Analysis of Chatter in Tandem Cold Rolling Mills

Yukio KIMURA, Yasuhiro SODANI, Nobuo NISHIURA,¹⁾ Naoki IKEUCHI²⁾ and Yutaka MIHARA³⁾

Materials & Processing Research Center, NKK Corporation, Kokan-cho, Fukuyama, Hiroshima-Pref. 721-8510 Japan.

1) Keihin Works, NKK Corporation, Minamiwatarida-cho, Kawasaki-ku, Kawasaki 210-0855 Japan.

2) Fukuyama Works, NKK Corporation, Kokan-cho, Fukuyama, Hiroshima-Pref. 721-8510 Japan.

3) Faculty of Engineering, Kagawa University, Hayashi-cho, Takamatsu, Kagawa-Pref. 761-0396 Japan.

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This paper is concerned with chatter in tandem cold rolling mills, which has been recognised to have close relation with friction conditions in roll bites. However, the relation between rolling conditions and the stability of vibration has not been clearly understood. Therefore, an attempt is made to understand how the rolling conditions can be related with vibration phenomena, through numerical simulations.

Firstly, a theoretical model is constituted to simulate the vibrational behaviour of a five-stand continuous rolling mill, and the stability of the mill vibration is evaluated, introducing a disturbance to the static rolling conditions. The results of the calculation show that the vibration is greatly influenced by rolling speed and friction coefficient. With increase in rolling speed, the mill vibration tends to be self-excited. Also, the results show that optimal range of friction coefficient exists in which the vibration is damped and the mill is stable against the disturbance.

Secondly, a simple self-excited vibration model is proposed and the stability index is proposed for further understanding of the phenomena. An analytical approach is made to predict the stability of the rolling process. With this model, it is shown that the rolling force and the delay of the response to the change in gap have a major effect on self-excited vibration. Also, the calculated optimal range of friction coefficient in the continuous rolling simulation can be explained by using the analytical model.

KEY WORDS: cold rolling; chatter; simulation; friction coefficient; stability of vibration.

1. Introduction

Chatter has been recognised as major restriction for the increase in productivity of cold rolling processes, limiting the rolling speed for thin steel strips with high strength. The relevant problems and the possible causes have been widely reported. Bending vibration caused by surface defects on backup rolls and torsion vibration of the drive system are the examples.^{1,2)} The most likely instability caused in tandem cold rolling mills is due to vertical oscillations of rolls, and the typical frequency of chatter ranges from 150 to 250 Hz. Also, it has been recognised to have close relation with friction conditions in roll bites.

However, the influence of friction on chatter has not been clearly understood and the conclusions reached in investigations reported in literature are sometimes contradictory. Some investigations indicate low friction by excess lubrication causes chatter, and others suggest high rolling force leads vibration instability.

For example, Suga *et al.* measured temperature of the strip, which influenced friction coefficient, and concluded that chatter is caused by slip in the roll bite due to excessive lubrication.³⁾ Also, Chefneux *et al.* reports experimental findings at a five-stand tandem mill for tinplate, where chatter generally starts at No. 4 or 5 stand and appears more frequently with low rolling force by using polished rolls or excess lubrication.⁴⁾ On the other hand, Yaritha *et al.* investi-

gated chatter vibration at a three-stand tandem mill and they concluded that chatter occurs at reductions close to the limit for rolling, when coupled with inadequate lubrication.⁵⁾ Furukawa *et al.* suggested that poor lubrication, which increases gauge and decreases the bite angle, lead to a vicious cycle of load change which causes the self-excited vibration.⁶⁾ Chefneux *et al.* also reported an occasion that sudden reduction of lubrication caused vibration instability in an experimental operation.⁴⁾

Then, it appears either too low or too high friction increases the risk of chatter and there must be optimum conditions for the friction where chatter can be prevented. Then, practical action can be lead by the empirical knowledge in production mill operations, adjusting lubrication conditions to the optimum range.

However, from the theoretical viewpoint, no satisfactory explanation has been made for the existence of the optimum friction condition. Then, the questions are how the critical conditions can be identified, and how they change with rolling speed, progress of roll wear, rolled materials and other rolling conditions. Also, more fundamentally, since chatter is a phenomenon caused by vibration instability, the problem to be clarified is how the friction and the lubrication conditions can be related with such vibrational behaviour of rolling mill system.

