Iron Ore Granulation Model Supposing the Granulation Probability Estimated from Both Properties of the Ores and Their Size Distributions

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It is important to reduce fluctuation of the ore granulation for a stable operation of iron ore sinter plant, because the fluctuation affects directly productivity and productive yield of the operation. To clarify the granulation phenomena in a rotating granulator, an advanced mathematical model that is different from previously proposed ones was developed considering the probability theory. Mathematical characteristics of the model and granulation simulation by the model are summarized as follows:

1) Since it is difficult to analyze the phenomena by the models based on the motion of quasi-particles due to the complicated movement of large amounts of the quasi-particles, the present model treated it as a kind of probability phenomena.

2) This model is basically composed of matrix algebras defining following two granulation parameters, 1) overall granulation probability resulted from ore properties, and 2) granulation and disintegration probabilities at each size range of the ores.

3) This model is useful for evaluation of the granulation phenomena, because the simulation results using the appropriate granulation parameters agree with those from the granulation operation.

4) It is important to make clear the granulation conditions for defining the granulation parameters, because the parameters depend on the conditions of the granulation dynamics and the size of the granulator.

KEY WORDS: granulation; simulation model; iron ore sinter; probability theory; Markov chain.

1. Introduction

More than 50% of iron ore resources used in Japan depend upon Australian ores. This trend will increase more and more, due to the low transportation costs. As to the Australian ores, supply of coarse Block-man ores containing low phosphorous elements, which is the main ores used in Japanese sinter plants, tends to decrease because of the shortage of the resource. Instead of the Block-man ores, relatively fine Mara-mamba ores containing crystallized water ranging from 4 to 6 mass% will be supplied to Japan in future by their large amounts of ore deposits. Since the Mara-mamba ores are high iron grade as compared to other Australian ores, the ores can contribute to produce iron ore sinter containing low silica, which is evaluated as high quality burdens for blast furnace operation with high-pulverized coal injection. However, recent fundamental studies reported that in addition to the fine size, granulation characteristics of them are inferior to other ores, due to the weak adhesion properties between the fine particles.

Previously, maximum productivity of sinter operation depended mainly on the combustion efficiency of added fine coke in raw materials. Presently, permeability in the sinter bed limits the productivity. It is derived from some advanced technologies, such as the method of fine coke coating on the surface of quasi-particles and introduction of segregative charging system of fine coke in the depth direction of sinter bed. These technologies have reduced the coke consumption remarkably in the sinter operation. These technological trends represent that it is important in the future to improve the bed permeability in the sinter bed, in order to achieve high productivity. To keep the permeability using fine ores, it is necessary to improve granulation of the fine ores before sintering. The purpose of the present study is to propose an advanced mathematical model for simulating iron ore granulation phenomena in a disc-pelletizer or drum mixer.

2. Previously Proposed Granulation Models

To keep high permeability of granulated material bed in a grate process, granulation of iron ore fines is quite important for succeeding sintering process. Many mathematical models have been proposed for simulating the granulation phenomena, in addition to the empirical ones on the basis of granulation experiments.

Rumpf clarified firstly granulation mechanism of fine powders. He concluded that the mechanism depends on contact points, bridges and capillary forces between the powders. He also made clear that maximum shearing or tension forces between the powders influence the strength of the granulated materials. Ouchiyama et al. proposed a
granulation model in which fine particles coalesce mutually by both external and friction forces during rotating motion. Suzuki et al.\textsuperscript{9)} analyzed granulation phenomena formulating mass balance of iron ore fines in a drum mixer and disc-pelletizer. They concluded that size of quasi-particle is determined by total rotating distance in the granulators and granulation characteristics of a disc-pelletizer are superior to those by a drum mixer. Lister et al.\textsuperscript{10)} presented a population balance equation in which quasi-particles grow by the coating of fine ores on the surface of nuclei coarse ores. This equation made clear that it is important to control size of fine ores for size enlargement of the quasi-particles. Kanoh et al.\textsuperscript{11)} analyzed theoretically the granulation phenomena during rotating motion in a drum mixer, considering kinetic and colliding energy. They pointed out through the analysis that the granulation obtained in small-scaled tests does not agree with that in commercial plant operation. With recent advance of capacity of computers, analyses of granulation dynamics will progress remarkably introducing some advanced computational methods.\textsuperscript{11)}

3. Proposal of Granulation Model Based on the Probability Theory

When fine ores are granulated in a disc pelletizer or drum mixer, it is necessary to obtain information of the granulated quasi-particles considering both the ore properties and granulation conditions in a granulation operation. Figure 1 shows a relation among raw material conditions, granulation parameters and quasi-particle conditions. It must be considered for the constitution of the granulation model that size distribution of fine ores influencing the granulation characteristics, ore properties, binder, and granulation mode. In addition to information of average size of the quasi-particles, size distribution of the quasi-particles also needs for their evaluations in the succeeding sinter plant operation, because permeability of the sinter bed depends mainly on them.

