Critical Strain for Dynamic Recrystallization in Variable Strain Rate Hot Deformation

E. I. POLIAK and J. J. JONAS

Research Laboratories, Ispat Inland Inc., East Chicago, IN, USA.
1) Dept. of Metallurgical Engineering, McGill University, Montreal, PQ, Canada.

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In rolling, the strain rate in a rolling pass is not constant but depends on pass reduction \( \dot{\epsilon}_p \), decreasing or increasing along the arc of contact. In the present work, high temperature compression tests were performed with the rate varying according to strain rate profiles pertaining to various flat rolling pass reductions. Due to the high rate sensitivity of the stress at elevated temperatures, the stress follows such variations in strain rate. This can lead to peaks in the flow curves without regard to dynamic recrystallization (DRX). Nevertheless, critical strains for the onset of DRX can still be defined if the stresses and strains in variable strain rate deformation are normalized by the peak stresses and strains that would be observed if the deformation were being performed at a series of constant strain rates equal to that of successive points along the roll bite. Using plain carbon and Nb-bearing steels, it is demonstrated that the DRX critical strains are lower when \( \dot{\epsilon}_p < 30\% \) and higher when \( \dot{\epsilon}_p > 30\% \) than in constant \( \dot{\epsilon} \) deformation at the same initial strain rate. The present method permits the more accurate extrapolation of laboratory test results to industrial conditions and enables rolling loads to be analyzed with greater precision.

KEY WORDS: hot flat rolling; simulation; austenite; dynamic recrystallization.

1. Introduction

The strain at which dynamic recrystallization (DRX) is initiated is of considerable importance in the modeling of hot rolling mills, as it determines the stage of the process at which metadynamic recrystallization (MDRX) becomes possible. The latter is much more rapid than conventional static recrystallization (SRX) and can thus be responsible for sudden load drops during rolling below the conventional SRX stop temperature. This pertains especially to the hot rolling of steel strips at relatively high strain rates, when the short time intervals between rolling passes can be less than the incubation period for SRX and softening of the steel can only occur as a result of DRX and MDRX.

The understanding and interpretation of DRX is still largely grounded on laboratory simulations performed at constant strain rates that are considerably lower than in most industrial hot working processes. However, the strain rates in industrial forming are never constant even during a single operation. Although the DRX behavior at high constant strain rates can be predicted from low constant strain rate tests, the effects of strain rate variations on DRX remain unclear. Even though the importance of strain rate history in high temperature mechanical behavior was emphasized more than two decades ago, few examples of laboratory tests carried out at variable strain rates can be found in the literature (for example, Refs. 3, 4) and none of them addresses the issue of the effect of variable strain rate during a forming operation on the initiation and progress of DRX.

The traditional view regarding constant strain rate deformation considers that DRX manifests itself through stress peaks on the flow curves. Additionally, the onset of DRX can be identified from inflections in the plots of strain hardening rate versus flow stress \( \tau \). Now it has been shown that the absence of stress peaks in constant strain rate flow curves does not necessarily indicate the absence of DRX. Instead, inflections in the strain hardening rate plots (or in the equivalent \( \ln \tau - \ln \sigma \) and \( \ln \theta - \dot{\epsilon} \) plots) applicable to these testing conditions are better indicators of the occurrence of DRX than stress peaks. This was exemplified using a Nb bearing steel and an austenitic stainless steel, materials that do not display any well-defined stress peaks in constant strain rate compression tests.

Under variable strain rate conditions, the mechanical behavior is clearly more complicated than at constant strain rate, so that identification of the occurrence of DRX is significantly more difficult. For one thing, stress peaks can be produced that are entirely caused by the strain rate variation (i.e., without any contribution from DRX). Furthermore, the presence of such peaks unavoidably involves points of inflection in the strain hardening rate that, in this case, are not associated with the introduction of a new softening mechanism, such as DRX.