Several theoretical approaches have been made to investigate the mechanisms and to prevent chatter by using

mathematical vibration models.^{4,7-11)} However, most of the attempts did not succeed in predicting the existence of the optimum range of friction conditions. Only a report by Kong *et al.* shows such a calculation result, indicating both too high and too low friction coefficients increase the risk of vibration instability, but no further explanations of the causes were mentioned.⁹⁾

Therefore, the purpose of the paper is to investigate the fundamental mechanism of chatter, as well as to understand how the rolling conditions can be related with the vibration stability, especially dependence on friction conditions. Firstly a theoretical model to simulate the vibration behaviour of a five-stand continuous rolling mill is presented. The mill stands are represented as a mass-spring system similar to other analyses.⁷⁻⁹⁾ In order to calculate the rolling force and material velocity, a gap model proposed by Pawelski *et al.*^{10,11)} is modified into a simplified form. The model describes the dynamic response of the rolling force and the material velocity with oscillational change in the roll gap. With this numerical model, the stability of mill vibration is evaluated in order to see whether the vibration is damped or self-excited in a certain rolling condition. Also, the results of the simulation are compared with a production mill data. Secondly, a simple self-excited vibration model is proposed and the stability index is defined for further understanding of the phenomena and for explaining the possible reason why the optimum friction range exists.

2. Mathematical Models

2.1. Continuous Rolling Model

In order to simulate vibration behaviour of a five-stand rolling mill, continuous rolling is modelled, as schematically represented in **Fig. 1**. The roll gap model provides the rolling force response and the material velocity. With the tension model, inter-stand tension is calculated from the elongation of material between the adjacent stands. The material transfer model accounts for the time delay as the material moves to the next stand. The stand model depicts the vibration behaviour of the mill elements in response to the rolling force. Each model in the figure is correlated with others, and vibration in a certain stand has influence on rolling conditions of the adjacent stands by the change in thickness, tension or material velocity.

2.2. Roll Gap Model

For the calculation of the rolling force and the material velocity, a theoretical model proposed by Pawelski *et al.*^{10,11)} was modified into a simplified form. The model describes the dynamic response against the periodical change of roll gap, which results in the damping effect of material in the roll bite to the whole system. **Figure 2** shows the geometry of a roll bite when the roll gap oscillates. The idea is that the exit position of the bite moves forward or backward when the gap changes with rather high speed in vertical direction. If the change of the gap is quick or speed of the strip is slow, the extension or the reduction of the bite length is significant, resulting large change in rolling force. Therefore, the response of the rolling force is determined by not only static conditions but also relationship between the vertical speed of the vibrating rolls and the material

Fig. 1. Schematic of continuous rolling model.

Fig. 2. Mathematical model of roll gap.

travelling speed, which means the bite has damping effect in the system.

With this idea, the model is described as follows. Assuming the contour of the roll gap is parabolic and the condition of vanishing strain rate gives the exit position of the gap as the solution of the following quadratic equation,

$$x^2 + 2 \left\{ \frac{u_{in} h_{in}}{(\partial h_a / \partial t)} - x_{in} \right\} x - h_a R' = 0 \quad \text{.....(1)}$$

The entrance position of the roll gap is given by,

$$x_{in} = \sqrt{\{h_{in}(t) - h_a(t)\} \cdot R'(t)} \quad \text{.....(2)}$$

Pressure distribution in the bite can be derived from the momentum equation as,

$$\frac{\partial p}{\partial x} \mp \frac{2\mu p}{h} - \frac{1}{h} \cdot \frac{\partial}{\partial x} (h k_f) = 0 \quad \text{.....(3)}$$