3.1. Growing Phenomena of Quasi-particles in a Rotating Granulation Process

It is difficult to clarify quantitatively the growing phenomena of quasi-particles during rotation in a granulating device under moisturizing conditions. Figure 2 shows typical granulation phenomena in a rotating drum mixer. Fine ores, which partially remain unchanged during the rotation, grow through a unit granulating operation to quasi-particles by coalescence of fine ores and coating of fine ores on the surface of nuclei ores. In addition to growing of the quasi-particles, a portion of the quasi-particles reduces their sizes.
because of the disintegration. Quasi-particles substantially increase in their sizes by progressing the granulation times. The average size and size distribution of the quasi-particles, therefore, depend upon the contributing ratio determined by these granulation and disintegration motions.

### 3.2. Formulation of Mathematical Model Simulating Granulation Phenomena

Since complicated granulation occurs simultaneously in a granulation system, there is a limit to formulate a mathematical model taking account of the motion based on the mutual action between particles. The author tries to develop a probability model supposing granulation probabilities in a disc pelletizer or drum mixer, because it is difficult to determine the size and size distribution by the previous models due to the limit mentioned above. The same approach as the probability model was applied for analyzing coal grinding mechanism by Broadbent et al. They clarified the mechanism introducing a grinding matrix and elements constituting the matrix.

**Figure 3** shows a concept of the present granulation model on the basis of the probability theory. Initial fine ores are separated to granulated and un-granulated portions after a unit rotating operation. Size distribution of the fine ores, \( F = (f_1, f_2, \ldots, f_n) \) and the granulated quasi-particles, \( G = (g_1, g_2, \ldots, g_n) \) after \( N \)-times of the rotation are expressed using column vectors according to the size distribution as follows:

\[
F = G_0 = (f_1, f_2, \ldots, f_n) \quad \cdots (1)
\]
\[
G_n = (g_1, g_2, \ldots, g_n) \quad \cdots (2)
\]

Assuming the overall granulation probability of ore, \( \pi \), which depends on the ore properties defined to the unit rotating operation, Fig. 3 indicates that finest portion of the initial ores, \( f_1 \) changes to following granulated and un-granulated sections after the unit rotating operation:

\[
\pi \cdot f_1 \quad \text{(Granulated section)} \quad \cdots \cdots (3)
\]
\[
(1-\pi) \cdot f_1 \quad \text{(Un-granulated section)} \quad \cdots \cdots (4)
\]

Equation (3) gives the granulated section regarded as mass of quasi-particles that includes various sized quasi-particles. In case that the initial ores grow to the quasi-particles by the rotating operation, the size distribution of the quasi-particles depends on the granulation probability characterized by each size range of the ore. If the probability is defined as \( q_{ij}(\cdot) \), it means the probability elements of each sized quasi-particles in which the finest ore portion, \( f_1 \) is granulated to the quasi-particle having size of \( i \)-th sieve mesh. The granulation probabilities, \( q_{ij} \), accordingly range from 0 to 1.0.

When the finest ore portion, \( f_1 \) having size of less than 0.125 mm is granulated, the portion changes as follows by the every one time of rotation:

\[
(1-\pi) + \pi q_{11} \quad \text{(Sum of weight ratio of non-granulated particles and granulated quasi-particles less than 0.125 mm in size)} \quad \cdots \cdots (5)
\]
\[
\pi q_{21} \quad \text{(Weight ratio of quasi-particles less than 0.125 mm in size)} \quad \cdots \cdots (6)
\]
\[
\pi q_{j1} \quad \text{(Weight ratio of quasi-particles ranged sieve mesh from (i)-th to (i+1)-th size)} \quad \cdots \cdots (7)
\]
\[
\pi q_{n1} \quad \text{(Weight ratio of quasi-particles more than the maximum size)} \quad \cdots \cdots (8)
\]

