1. Introduction

Rotating flow fields are known to be useful for removing non-metallic inclusions and fine bubbles from molten metal. Novel refining processes are also expected by using cylindrical vessels containing slag and molten metal suddenly set in rotation. This is because the slag and molten metal are highly mixed on the initial stage in the transient period and then separated rapidly on the final stage.

Many investigations have been carried out on the fluid flow phenomena contained in a cylindrical vessel suddenly set in rotation. In particular, much information is available on the flow establishment time of a single liquid in the laminar and transitional flow regimes. Some empirical equations for the flow establishment time were proposed. However, investigations on the transient behavior of stratified two liquid layers are limited. When the difference between the kinematic viscosities of the two liquids is large, the interfacial area between the two liquids becomes much greater than its initial value. The interfacial area is highly increased by using a baffled vessel. This phenomenon is beneficial for designing a novel refining process.

In a previous study, the authors proposed an empirical equation for the transient period of stratified two liquid layers contained in a cylindrical vessel suddenly set in rotation introducing an apparent kinematic viscosity, \( \nu_{\text{mix}} \). The diameter and height of the test vessel were fixed to be 46 mm and 120 mm, respectively. In this study additional two vessels were used to investigate the effect of the vessel diameter on the flow establishment time in the laminar flow regime. In addition, the difference between the densities of two liquids was changed over a wide range to clarify a change in the interfacial area in the transient period. Also, we focused on the effect of a baffle on the mixing and separation of two liquids.

2. Experimental Apparatus and Procedure

Figure 1 shows a schematic of the experimental apparatus. The inner diameter of the cylindrical vessel, \( D \), was 25 mm, 46 mm, and 74 mm, while the height of the liquid column, \( H \), was fixed to be 120 mm. The aspect ratio, \( H/D \), therefore ranged from 1.62 to 4.80. The cylindrical test vessel was enclosed with another vessel of a square cross-section.
tion and water was filled between the two vessels to decrease the distortion effect as much as possible. The two vessels were made of transparent acrylic resin. Immiscible two liquids were filled in the cylindrical vessel and the vessel was suddenly rotated with a motor. The physical properties of the liquids are listed in Table 1. The volume ratio, \( R = \frac{V_2}{V_1} \), was changed from 0.5 to 2.0, \( V_1 \) and \( V_2 \) being the volumes of the upper and lower liquids, respectively. The subscripts, 1 and 2, denotes the upper and lower liquids, respectively. Combinations of the upper and lower liquids are listed in Table 2. The density ratio, \( \rho_1/\rho_2 \), ranged from 0.513 to 0.937 and the kinematic viscosity ratio, \( \nu_1/\nu_2 \), ranged from 0.0714 to 100. These ratios nearly cover the values encountered in the real processes. The angular frequency of rotation, \( \omega \), was varied from 2.36 rad/s to 47.14 rad/s.

The upper wall of the rotating vessel is, of course, needed when the vessel is applied to the real processes. However, direct contact of the upper liquid layer (slag) with the upper wall is necessarily required because large deformation of the interface between the upper and lower liquids takes place in the presence of the gas phase above the upper liquid layer. Also, the choice of the vessel size and the rotating speed of the vessel is of essential importance in practical applications. Investigations on this subject should be carried out in a future study.

The Reynolds number, Re, and the Eckman number, Ek, are defined as follows:

\[
\begin{align*}
\text{Re}_1 &= \frac{\rho \omega v_1}{\nu_1} \sqrt{\frac{W}{2}} \\
\text{Re}_2 &= \frac{\rho \omega v_2}{\nu_2} \sqrt{\frac{W}{2}} \\
\text{Ek}_1 &= \frac{v_1}{(\omega H)^2} \\
\text{Ek}_2 &= \frac{v_2}{(\omega H)^2}
\end{align*}
\]

where \( R (=D/2) \) is the radius of the vessel and \( \nu \) is the kinematic viscosity of liquid. These dimensionless parameters were introduced to correlate the flow establishment time. Under the present experimental conditions, Re_1 and Re_2 ranged from 3.53 to 157.9, and Ek_1 and Ek_2 ranged from 1.47×10^{-6} to 4.12×10^{-6}.

The tangential velocity component of liquid flow, \( v_{\text{tang}} \), was measured with PIV (Particle Image Velocimetry). The PIV system is schematically shown in Fig. 2. Seeding particles made of high porous polymer were dispersed in the bath. The density and diameter of particles are 1013 kg/m^3 and 75–150 \( \mu \)m respectively. The particles on a horizontal plane were illuminated with a laser sheet, and the motions of them were recorded with a CCD camera. The position at which the liquid in the vessel reaches a steady state most slowly is around the middle height of the liquid layer of smaller kinematic viscosity. Accordingly, the laser sheet was set so as to pass through that position. The deformation of the interface between the two layers was observed from the side of the vessel with a video camera at 30 frames/s.