Equation (3) is a close resemblance to v. Karman differential equation, but the expression is different with using partial derivatives. Rolling force can be calculated by integrating the pressure throughout the bite. Since it takes quite long computational time, an approximate expression for the pressure is derived in a similar manner that the Bland & Ford equation¹²⁾ is derived from the v. Karman differential equation. The pressure distribution in the forward and backward slip region can be written as,

$$p^+ = \left(\frac{k_f h}{h_{out}} \right) \left(1 - \frac{t_f}{k_f} \right) \exp\{\mu(H - H_{out})\} \quad \text{.....(4a)}$$

$$p^- = \left(\frac{k_f h}{h_{in}} \right) \left(1 - \frac{t_b}{k_f} \right) \exp\{\mu(H_{in} - H)\} \quad \text{.....(4b)}$$

where,

$$H = 2 \sqrt{\frac{R'}{h_a}} \cdot \tan^{-1} \left(\phi \sqrt{\frac{R'}{h_a}} \right) \dots\dots\dots (5)$$

Also, the neutral point is calculated by,

$$\phi_n = \sqrt{\frac{h_a}{R'}} \tan \left(\frac{H_n}{2} \cdot \sqrt{\frac{h_a}{R'}} \right) \dots\dots\dots (6)$$

where,

$$H_n = \frac{H_{in} + H_{out}}{2} - \frac{1}{2\mu} \ln \left\{ \frac{h_{in}}{h_{out}} \left(\frac{1 - t_f / k_f^{out}}{1 - t_b / k_f^{in}} \right) \right\} \dots\dots (7)$$

In the following simulation, the Eqs. (4a) and (4b) are used to calculate the rolling force, integrating the pressure along the roll bite, which can be derived from the Eqs. (1) and (2). Also, the inlet and outlet velocity of the strip can be calculated by using the Eq. (6), coupled with a continuity law of material.

2.3. Model of Rolling Mill Stand

A rolling mill stand is represented by a mass-spring system with five degree of freedom to calculate the vertical vibration, as shown in **Fig. 3**. It consists of five mass elements, corresponding to the top and bottom work rolls, the top and bottom backup rolls and the housing. The elastic deformation behaviour of the rolls is represented by springs. Since the rolling force can not be represented by a spring due to its non-linear behaviour, it is applied as an external force to the system. Then, the equations of motion are given by,

$$\begin{cases} m_1 \ddot{y}_1 = -k_1 y_1 + k_2 (y_2 - y_1) \\ m_2 \ddot{y}_2 = -k_2 (y_2 - y_1) - k_R y_2 - P(t) \\ m_3 \ddot{y}_3 = -k_R y_3 + k_3 (y_4 - y_3) + P(t) \dots\dots\dots (8) \\ m_4 \ddot{y}_4 = -k_3 (y_4 - y_3) + k_4 (y_5 - y_4) \\ m_5 \ddot{y}_5 = -k_4 (y_5 - y_4) - k_5 y_5 \end{cases}$$

The spring constants k_2 and k_3 are determined by the elastic contact deformation of two cylinders.

3. Numerical Simulation

3.1. Method of Calculation

In order to simulate vibration behaviour by using the above models, a computer program is coded with a simulation language for continuous system. Then, the second-order differential equations, Eq. (8) are solved numerically by using fourth-order Runge-Kutta method, and the incremental time step is 0.02 ms.

With this computer program, a disturbance is introduced to the static rolling conditions, in order to evaluate the stability of mill vibration. It is of special interest to know whether the vibration is damped or self-excited in a certain rolling condition.

The standard conditions used for the simulation are shown in **Table 1**. The width of strip is 900 mm, and the distance between the adjacent stands is 4.5 m. The deformation resistance of the material to be rolled k_f (MPa) is ex-

Fig. 3. Mill stand model.

Table 1. Calculation conditions.

Stand	#1	#2	#3	#4	#5
Entry thickness h_{in} , mm	1.90	1.31	0.76	0.45	0.28
Exit thickness h_{out} , mm	1.31	0.76	0.45	0.28	0.19
Back tension t_b , MPa	98	137	137	137	137
Front tension t_f , MPa	137	137	137	137	98
WR radius R_w , mm	260	253	272	276	291
BUR radius R_b , mm	700	700	700	700	700
WR mass m_2, m_3 , kg	7700	7700	7700	7700	7700
BUR mass m_1, m_4 , kg	53000	53000	53000	53000	53000
Housing mass m_5 , kg	70000	70000	70000	70000	70000
Spring Const. k_R , MN/mm	92.8	93.6	88.7	86.2	86.3
Spring Const. k_2, k_3 , MN/mm	46.3	47.0	46.3	46.1	46.7
Spring Const. k_1, k_4 , MN/mm	15.5	15.5	15.5	15.5	15.5
Spring Const. k_5 , MN/mm	32.3	32.3	32.3	32.3	32.3
Friction coefficient μ	0.075	0.045	0.025	0.018	0.015
Roll velocity V_r , m/s	5.1	8.8	14.9	23.9	35.0