By the same procedure as shown in Fig. 3, **Fig. 4** shows the distribution of quasi-particles after granulation of the ores ranged from 0.125 to 1.00 mm in size. Weight portion of \( f_2 \), located in the secondary sieve mesh (0.125–1.00 mm...
As material balance before and after the granulation must be
fined as follows:

\[
\begin{align*}
\pi q_{12} &= \text{(Weight ratio less than 0.125 mm in size by the disintegration probability)} \quad \ldots \ldots (9) \\
(1 - \pi) + \pi q_{22} &= \text{(Sum of weight ratio of non-contributing granulation and incomplete granulation ranged from 0.125-1.00 mm in size)} \quad \ldots \ldots (10) \\
\pi q_{n2} &= \text{(Weight ratio of quasi-particles more than 20.65 mm in size)} \quad \ldots \ldots (11)
\end{align*}
\]

Size distribution of the quasi-particles at the first unit rotation operation can be determined by the similar mathematical procedure from \( j = 1 \) to \( n \).

A square matrix \( B \) of \((n, n)\), composed of the probability elements governing granulation and disintegration is defined as follows:

\[
B = \begin{bmatrix}
q_{11} & q_{12} & \cdots & q_{1n} \\
q_{21} & q_{22} & \cdots & q_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
q_{n1} & q_{n2} & \cdots & q_{nn}
\end{bmatrix} \quad \ldots \ldots (12)
\]

As material balance before and after the granulation must be conserved in Eq. (12), a following equation is obtained:

\[
\sum_{j=1}^{n} q_{ij} = 1.0 (-) \quad (j = 1 - n) \quad \ldots \ldots (13)
\]

where, \( q_{ij} \) defines the probability elements constituting the \( n \times n \) matrix \( B \).

A portion of quasi-particles located less than a minimum sized sieve mesh is presented by the following equations based on Eq. (5):

\[
g_{i} = f_{i}(1 - \pi) + f_{i}\pi q_{11} \quad \ldots \ldots (14)
\]

\[
g_{2} = f_{2}(1 - \pi) + f_{2}\pi q_{21} + f_{2}\pi q_{22} + \cdots + f_{2}\pi q_{2n} \quad \ldots \ldots (15)
\]

\[
g_{j} = f_{j}(1 - \pi) + f_{j}\pi q_{j1} + f_{j}\pi q_{j2} + \cdots + f_{j}\pi q_{jn} = f_{j}(1 - \pi) + \pi \sum_{m=1}^{n} f_{m} \pi q_{jk} \quad \ldots \ldots (16)
\]

Equation (14) means that \( g_{1} \) is given by summing the terms of both non-granulating portion and granulated portion less than 0.125 mm in size, in spite of the granulation operation. Equation (15) also means that \( g_{2} \) is expressed by the sum of non-granulating portion and granulated portion resulted from \( f_{1} \) fraction and granulated one between 0.125 to 1.00 mm in size. The same mathematical procedure as Eqs. (14) and (15) can introduce to the generalized formulation. A following equation can be given from Eq. (16) using a matrix expression.

\[
G = (1 - \pi)E \cdot F + \pi B \cdot F \quad \ldots \ldots (17)
\]

Where, \( G \) and \( F \) mean a column vector composed of \( g_{i} \) and \( f_{i} \), respectively, and \( E \) shows a \( n \times n \) unit matrix.

\[
G = \begin{bmatrix}
g_{1} \\
g_{2} \\
\vdots \\
g_{n}
\end{bmatrix}, \quad F = \begin{bmatrix}
f_{1} \\
f_{2} \\
\vdots \\
f_{n}
\end{bmatrix}, \quad E = \begin{bmatrix}
1 & 0 & \cdots & 0 \\
0 & 1 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 1
\end{bmatrix} \quad \ldots \ldots (18)
\]

Size distribution of quasi-particles, \( G_{n} \), obtained after the secondary unit granulating operation represents as follows by the same procedure described in Eq. (17):

\[
G_{n} = ((1 - \pi) \cdot E + \pi B)^{n} \cdot F \quad \ldots \ldots (19)
\]

The size distribution, \( G_{n} \), after \( N \)-times of the unit granulating operation is expressed by Eq. (20):

\[
G_{n} = ((1 - \pi) \cdot E + \pi B)^{N} \cdot F \quad \ldots \ldots (20)
\]