A baffle was attached to the inner wall of the vessel as shown in Fig. 3 to enhance the mixing and subsequent separation of the upper and lower liquids. The width and the thickness were 10 mm and 3 mm, respectively.
3. Experimental Results and Discussion

3.1. Classification of the Deformation Pattern of Interface

The deformation of the interface is schematically shown in Fig. 4. We assume that the kinematic viscosity of the upper layer is much higher than that of the lower layer, while the density ratio $\rho_1/\rho_2$ is much smaller than unity. The initial interface is nearly flat (Fig. 4(a)). With a lapse of time, the interface becomes parabolic in shape and finally approaches the shape in the steady state. The momentum transferred from the side wall of the vessel to the liquid is dependent on the kinematic viscosity of the liquid. The transfer rate increases with an increase in the kinematic viscosity. As a result, on an earlier stage of rotation the momentum is transferred faster to the liquid in the upper layer, and the liquid becomes to rotate earlier than that in the lower layer. The centrifugal force acts more strongly on the liquid in the upper layer and the liquid in that layer is forced to move outward as shown in Fig. 4(b). On the contrary, the liquid in the lower layer is forced to move inward. After a while, the momentum is also completely transferred to the liquid in the lower layer. As the density of the liquid in the lower layer is higher than that in the upper layer, the centrifugal force becomes to act more strongly on the liquid in the lower layer. As a result, the lower layer is forced to move outward and the upper layer inward. Finally, a steady state is reached, as shown in Fig. 4(d).

Three types of interface deformation were observed under the present experimental conditions, as shown in Fig. 5. When the kinematic viscosity is much higher and the density is slightly lower in the upper layer than in the lower

![Fig. 4. Mechanism of liquid–liquid interface deformation.](image)

<table>
<thead>
<tr>
<th>Type</th>
<th>Condition</th>
<th>Time (s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Silicone oil-Salt water</td>
<td>$\omega=11.0\text{rad/s}$</td>
<td>t=0s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t=2s</td>
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<tr>
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<td>t=5s</td>
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<tr>
<td></td>
<td></td>
<td>t=10s</td>
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<tr>
<td></td>
<td></td>
<td>t=30s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t=60s</td>
</tr>
<tr>
<td>(b) Fluorinert-Salt water</td>
<td>$\omega=23.57\text{rad/s}$</td>
<td>t=0s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t=2s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t=5s</td>
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<td></td>
<td></td>
<td>t=10s</td>
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<tr>
<td></td>
<td></td>
<td>t=30s</td>
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<tr>
<td></td>
<td></td>
<td>t=60s</td>
</tr>
<tr>
<td>(c) Fluorinert-Silicone oil</td>
<td>$\omega=23.57\text{rad/s}$</td>
<td>t=0s</td>
</tr>
<tr>
<td></td>
<td></td>
<td>t=2s</td>
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<td></td>
<td></td>
<td>t=30s</td>
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<tr>
<td></td>
<td></td>
<td>t=60s</td>
</tr>
</tbody>
</table>

![Fig. 5. Photographs of liquid–liquid interface deformation.](image)
layer, the interface shows a projection into the upper layer on the initial stage and finally reaches a nearly flat distribution, as shown in Fig. 5(a). On the contrary, when \( n_1 \) is lower than \( n_2 \) and \( r_1 \) is much lower than \( r_2 \), the interface shows a projection into the lower layer (see Fig. 5(b)) all the way. When \( n_1 \) is higher than \( n_2 \) and \( r_1 \) is much lower than \( r_2 \), the interface shows a projection into the upper layer on the initial stage and finally shows a projection into the lower layer, as can be seen in Fig. 5(c).

Figure 6 shows a relationship between the vertical distance for the maximum projection, \( z_{\text{max}} \), and the angular frequency of rotation, \( \omega \). With an increase in the angular frequency of rotation, \( \omega \), the absolute value, \( |z_{\text{max}}| \), became large. Figure 6(b) shows the maximum values of the interfacial area, \( S_{\text{i,max}} \), calculated from the \( z_{\text{max}} \) values shown in Fig. 6(a). It is evident that \( S_{\text{i,max}} \) becomes much greater than the initial value in every case as \( \omega \) increases.