pressed by the following equation,

$$k_f = 811.4 (\varepsilon + 0.002)^{0.24} \dots\dots\dots (9)$$

The external disturbance introduced to the calculation is a small reduction of the entry gauge, 30 μ m, at the first stand for a small period of time, 10 ms, from the static condition. For the series of calculation, the rolling speed and the friction coefficient at the last stand are changed, and the other conditions are fixed.

3.2. Results of Calculation

The simulation results are shown in **Figs. 4** and **5**. The calculation stops after 3 sec. Although the amplitude does not become large enough to generate serious thickness deviations, it is possible to judge whether the vibration will be diverged or damped for the longer time period.

Figure 4 shows the vibration behaviour of each mass element when the rolling speed is 35 m/s and the friction coefficient of the last stand μ_5 is 0.017. The vibration of each mass element in the down stream stands increases with time. It can be seen from Fig. 4(f) in which time scale is magnified that its vibration mode is that the top work roll and the top backup roll oscillate in the same phase, which is in the opposite phase with respect to the housing. The bottom work and the bottom backup rolls also vibrate together, but their phase is opposite to that of the top rolls, representing the third-octave mode of vibration in five-degree of freedom system. The calculated frequencies are about 170 Hz.

 Fig. 4. Simulation results for rolling speed 35 m/s and friction coefficient $\mu_s=0.017$.

 Fig. 5. Simulation results for rolling speed 35 m/s and friction coefficient $\mu_s=0.015$.

 Fig. 6. Simulation results for rolling speed 35 m/s and friction coefficient $\mu_s=0.012$.

When the friction coefficient of the last stand is changed to 0.015, the vibration is damped, as shown in Fig. 5. It means the continuous rolling mill system becomes more stable, if the friction condition is properly controlled. Also, the interesting result obtained by the calculation is shown in Fig. 6. When the friction coefficient μ_s is reduced further to 0.012, the vibration is self-excited again. Then, the calculation results indicate there is a certain range of friction condition where the mill vibration is stable.

Series of the calculation are summarised in Fig. 7, which shows the influence of the friction coefficient of the last stand and the rolling speed on the vibration stability. The vibration tends to diverge as the rolling speed increases. The allowable range of the friction coefficient of the last stand to ensure the stability becomes narrow under high-speed rolling. Neither too high nor too low friction coefficient stabilises the vibration.

Fig. 7. Influence of rolling speed and friction coefficient μ_5 on vibration stability.

3.3. Comparisons with Production Mill Data

The calculation results can be evaluated by comparing with data obtained in a production rolling mill. The data have been obtained in No. 2 TCM in Fukuyama Works in NKK, which is a five-stand tandem cold rolling mill with 4-high stand for the production of tin gauge strips. The results of the measurement in the mill are shown in Fig. 8. The data are selected from those the strip thickness is less than 0.25 mm and the temper grade is T5, where the chatter sometimes causes trouble to the operation. The friction coefficient of the last stand is calculated by Bland & Ford equation using the measured forward slip coupled with the rolling force.

It can be seen from the figure that the chatter tends to occur when the rolling speed is high and the friction coefficient of the last stand is either too high or too low at high rolling speed. This tendency is consistent with the results of calculation shown in Fig. 7. It is convincing that the results of the simulation show the correct tendency in the production mill system.

However, it is not possible to conclude that the simulation provides quantitatively right results at this stage. Most of the rolling conditions of the mill operation here are similar to ones listed in Table 1, but the friction coefficients are not completely the same with the calculation conditions. Since the friction coefficient is strongly influenced by lubricant, rolling speed and roll surface conditions, it is quite difficult to realise the same conditions for all the strips.