From Eqs. (12) and (18), the first term of the right side in
Eq. (20) is also expressed by a matrix \( B' \) of \((n, n)\), as follows:

\[
B' = \{(1 - \pi) \cdot E + \pi B\}^N = \begin{bmatrix}
q_{11}' & q_{12}' & \cdots & q_{1n}' \\
q_{21}' & q_{22}' & \cdots & q_{2n}' \\
\vdots & \vdots & \ddots & \vdots \\
q_{n1}' & q_{n2}' & \cdots & q_{nn}'
\end{bmatrix}
\] (21)

Both granulation and disintegration phenomena are considered in the matrix \( B' \), due to the composition of the matrix \( B \). Equation (20) must satisfy the law of conservation of mass before and after the granulation.

\[
G = \begin{bmatrix}
g_1 \\
g_2 \\
\vdots \\
g_n
\end{bmatrix} = B' \begin{bmatrix}
f_1 \\
f_2 \\
\vdots \\
f_n
\end{bmatrix}
\] (22)

From Eq. (22), a series of following equations is given:

\[
g_1 = q_{11}' f_1 + q_{12}' f_2 + \cdots + q_{1n}' f_n \\
g_2 = q_{21}' f_1 + q_{22}' f_2 + \cdots + q_{2n}' f_n \\
\vdots \\
g_n = q_{n1}' f_1 + q_{n2}' f_2 + \cdots + q_{nn}' f_n
\]

Equation (23) can be obtained by summing above the equations:

\[
\sum_{k=1}^{n} g_k = \sum_{k=1}^{n} q_{k1}' f_1 + \sum_{k=1}^{n} q_{k2}' f_2 + \cdots + \sum_{k=1}^{n} q_{kn}' f_n
\] (23)

Following relation is given by Eq. (13):

\[
\sum_{k=1}^{n} q_k = f_1 + f_2 + \cdots + f_n = \sum_{k=1}^{n} f_k
\] (24)

Equation (24) indicates that total weight of fine ores is equivalent to that of granulated quasi-particles.

### 3.3. Simulation of the Granulation Dynamics

**Figure 5** shows a procedure for the simulation using the equations described in 3.2. For conducting the simulation, granulation conditions must be determined, such as total numbers of rotation times, \( N_{\text{max}} \), numbers of sieve mesh providing both the size distribution of fine ores and granulated quasi-particles, \( n - 1 \), size distribution vector of initial ores, \( F \), overall granulation probability depending on the ore properties, \( \pi \), and matrix elements, composed of granulation and disintegration probability of every ore size range, \( B \). Matrix calculation at each granulation operation is carried out using Eq. (20) under the above granulation conditions. The matrix calculation gives information of the new granulation matrix, \( B' \), corresponding Eq. (21) and the size distribution of quasi-particles, \( G \) at each unit granulating operation. Figure 4 indicates when fine ores less than 1mm in size are consumed completely by the coating on the surface of nuclei coarse ores, the calculation procedure ends. From a viewpoint of the simulation, this computational treatment is based on the assumption that enlargement of quasi-particles is controlled by the shortage of fine ores. The limit of the fine ore size, accordingly, must change, corresponding to ore properties and granulation conditions.

**Figure 6** shows typical simulation results assuming the granulation parameters of \( \pi \) and \( B \). **Table 1** shows sieve meshes and the size distribution of ore fines. The distribution is characterized as finer ores in comparison with typical ores used in other sinter plants in terms of the contents of size less than 0.125 mm. To characterize the granulation phenomena, typical two kinds of parameters, matrix \( B \) were selected under the same parameter of \( \pi \). **Table 2** indicates the granulation parameters, \( \pi \) and \( B \) for the simulation. Matrix \( B \) in Case 1 is characterized by the upper triangular matrix bordered on the diagonal matrix. This controls the disintegration of the quasi-particles during the granulation. Matrix \( B \) also indicates that the probability elements from \( q_{ij} \) to \( q_{ij} \) are relatively larger than other row elements. This consideration increases in the yield of quasi-particles having 3.0 to 10.3 mm in size. To widen the size distribution of the quasi-particles during the simulation, matrix \( B \) in Case 2 is characterized by the reversed constitutions of the matrix as shown in Case 1. Since these parameters are determined as assumed values to verify the validity of the mathematical model, the appropriate parameters must be defined by granulation tests using the target ores. The average data of initial size distribution of ores that are used at Fukuyama No. 5 Sinter Plant were adopted for the simulation. This data was subdivided according to the numbers of sieve mesh, because the original data of the size distribution at the sinter plant is divided roughly. Case 1 in Fig. 6 indicates the suitable granulation condition, because fine ores are consumed quickly. As a result of the condition, size distribution of the quasi-particles gradually concentrates to that from 5 to 10 mm in size. Case 2 suggests the unsuitable condition for the granulation, because reduction of the fine ore portion delays, due to the assumed probability elements caused by inferior granulation characteristics. Kinds of iron ores as mentioned above will influence the characteristics by their different physical and mineralogical properties, even though the size distributions are
Figure 7 shows the simulation result of the influence of the granulation characteristics of ores on the size distribution of quasi-particles according to the granulation times. In this simulation, B is composed of the same matrix as described in Case 1. With increase in the number of unit granulating operation, the size distribution of quasi-particles finally becomes similar, independent of their granulation characteristics. However, there is a significant difference in the size distributions during the granulation. Although the initial size distribution of ores is the same in all simulation cases, granulation of finer ores delays in case of ores having low value of $\pi$.

