9, as seen from Table 4.

Furthermore, the measured temperature distribution in the water model can be scaled up, via Eq. (13), into the temperature distribution in the steel ladle, so that the development rate of thermal stratification in the steel ladle may be predicted by using the water model. The thermal stratification rate (°C/min) is defined as the rate of change in the difference between the temperature of the hottest liquid near the top of the bath and the temperature of the coldest liquid near the bottom of the bath, i.e.,

\[ \eta = \frac{T_{\text{top}} - T_{\text{bot}}}{\Delta t} \] ..........................(21)

Thus, according to this definition, the ratio between the thermal stratification rate in the steel ladle and that in the water model is the following:

\[ \frac{\eta_p}{\eta_m} = \left( \frac{T_{\text{top}} - T_{\text{bot}}}_p \right) \frac{\Delta t_p}{\Delta t_m} \] ..........................(22)

According to Eq. (13), it is easy to prove that there is

\[ \beta_p (T_{\text{top}} - T_{\text{bot}})_p = \beta_m (T_{\text{top}} - T_{\text{bot}})_m \] ..........................(23)

Combination of Eq. (22) with Eq. (23) with consideration of Eq. (11), after rearrangement, yields the relation between the thermal stratification rate in the prototype steel ladle and that in the water model as

\[ \eta_p = \eta_m \left( \frac{\beta_m}{\beta_p} \right) \sqrt{f_G} \] ..........................(24)

A comparison between Eq. (24) and Eq. (20) shows that they are in the same form. A combination of these two equations leads to:

\[ \eta_p = \eta_m \left( \frac{\beta_m}{\beta_p} \right) \sqrt{f_G} = \text{constant} \] ..........................(25)

Consequently, it may be deduced from Eq. (25) that the thermal stratification rate could be linearly proportional to the bulk-cooling rate for both the water model and the steel ladle.

According to Eq. (21), the thermal stratification rate in the water model is calculated by taking, respectively, \( T_{\text{top}} \) and \( T_{\text{bot}} \) as the average of the temperatures measured by the upper TCs No. 7, No. 15, and No. 23 and that of the temperatures measured by the lower TCs No. 1, No. 9 and No. 17 (cf. Fig. 1(b)). The thermal stratification rate in the prototype steel ladle is calculated directly from that in the water model by using Eq. (24). Or, alternatively, identical values of the thermal stratification rate in steel ladles can also be obtained using Eq. (21) after scaling up, via Eq. (13), \( T_{\text{top}} \) and \( T_{\text{bot}} \) in the water model into those in the steel ladle. Table 4 gives the thermal stratification rates obtained from the water model and those calculated for steel ladles. Using these data, a plot of the thermal stratification rate, \( \eta_m \), against the bulk-cooling rate, \( \dot{\phi}_m \), for 107-tonne steel ladles, is shown in Fig. 10. This figure also includes the line proposed by Austin et al.\(^\text{23}\) for comparison. As seen, a nearly linear relationship between the thermal stratification rate and the bulk-cooling rate is suggested by the experimental

---

**Table 4. Conditions and results of hot-water model tests.**

<table>
<thead>
<tr>
<th>Test No.</th>
<th>Side-cooling chamber</th>
<th>Bottom-cooling chamber</th>
<th>Water model</th>
<th>Steel ladle</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flowrate (l/min)</td>
<td>Flowrate (l/min)</td>
<td>Flowrate (l/min)</td>
<td>Flowrate (l/min)</td>
</tr>
<tr>
<td></td>
<td>Temperature (°C)</td>
<td>Temperature (°C)</td>
<td>Temperature (°C)</td>
<td>Temperature (°C)</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>10.8</td>
<td>4.1</td>
<td>10.9</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>9.4</td>
<td>7.5</td>
<td>9.0</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>9.6</td>
<td>10.5</td>
<td>9.8</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>12.7</td>
<td>4.1</td>
<td>15.5</td>
</tr>
<tr>
<td>5</td>
<td>12</td>
<td>13.0</td>
<td>7.6</td>
<td>13.6</td>
</tr>
<tr>
<td>6</td>
<td>15.3</td>
<td>10</td>
<td>15.2</td>
<td>15.3</td>
</tr>
<tr>
<td>7</td>
<td>20.4</td>
<td>4</td>
<td>20.5</td>
<td>20.5</td>
</tr>
<tr>
<td>8</td>
<td>12.0</td>
<td>7.7</td>
<td>20.2</td>
<td>11.2</td>
</tr>
<tr>
<td>9</td>
<td>20.7</td>
<td>10</td>
<td>20.4</td>
<td>11.4</td>
</tr>
<tr>
<td>10</td>
<td>9.3</td>
<td>10</td>
<td>9.8</td>
<td>11.1</td>
</tr>
<tr>
<td>11</td>
<td>10.2</td>
<td>10</td>
<td>10.3</td>
<td>11.1</td>
</tr>
<tr>
<td>12</td>
<td>9.1</td>
<td>10</td>
<td>10.1</td>
<td>10.8</td>
</tr>
</tbody>
</table>

---

© 2002 ISIJ
data, which confirms the deduction made from Eq. (25),
giving a slope of 2.34 that is close to the value of 2.0 re-
ported by Austin et al.\textsuperscript{8} who performed CFD simulations
on industrial steel ladles of different sizes. However, the
present water model predicts generally higher thermal strat-
ification rates than those presented by Austin et al. This is
because, as seen from Fig. 6(a), the water model has a
smaller heat loss flux in the upper section of the sidewall
but a larger heat loss flux in the lower section of the side-
wall, while Austin et al. applied uniformly distributed (av-
verage) heat loss fluxes in their modelling work. Therefore,
it can be expected that, even though with the same bulk-
cooling rate (i.e., average heat loss rate) in both works, the
water model in the present work should have a larger extent
of thermal stratification than in Austin et al.’s work. This
result signifies that thermal stratification phenomenon is
influenced not only by the magnitude of the average heat loss
flux but also, more importantly, by the distribution of the
heat loss flux on the sidewall.