Then, it is necessary to be careful in looking at the production mill data that not only the friction coefficient of the last stand but also friction coefficient of the other stands may be different. Further comparisons and the modifications should be continued to justify the evaluation of the simulation.

4. Discussions

The results of the five-stand continuous rolling simulation show that the vibration is greatly influenced by rolling speed and friction coefficient of the last stand. When the rolling speed increases, the mill vibration tends to be self-excited. The optimal range of friction coefficient exists in which the vibration is damped. These tendencies are consistent with those observed in production rolling mills. However, the following should be examined for quantitative estimation. Firstly, the spring constants and damping coefficients

Fig. 8. Friction coefficient at which chatter occurred in a production tandem mill.

Fig. 9. Self-excited vibration model.

of the production rolling mill should be accurately estimated. Secondly, the dependency of the friction coefficient on rolling speed should be considered. Also, the change in friction coefficient arising from the change in the bite angle should be incorporated.

Despite the simple approximation employed in the present simulation, one of the most interesting results obtained here is the dependency of the vibration stability on friction coefficient of the last stand. In order to understand the chatter mechanism in more details, a simple self-excited model is proposed.

4.1. Self-excited Vibration Model

The motion of a simple vibration system as shown in Fig. 9 is governed by the equation,

$$m\ddot{y}(t) + c_M\dot{y}(t) + k_M y(t) = \Delta P(t) \quad \text{.....(10)}$$

Assuming that the response of the rolling force to the vertical motion of the roll is that of a spring element, and that a slight time delay exists in the response, the external force can be approximated as,

$$\Delta P(t) = -k_p y(t - T_d) \approx -k_p y(t) + k_p T_d \dot{y}(t) \quad \text{.....(11)}$$

Then, Eq. (10) is written as,

$$m\ddot{y}(t) + (c_M - k_p T_d) \dot{y}(t) + (k_M + k_p) y(t) = 0 \quad \text{.....(12)}$$

The self-excited vibration occurs when the damping coefficient becomes negative as,

$$c_M - k_p T_d < 0 \quad \text{.....(13)}$$

The conditions for self-excitation can be rephrased from rolling conditions as: the damping effect of the hydraulic jacks is small, and the force change against a vertical displacement of the rolls and its delay in response are large.

 Fig. 10. Schematic of calculation method of stability index Q in Eq. (14).

In order to estimate the influence of rolling conditions on the stability of the system, the stability index Q is defined as,

$$Q \equiv k_p T_d = \left| \frac{\Delta P}{\Delta S} \right| \cdot T_d \dots \dots \dots (14)$$

The value of the index can be determined by the rolling conditions, not influenced by rolling mill configuration. Then, the value indicates the stability against the vibration that is inherent in the rolling conditions and independent of the rolling mill facility. As the Q value increases, the system tends to be unstable. The main reason for the phase delay in the force response is caused by the difference in phase of inter-stand tension against the phase of the roll gap.

4.2. Evaluation of Stability

Assuming a continuous rolling condition of two stands as shown in Fig. 10, the response of rolling force against cyclic change in roll gaps with the same amplitude and frequency. Then, the corresponding stability index Q , which is defined in the Eq. (14), for each stand Q_1 and Q_2 can be calculated. The details of the calculation method are shown in Appendix.

With the calculation conditions in Table 2, which correspond to the typical condition of the last two stands in a tandem cold rolling, the calculated results of the stability index are shown in Fig. 11. It can be seen from the figure that when the friction coefficient of the second stand is low, the value of the parameter at the second stand Q_2 is larger than the first stand Q_1 . It means the second stand is less stable. On the other hand, if the friction coefficient of the second stand is high, Q_2 is smaller than Q_1 of the first stand, suggesting the first stand is less stable. Also, there is a certain value of friction coefficient μ_2 where the both Q_1 and Q_2 take small values. Therefore, if the friction coefficient of the second stand μ_2 falls into such a range, both first and second stands may be stable.

The evaluation based on the stability index Q , which is derived from the simple self-excitation model, makes the explanation possible that the optimal range in friction coefficient exists in high rolling speed. It can be represented as; when the friction coefficient of a downstream stand is low, the stand itself becomes vibrationally unstable, and when

 Table 2. Calculation conditions of stability index Q .