3.4. Application of the Model to the Analysis of Granulation Experiment

In order to evaluate validity of the mathematical model for simulating the granulating phenomena, the model was verified by comparing the granulation test data. In the test data, numbers of sieve mesh are divided into following six parts; –0.125, 0.125–0.50, 0.50–1.0, 1.0–2.0, 2.0–5.0, 5.0–10.0 mm, respectively. Figure 6 indicated that the elements, $q_{ij}$ influence the final size distributions. The distributions depend on the $q_{ij}$, located in the upper (I) or lower area (II) bordered on the diagonal matrix, $q_{ii}$. Equation (17) characterizes that the elements in the area (I) contribute the granulation and those in the area (II) deteriorate it, due to the disintegration probabilities. The data characterizes that the reduction of fine portion less than 0.25 mm in size and increase of the portion between 2.0 to 5.0 mm in size are remarkable. Since the granulation test does not give the information of the overall granulation probability, $\pi$, it is apparently determined as $0.4(-)$, shown in Eq. (25). This means that contribution ratio of the granulation is defined as 40%.
4. Discussion

4.1. Mathematical Evaluation of the Granulation Parameters

Granulation matrix, B, composed of the granulation and disintegration probabilities at the each size range of ores and overall ore granulation properties, π are the main parameters in the present mathematical model. Theoretical meaning of these parameters and the influence of the parameters on the granulation phenomena are discussed as follows:

4.1.1. Granulation Matrix (B)

Following extreme cases can assume in the granulation matrix, B, described in Eq. (12).

\[
\begin{bmatrix}
q_{11} & q_{12} & \cdots & q_{1n} \\
0 & q_{22} & \cdots & q_{2n} \\
0 & 0 & \ddots & \vdots \\
0 & 0 & \cdots & q_{nn}
\end{bmatrix}
\]

B shows a triangular matrix where probability elements located upward on the diagonal matrix, \(q_{kk}\) are zero. This means that granulated quasi-particles never disintegrate by the granulation operations. B shows a matrix, which is composed of the same valued elements under the below condition:

\[
r = 1.0/n
\]

This means granulation and disintegration of quasi-particles occur equally because of the same probability elements during the granulation operation. B means a triangular matrix in which the elements are arranged inversely as compared to those of B matrix. This matrix means that the quasi-particles continue to be disintegrated in spite of the granulating operation.

The parameter, π is regarded as a constant probability during the granulation operation in the mathematical model. This is based on an assumption that the parameter is independent of the operation conditions, such as size distributions of quasi-particles after N-times of granulation or rotation times of the granulation systems. However, with progress in the granulation, there is a possibility that π gradually decreases by changing the ratio of fines to quasi-particles. An appropriateness of the assumption must be ultimately evaluated by experimental procedures, in addition to a clarification of the physical meanings.