3.2. Flow Establishment Time

The flow establishment time, \( T_{S} \), is defined as a period from the start of rotation to the instance at which the tangential velocity in the layer of lower kinematic viscosity reaches 0.95 times as large as its steady value. Further details of the definition should be referred to the previous paper. As mentioned above, the deformation of the interface has a close relationship with the history of the tangential velocity component of liquid flow in the bath. The flow establishment time can also be determined from the history of the deformation. Unfortunately, it is difficult to determine the flow establishment time in a similar manner to that chosen for the tangential velocity component. The following method was chosen in this study.

Figure 7(a) shows a temporal change in the central position (\( r=0 \)) of the interface, where \( r \) is the radial distance measured from the vertical axis of the vessel. The area enclosed with the \( z \) curve, the minimum value of \( z \), \( t=0 \) and \( t=t \) is denoted by \( S_d \). With a lapse of time, \( S_d \) becomes to follow the history shown schematically in Fig. 7(b). After a steady state is attained, \( S_d \) increases linearly with respect to time, \( t \). According to the trial and error method, the 97% criterion was found to give the flow establishment time based on the deformation, \( T_{S,I} \), closest to that determined from the tangential velocity component of liquid flow, \( T_{SV} \). For example, the measured value of \( T_{S,I} \) was 31.2 s for \( D=46 \text{ mm}, \text{Silicone oil 100}, \text{Salt water}, R_v=1.0 \) and \( \omega=11.0 \text{ rad/s} \). This value is in good agreement with the value calculated based on the velocity distribution, \( T_{SV}=33.4 \text{ s} \), as shown in Fig. 7(b).

Figure 8 shows a comparison between the measured val-
ues of flow establishment time and the following empirical equation proposed previously for \( D/H \leq 100 \) mm.

\[
T_s w / H = 5.61 \frac{n_{mix}}{(w L^2)^{0.67}} \quad \text{ ...........(5)}
\]

Substitution of Eq. (10) into Eq. (9) yields

\[
T_s w / H = 3.0 \left( \frac{D}{H} \right)^{0.67} \quad \text{ ...........(11)}
\]

The measured value of \( T_s w \) are compared with Eq. (9) in Fig. 11. All the measured values of \( T_s w \) were approximated by Eq. (11) within a scatter of \( \pm 35\% \). Equation (11) is expected to be valid for a cylindrical vessel of an arbitrary cross-section.

3.4. Derivation of Empirical Equation for Flow Establishment Time (2)

Equation (11) is useful for predicting the flow establishment time, but its physical meaning is not necessarily clear. In this section, the effects of the upper and lower ends of the vessel on the flow establishment time will be discussed more in detail. We newly define the following Reynolds ratio, \( H/D \), was further introduced. The measured values for 25 and 74 mm in Fig. 9 decreased with \( v_{mix}/(oL^2) \) at the same gradient for \( D=46 \) mm. The relationship between \( T_s \omega \) and \( v_{mix}/(oL^2) \) can be expressed as follows:

\[
T_s \omega = k \left( \frac{v_{mix}}{(oL^2)} \right)^{0.67} \quad \text{ ...........(9)}
\]

where \( k \) is assumed to be a function of the aspect ratio, \( H/D \). The relationship between \( k \) and \( H/D \) is shown in Fig. 10. The following equation was derived.

\[
k = 3.0 (H/D)^{0.67} \quad \text{ ...........(10)}
\]
number, Remix, and Eckman number, Ekmix.

\[ \frac{\text{Remix}}{H} = 0.35 \left( \frac{\text{H}}{D} \right) \] ........................(15)

Combination of Eqs. (14) and (15) yields

\[ \frac{T_s}{H} = 0.35 \left( \frac{\text{H}}{D} \right) \frac{\text{Re}^{1.5}}{\text{mix}} \] ............................(16)

Equation (16) reduces to

\[ \frac{T_s}{H} = 0.175 \frac{\text{Re}^{0.5}}{\text{mix}} \] ............................(17)

According to the previous study, the effects of \( \Pi_6 - \Pi_8 \) on the flow establishment time, \( T_s \), are negligible, and these parameters are omitted in deriving empirical equations for \( T_s \).

By referring to the previous paper, the dimensionless parameters, \( \Pi_6 - \Pi_8 \), can be summarized in the following parameter.

\[ \Pi_6 = \alpha L^{1/3} / \nu_{\text{mix}} \] ..........................(28)

The quantities, \( H_1, \rho_1, \rho_2, \mu_1 \), and \( \mu_2 \), are included in \( \nu_{\text{mix}} \), as can be seen in Eqs. (6) and (8).