Stand	#1	#2
h_{in} (mm)	0.41	0.261
h_{out} (mm)	0.261	0.183
t_b (MPa)	137	137
t_f (MPa)	137	98
R_w (mm)	276	291
μ	0.012-0.014	0.009-0.02
V_r (m/s)	24.8	35.0

amplitude of gap change, Y_1, Y_2 (μ m)	1.0
difference in phase between 1st and 2nd stand, α (deg)	180
width of strip, W (mm)	900
thickness of mother strip (mm)	1.90
deformation resistance, k_f (MPa)	Eq. (9)
inter-stand distance, L_1, L_2, L_3 (m)	4.5

 Fig. 11. Influence of friction coefficient on stability index Q .

the friction coefficient of a downstream stand is high, the first stand becomes unstable. That is the reason why the vibration is damped at a certain range of friction coefficient of the last stand in the five-stand continuous rolling simulation in Fig. 7.

5. Conclusions

A continuous rolling simulation was conducted to evaluate vertical vibrations in a five-stand tandem mill. The following findings were obtained: As the rolling speed increases, rolling mill vibrations become less stable. The optimum range of the friction coefficient for the last stand exists with regards to the stability.

These phenomena were examined using a self-excitation

model with a single degree of freedom. The results indicated that the stability of a rolling mill system is affected by the phase difference between the vertical motion of rolls and the force response. The stability of a tandem cold rolling mill is determined by the rolling conditions, especially the friction coefficient. Neither too high nor too low friction coefficient stabilises the self-excited vibration in a tandem rolling.

Nomenclature

c_M :	Damping coefficient of rolling mill (MN/m)
D :	Damping coefficient in roll bite (MN/m)
E :	Young's modulus of strip (MPa)
h, h_{in}, h_{out} :	Strip thickness, at entry and exit (m)
h_a, h_n :	Thickness at minimum gap and neutral point (m)
k, k_M :	Spring constant of mill stand (MN/m)
k_f :	Deformation resistance (MPa)
k_p :	Spring constant of rolling force response (MN/m)
L :	Distance between adjacent stands (m)
m :	Mass of roll (kg)
P :	Rolling force (MN)
p :	Pressure in roll bite (MPa)
R_w, R_b, R' :	Work roll radius, back-up roll radius, flattened work roll radius (m)
S :	Roll gap (m)
t_b, t_f :	Back and front tension (MPa)
T_d :	Time delay of rolling force response against gap (s)
u, u_{in}, u_{out} :	Strip speed, at entry and exit (m/s)
V_r :	Circumference velocity of work roll (m/s)
W :	Width of strip (m)
x_{in}, x_{out} :	Entrance and exit position of roll bite (m)
y :	Displacement of roll mass centre from the statically balanced position (m)
ϕ, ϕ_n :	Angle between roll and strip, neutral point (rad)
μ :	Friction coefficient
ε :	Cumulative strain
Subscript	
i :	Stand number
in, out:	Inlet, exit of roll bite

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Appendix

The stability index Q defined in Eq. (14) can be calculated by using the following assumptions.

(a) Cyclic changes of the roll gap at the first and the second stand are given by,

$$\Delta S_1 = Y_1 \sin(\omega t - \alpha) \dots\dots\dots(A-1)$$

$$\Delta S_2 = Y_2 \sin \omega t \dots\dots\dots(A-2)$$

where the frequencies are the same, and the phase deference is given by α .

(b) The entry thickness of each stand is constant, even if the delivery thickness changes with time.

(c) The damping effect in roll bite is approximated as,

$$D_i = -\left(\frac{\partial P}{\partial S}\right)_i \approx \left(\frac{k_f \cdot R_w}{2V_r}\right)_i \cdot W \dots\dots\dots(A-3)$$

(d) As a boundary condition, the entry speed away from the first stand and the exit speed away from the second stand are always constant.