Figure 9 shows the simulation results using the granulation matrixes described in Eq. (27) under the constant ore granulation properties, π=0.4(−). In case of using B, granulation phenomena are influenced by initial ore size distribution if numbers of the granulation times are limited. The granulation gradually proceeds, with increase in numbers of the unit granulation operation. Finally, ores are
completely granulated due to the operations. As a result of the granulation simulation, size of the granulated quasi-particles becomes larger more than the maximum sieve mesh, 20.7 mm in size. In case of $B_2$, all probability elements constituting the matrix are the same, $r=0.10(-)$ under the condition described in Eq. (28). In this case, as both granulation and disintegration of quasi-particles occur by the simultaneous probability during the operation, weight ratio of each sized quasi-particles focuses on the same value, $g(i)=0.1(-)$. When matrix $B_1$ that is conversely composed of $B_2$ is used for the simulation, quasi-particles continue to be disintegrated by progressing the operation. Size of the quasi-particles focuses on the minimum sieve meshes less than 0.125 mm in size. This means that quasi-particles reduce their sizes due to the matrix $B_3$, in spite of the granulation operation. This mathematical procedure is approximately the same as that of the grinding model reported by Broadbent et al.\textsuperscript{12)}

The proposed model, therefore, provides size distribution of quasi-particles, $G$, according to introducing the granulation and disintegration matrix, $B$. There are no cases that size of all quasi-particles becomes larger than the maximum sieve mesh, $B_i$ or passes the minimum one, $B_j$ in a commercial granulation plant. Actual granulation matrix $B$ used in the mathematical model accordingly exists between the matrix $B_i$ and $B_j$. Broadbent et al.\textsuperscript{12)} propose the empirical formula of the grinding matrix, $B$ by grinding tests. Similar mathematical procedure may be introduced to the granulation model, although the physical phenomena are different. Detailed discussions are left over as a future problem to be solved.

4.1.2. Granulation Property Depending on Ore Characteristics

Figure 7 clarified that the influence of overall granulation property of ores, $\pi$ on the size distribution of quasi-particles. With a decrease in the value of $\pi$, it takes relatively long times to attain the final size distribution of quasi-particles which are defined by the granulation matrix $B$. These granulation parameters are, therefore, regarded that $B$ defines the final size distribution and $\pi$ determines a granulation rate until attaining the final distribution.

Figure 10 shows the simulation results showing influence of granulation operation times on the change of matrix $B'$ obtained from Eq. (21), using the initial granulation parameters described in Case 1 of Table 2. With the increase in the numbers of operation times, the matrix showing both granulation and disintegration of quasi-particles tends to become similar. From a viewpoint of the probability theory, this tendency depends on a transition probability matrix $B$ under the condition of Eq. (13). In case of the transition probability matrix, probability elements after the operation of $(N+1)$ times, $q_{ij}^{(N+1)}$ is only influenced by the elements of $q_{ij}^{(N)}$, as described in Eq. (29) based on Eq. (20).

$$q_{ij}^{(N+1)} = \sum_{r=1}^{k} g_{ir}^{(N)} q_{ri}^{(N)} \quad \cdots \quad (29)$$

![Fig. 9. Relation between granulation matrix and calculated size distribution of quasi-particles.](image)

![Fig. 10. Influence of granulation operations on the change of the granulation matrix.](image)
Equation (29) means that the matrix $B$ is regarded as Markov chain, due to the change of the elements $q_{ij}$. If the elements are controlled by Markov chain process, the elements converge on following constant ones by the probability theory:

$$p_i = 1.0$$

As shown in Fig. 10, when the granulation operation exceeds more than 3-times ($N \geq 3$), $q'_{ik}$ ($k$: constant) approaches to a probability column vector on the basis of Eq. (29). Therefore, simulation result, such as the result in Fig. 7, by the present probability model necessarily is focused on a constant column vector $G$, according to the initial granulation matrix. From a viewpoint of probability model, the present mathematical model leads that the simulation result of the size distribution after a large number of granulation operations necessarily approaches a constant value, $G$ without divergence.

The simulation results made clear that there are an intimate correlation between the granulation parameters and granulation characteristics. It is, accordingly, very important to establish sieving conditions of initial ores incompletely granulated and quasi-particles after every granulation operation.

4.2. Concept of Application of the Model to the Control of Granulation Operation

Granulation operation using a large amount of fine ores frequently fluctuates by changing the ore pile, even under the constant operating conditions. Sintering operation inevitably shifts unstably by the fluctuation. It is true that there are many reasons for taking place it, but physical and mineralogical properties of ores seem to be one of the main reasons. The fluctuation results from the change of mixing ratio of ores having different granulation properties, even if fluctuation of size distribution and chemical compositions of the pile are controlled to minimize. As for the granulation model, the fluctuation corresponds to the change of the parameters, $q_{ij}$ and $\pi$, used in the model. In order to reduce the fluctuation, it is necessary to introduce the operation considering the granulation characteristics of typical ores. From the viewpoint of simulation methods for reducing the fluctuation, the granulation parameters must be established to clarify quantitatively the granulation phenomena by granulation tests. Granulation characteristics of the pile, composed of many kinds of ores, can estimate by the granulation model using the mixing ratio of ores and these obtained parameters.