It is also reasonable to choose the following dimensionless parameter in place of Eq. (28).

\[ \Pi_9 = \alpha R^{1/2} / \nu_{\text{mix}} \] ..........................(29)
Equation (11) was derived by using $P_1$, $P_2$, and $P_3$, while Eq. (17) was derived on the basis of $P_1$, $P_2$, and $P_{10}$. The deviations of the measured values of flow establishment time around Eqs. (11) and (17) were approximately the same, as demonstrated in the preceding sections. Equation (17) was presented to clearly show the role of the side wall and the upper wall on the flow establishment time, $T_s$, in terms of the Reynolds number and the Ekman number. Further investigations using larger vessels and molten metals are desirable to make clear the effects of the interfacial tension and other parameters on $T_s$.

3.6. Effect of Baffle on the Transient Fluid Flow Phenomena

Figure 15 shows photographs of the bath in the baffled vessel. The interface between the silicone oil and salt water was highly deformed and the interfacial area became much greater than the initial value of approximately $\pi D^2/4$. Figure 16 compares the measured values of the flow establishment time, $T_s$, in the baffled vessel and Eq. (17) derived for the cylindrical vessel. The flow establishment time became much shorter in the presence of the baffle.

3.7. Case Study for Molten Slag and Metal

When this device is applied to real refining processes using slag–metal reactions, the slag and molten metal would be highly mixed in the first half period due to the centrifugal force and the interfacial area between them would become more than ten times as large as the initial value. In addition, the slag and molten metal thus mixed could be completely separated in the second half period. This device is therefore useful for the development of a novel refining process in which rapid mixing and subsequent separation of molten metal and slag can be achieved without introducing additional devices. However, the baffled vessel presented in this study is solely a primary model for real refining processes using slag-metal reactions. The details of the processes must be left for a future study.

As a representative case, the diameter and height of a cylindrical vessel and the physical properties of molten slag and metal are assumed as follows: $D=1000$ mm, $H=1500$ mm, $R_s=1.0$, $N=100$ rpm ($\omega=10.47$ rad/s), $v_{\text{max}}=2$ mm$^2$/s. The Reynolds number and Eckman number are calculated from Eqs. (12) and (13) to give

\[
\begin{align*}
\text{Re}_{\text{max}} &= 1140 \\
\text{Ek}_{\text{max}} &= 8.49 \times 10^{-8}
\end{align*}
\]

Although these values do not fall in the applicable range of Eq. (17), the deviations are not so large. Equation (17) therefore is assumed to be valid under this condition, too. The flow establishment time is given by

\[
T_s = 1940 \text{ (s)}
\]

When a baffled vessel is used, the flow establishment time would be significantly decreased, say less than 600 s. Further investigation is necessary on the applicability of Eq. (17) for large scale vessels.

4. Conclusions

The flow establishment time was measured for stratified two liquid layers contained in a cylindrical vessel and a baffled vessel suddenly set in rotation. Two empirical equations were proposed for the flow establishment time in a cylindrical vessel. The flow establishment time was mainly dependent on the kinematic viscosities of the two liquids. The densities of them are nearly independent of the flow establishment time. The baffled vessel was found to shorten the flow establishment time significantly and to increase the interfacial area between the two layers. The baffled vessel suddenly set in rotation therefore is useful for rapid mixing and subsequent separation of molten slag and metal.

Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>Vessel diameter (mm)</td>
</tr>
</tbody>
</table>

Fig. 15. Photographs of liquid–liquid interface in baffled vessel. ($\omega=11.0$ rad/s, Silicone oil 100–salt water, $R_s=1$, $D=46$ mm).

Fig. 16. Comparison of Eq. (17) with measured values in baffled vessel.
Ek_1 : Eckman number for liquid 1 (−)
Ek_2 : Eckman number for liquid 2 (−)
H : Vessel height (mm)
R : Radius of vessel (mm)
Re_1 : Reynolds number for liquid 1 (−)
Re_2 : Reynolds number for liquid 2 (−)
R_1 : Volume ratio (−) = V_2 / V_1
r : Radial distance (mm)
T_0 : Flow establishment time (s)
t : Time (s)
V_1 : Volume of liquid 1 (mm³)
V_2 : Volume of liquid 2 (mm³)
z : Vertical distance for the projection of interface (mm)
z_max : Vertical distance for the maximum projection of interface (mm)
v_q : Tangential velocity component (mm/s)
ν_1 : Kinematic viscosity of liquid 1 (mm²/s)
ν_2 : Kinematic viscosity of liquid 2 (mm²/s)
ν_mix : Apparent kinematic viscosity (mm²/s)
ρ_1 : Density of liquid 1 (kg/m³)
ρ_2 : Density of liquid 2 (kg/m³)
ω : Angular frequency of rotation (rad/s)

REFERENCE