With the above assumptions, the deviation of exit thickness and the rolling force at each stand can be written as,

$$\Delta h_{out,i} = \Delta S_i + \Delta P_i / K_{R,i} \dots\dots\dots(A-4)$$

$$\Delta P_i = -M_i \Delta h_{out,i} - N_i \Delta t_{b,i} - O_i \Delta t_{f,i} - D_i \Delta \dot{S}_i \dots\dots(A-5)$$

where K_R is spring constant of roll flattening deformation, and

$$M_i = -\left(\frac{\partial P}{\partial h_{out}}\right)_i, \quad N_i = -\left(\frac{\partial P}{\partial t_b}\right)_i, \quad O_i = -\left(\frac{\partial P}{\partial t_f}\right)_i \dots\dots\dots(A-6)$$

Thickness change at neutral point is represented by,

$$\Delta h_{n,i} = \left(\frac{\partial h_n}{\partial h_{out}}\right)_i \Delta h_{out,i} + \left(\frac{\partial h_n}{\partial t_b}\right)_i \Delta t_{b,i} + \left(\frac{\partial h_n}{\partial t_f}\right)_i \Delta t_{f,i} \dots\dots\dots(A-7)$$

Also, the deviation of material velocities is represented by,

$$\begin{cases} \Delta u_{in,i} = V_{r,i} \cdot \frac{\Delta h_{n,i}}{h_{in,i}} \\ \Delta u_{out,i} = V_{r,i} \cdot \frac{\Delta h_{n,i} - h_{n,i} \Delta h_{out,i} / h_{out,i}}{h_{out,i}} \end{cases} \dots\dots\dots(A-8)$$

The inter-stand tension can be calculated with using the material velocities by,

$$\begin{cases} \Delta t_{b,1} = (E / L_1) \int \Delta u_{in,1} dt \\ \Delta t_{b,2} = \Delta t_{f,1} = (E / L_2) \int (\Delta u_{in,2} - \Delta u_{out,1}) dt \dots\dots(A-9) \\ \Delta t_{f,2} = (E / L_3) \int (-\Delta u_{out,2}) dt \end{cases}$$

With the Eqs. (A-1) to (A-9), each tension can be calculated, and the amplitude T_1 , T_2 , T_3 and the phase delay from the gap α , γ , λ in the following equations are determined.

$$\begin{cases} t_{b,1} = T_1 \sin(\omega t - \beta) \\ \Delta t_{b,2} = \Delta t_{f,1} = T_2 \sin(\omega t - \gamma) \\ \Delta t_{f,3} = T_3 \sin(\omega t - \lambda) \end{cases} \dots\dots\dots (A-10)$$

Then, rolling force response at the first stand can be given by,

$$\Delta P_1 = -g_1 \sin(\omega t - \eta_1) \dots\dots\dots (A-11)$$

where

$$\begin{cases} g_1 \cos \eta_1 = \frac{K_{R,1}}{K_{R,1} + M_1} (M_1 Y_1 \cos \alpha + N_1 T_1 \cos \beta \\ \quad + O_1 T_2 \cos \gamma + \omega D_1 Y_1 \sin \alpha) \\ g_1 \sin \eta_1 = \frac{K_{R,1}}{K_{R,1} + M_1} (M_1 Y_1 \sin \alpha + N_1 T_1 \sin \beta \\ \quad + O_1 T_2 \sin \gamma - \omega D_1 Y_1 \cos \alpha) \end{cases} \dots\dots\dots (A-12)$$

The rolling force response at the second stand can be written as,

$$\Delta P_2 = -g_2 \sin(\omega t - \eta_2) \dots\dots\dots (A-13)$$

where

$$\begin{cases} g_2 \cos \eta_2 = \frac{K_{R,2}}{K_{R,2} + M_2} (M_2 Y_2 + N_2 T_2 \cos \gamma + O_2 T_3 \cos \varepsilon) \\ g_2 \sin \eta_2 = \frac{K_{R,2}}{K_{R,2} + M_2} (M_2 T_2 \sin \gamma + O_2 T_3 \sin \varepsilon - \omega D_2 Y_2) \end{cases} \dots\dots\dots (A-14)$$

Then, the stability index Q is calculated by,

$$Q_1 = \left(\frac{g_1}{Y_1} \right) \cdot \frac{(\eta_1 - \alpha)}{\omega} \dots\dots\dots (A-15)$$

$$Q_2 = \left(\frac{g_2}{Y_2} \right) \cdot \frac{\eta_2}{\omega} \dots\dots\dots (A-16)$$