Suzuki et al. clarified experimentally the influence of pellet feeds in the sinter mix on the size of the quasi-particles by the consideration of their granulation characteristics. Figure 11 shows a concept of granulation control method using the model referring to the previous study. The method is based on an assumption that the granulation parameters of the pile are given from those of each mixing ore and the mixing ratio. The granulation parameters, $\pi$ and $q_{ij}$ are given from following equations considering numbers of ore kinds ($m$) and their mixing weight ratios ($w_1, w_2, w_3, \ldots, w_m$) under the assumption of constant ore densities.

$$\pi = \sum_{i=1}^{m} (w_i \cdot \pi_i), \quad q_{ij} = \sum_{i=1}^{m} (w_i \cdot q_{ij})$$

where; $\sum_{i=1}^{m} w_i = 1.0$ 

Equation (31) means mathematical strictness that $q_{ij}$ obtained from the sum of the product given by multiplying scalar $w_i$ by matrix $(q_{ij})$ is the same as that of Eq. (13). Mass conservation between ores and quasi-particles by the matrix calculation with Eq. (20), accordingly, is established before and after the granulation operation. Since the granulation characteristics of the mixed ores seem to depend on the mixing ratio of each ore possessing the different granu-
ulation parameters, the granulation parameters of main ores composing the pile must be determined experimentally. Following experimental granulation tests will clarify the parameters under the same dynamic similarity\(^{10}\) as that of commercial granulation plants: (1) Parameter, \(\pi\) can be determined by observing the change of mean size or cumulative distribution curve of the ore, because it is regarded as an index of the granulation rate. (2) As the quasi-particles converge constant size distributions after a large number of rotation times of the mixer, parameter \(q_{ij}\) can be estimated by the final distribution. When granulation conditions such as addition of binder or moisture are changed, the granulation parameters, \(\pi\) and \(q_{ij}\), must be reconsidered, because Eq. (31) can be applied under the same granulation conditions.

Columnar vector, \(\mathbf{G}\) indicating size distribution of the quasi-particles at the exit of a drum mixer is estimated giving \(\mathbf{F}\) of the mixed ores and retention time which is predicted by charging tracers with the ores in the mixer. It is well known that proper addition of moisture or binder in the mixed ores can improve the granulation characteristics, due to the change of the interfacial properties of the ores. The present model with modified granulation parameters, which are obtained experimentally as mentioned above, can evaluate this improvement. This model, therefore, shows a possibility to estimate the granulation phenomena and indicates the direction of the suitable granulation conditions before using the pile. To improve the accuracy of the model, it is necessary in future to establish the prediction methods of the granulation parameters.

4.3. Problems of the Proposed Mathematical Model to Be Solved in Future

Since the proposed model deals with only ore granulation phenomena based on the parameters, \(\pi\) and \(q_{ij}\), the effects of size and operating conditions of the granulator must be considered by another methods. If the conditions of commercial granulation plant operation are given, the Froude number\(^{15}\) is determined taking account of diameter of the granulator and rotating speed. The granulation parameters, thus, can be estimated by experimental tests under the same Froude number condition as that of the commercial operation.

Although this model seems to be effective for the evaluation of the granulation phenomena, there are still possibilities that the model limits its application by the following reasons: (1) Possibilities of changing the granulation parameters according to the type of granulators and their dimensions, in spite of arranging the dimensionless number. (2) Difficulties of introducing Eq. (31) by ore characteristics. (3) Dependence of the granulation parameters on the granulation time or ore characteristics, in spite of the independence, as described in Sec. 4.1.1. Both experimental and operational granulation trials are needed for evaluating these possibilities.

5. Conclusion

Supposing import of large amounts of fine ores containing crystallized water, ranging from 4 to 6%, occurred from Australia in near future, an advanced granulation model was proposed to use these ores effectively in the sintering process. The granulation model is based on the probability theory assuming the granulation can be regarded as a kind of probability phenomena. Simulation results using appropriate granulation parameters basically agree with those from the granulation operation. Further prediction method of the granulation parameters used in the model is quite important for improving the precision of the granulation simulation.