5. Conclusions

The present non-isothermal water model study has con-
firmed the validity of the dimensionless numbers Fr and
$\beta$ at as indices of key criteria governing the similarity be-
tween natural convection phenomena in hot-water models
and in prototype steel ladles.

Establishing a non-isothermal water modelling system is
useful for verification of CFD mathematical modelling re-
results. In addition to mathematical methods, the hot-water
model provides an alternative means of studying fluid flow
and heat transfer phenomena in steel ladles.

It is feasible to use a 1/4-scale hot-water model to simu-
late natural convection and resulting thermal stratification
phenomena in steel ladles during the holding period before
casting, as long as the heat loss fluxes and their distributions
in steel ladles are known.

The present water modelling results prove the fact that in
steel ladles the development rate of thermal stratification is
linearly proportional to the bulk-cooling rate, as has been
reported by other authors.\textsuperscript{8} In addition, these results also
indicate that the extent of thermal stratification in steel la-
dles is dependent not only on the magnitude of the average
heat loss flux but also, more importantly, on the distribution
of the heat loss flux on sidewall.

Acknowledgement

The authors wish to acknowledge the Computer Assisted
Materials and Process Development Association (CAMP-
DA) in Sweden for financial support for this study.

Nomenclature

\begin{itemize}
  \item $C$: Thermal capacity (J/kg K)
  \item $C_1, C_2, C_3$: Constants used in $k-\varepsilon$ turbulence model
  \item $f_G$: Geometry scale factor (\textdegree)
  \item $f_i$: Time scale factor (\textdegree)
  \item $F_r$: Froude number (\textdegree)
  \item $g$: Gravitational acceleration (m/s$^2$)
  \item $k$: Distance from the bottom of liquid bath (m)
  \item $H$: Height of liquid bath (m)
  \item $k$: Turbulence kinetic energy (m$^2$/s$^2$)
  \item $p$: Pressure (Pa)
  \item $Pr$: Prandtl number (\textdegree)
  \item $q_b$: Heat flux to bottom wall (W/m$^2$)
  \item $q_s$: Heat flux to side wall (W/m$^2$)
  \item $r$: Radial coordinate (m)
  \item $t$: Time (s)
  \item $\Delta t$: Cooling time interval (min)
  \item $T$: Temperature (K)
  \item $T_i$: Initial temperature (K)
  \item $T_{bc}$: Temperature close to bath bottom (K)
  \item $T_{fs}$: Temperature close to bath free surface (K)
  \item $\Delta T$: Difference of temperature from the initial tempera-
ture (K)
  \item $u$: Velocity (m/s)
  \item $x$: Cartesian coordinate (m)
  \item $z$: Axial coordinate (m)
  \item $\beta$: Thermal expansion coefficient (1/K)
  \item $\varepsilon$: Dissipation rate of turbulence energy (m$^2$/s$^3$)
  \item $\phi$: Bulk-cooling rate (°C/min)
  \item $\eta$: Thermal stratification rate (°C/min)
  \item $\lambda$: Thermal conductivity (W/m K)
  \item $\mu$: Viscosity (Pa s)
  \item $\theta$: Angular coordinate (rad)
  \item $\rho$: Density (kg/m$^3$)
  \item $\rho_{\text{ref}}$: Reference density (kg/m$^3$)
  \item $\sigma^t$: Prandtl number for turbulent kinetic energy (\textdegree)
  \item $\sigma^e$: Prandtl number for dissipation rate of turbulence
energy (\textdegree)
\end{itemize}

Subscripts

\begin{itemize}
  \item $l$: Laminary flow quantity
  \item $m$: Model parameter
  \item $p$: Prototype parameter
  \item $\text{ref}$: Reference quantity
  \item $t$: Turbulent flow quantity
\end{itemize}

REFERENCES

2) J. Petegnie, J. P. Brair, M. Larrecq, M. Bobrie, F. Lemiere and Y.
3) J. A. Wester: Internal technical report, MF 2/68, MEFOS, Luleå,
Sweden, (1968).
4) K. O. Jonsson: Ph.D. thesis in ferrous metallurgy, MEFOS, Luleå,
5) C. E. Grip: Steelmaking Conf. Proc., Vol. 77, ISS, Warrendale, PA,
(1994), 103.
563.
8) P. R. Austin, J. M. Camplin, J. Herbertson and I. J. Taggart: ISIJ Int.,
32 (1992), 196.
10) B. Olika, Y. Pan, B. Björkman and C. E. Grip: Scand. J. Metall., 26
12) J. de Barreto S., M. A. Barron Meza and R. D. Morales: ISIJ Int.,
36 (1996), 543.
18) P. Bradshaw: Turbulence, Vol. 12, Springer-Verlag, New York,
(1976), 244.
20) Technical data book of Avesta Sheffield Group, INF. 10100 GB,
21) J. Chen: Handbook of Diagrams and Data for Steelmaking, Publish
22) R. C. Weast: Handbook of Chemistry and Physics, 57th ed., CRC
Press, USA, (1977), F-5.

© 2002 ISIJ