The objectives of the present work were therefore to develop a procedure for determining the DRX critical strain under variable strain rate conditions and to study the effect of strain rate variations on the onset of DRX. Since DRX is of special interest during hot flat rolling, this work will focus on the strain rate variations pertaining to flat rolling.
2. Experimental

The experimental procedures used here are similar to those of the earlier work. Two steels were employed, both of which were shown to undergo DRX during constant strain rate hot compression. These were a plain low carbon aluminum killed (AK) steel (0.04% C, 0.22% Mn, 0.012% Si, 0.05% Al) and a low carbon Nb microalloyed steel (0.05% C, 1.41% Mn, 0.29% Si, 0.03% Nb, 0.06% Al). The former displays significant flow softening and pronounced peaks in constant strain rate testing while the latter does not exhibit any flow softening. The compositions are similar to those in flat rolling with negligible lateral spread.

Specimen preparation procedures and preliminary solution treatment were identical to those described earlier. The samples were tested in compression using a Gleeble® thermomechanical simulator. The simulator was programmed to perform compression tests with various strain rate profiles. These profiles were simulated by incremental changes of its speed became significant. The experimental procedures used here are similar to those described earlier. The samples were reheated to 1100°C for 30 s and then cooled to the test temperature (900–1050°C) at 5°C/s. After 20 s of soaking, the specimens were compressed at initial strain rates of 1.0–10.0 s⁻¹ to strains varying from 0.105 (10% height reduction) to 1.204 (70% reduction). A strain rate of 10 s⁻¹ was the highest that could be reasonably well controlled during a test because, at higher rates, the effects of ram inertia during incremental changes of its speed became significant.

3. Results and Discussion

3.1. Strain and Strain Rate Variations in the Roll Bite

In the present work, we consider strain rate variations similar to those in flat rolling with negligible lateral spread. No friction effects have been taken into account, including the effects of forward slip. That is, the strains, strain rates and flow stresses in the direction normal to the rolling plane have been assumed to be homogeneous. In this way, the strain rate at each point of a given section of the workpiece taken to be equal to the mean through-thickness strain rate, etc., and only variations in strain rate along the arc of contact were considered. Furthermore, uniaxial compression was assumed to provide reasonably good simulations of these variations. The reductions referred to below are therefore height reductions in the axial direction of the specimen.

Throughout the paper, the mechanical behavior is discussed in terms of von Mises equivalent stress, strain and strain rate. These strains and strain rates are 1.15 times lower, while the stresses are 1.15 times higher than the corresponding plane strain values in flat rolling. To apply to flat rolling, the present results must be converted into the respective plane strain values. Such conversions will not, however, alter the principal conclusions.

Under the above assumptions, the variation of the workpiece thickness in the roll bite can be described by the relation:

\[ h(\alpha) = h_0 + D(1 - \cos \alpha) \] ........................................(1)

where \( h_1 \) is the exit thickness, \( D = 2R \) is the flattened roll diameter, and \( \alpha \) is the roll contact angle. Since the angle \( \alpha \) is small, \( \tan \alpha = \alpha = y/R \) with \( y \) being the absolute distance along the roll bite counted from the exit side (Fig. 1). Then

\[ \alpha = \frac{l_x}{R}(1 - x') \quad , \quad x' = l_a - y \quad , \quad x = \frac{x'}{l_a} = 1 - \frac{y}{l_a} \] ............(2)

where \( l_a = \sqrt{R \Delta h} \) is the projected length of the arc of contact, and the relative distance \( x \) along the roll bite (0≤x≤1) is counted from the entry side for convenience. The current thickness of the workpiece at point \( x \) is (Fig. 1)

\[ h(x) = h_0 + D \left[ 1 - \cos \left( \frac{h_0}{R} (1 - x) \right) \right] \]

\[ = h_0 + D \left[ 1 - \cos \left( \frac{\Delta h}{R} (1 - x) \right) \right] \] ........................................(3)

Then the current reduction at \( x \) is

\[ r(x) = 1 - \frac{h(x)}{h_0} = 1 - \frac{h_1}{h_0} - \frac{D}{h_0} \left[ 1 - \cos \left( \frac{\Delta h}{R} (1 - x) \right) \right] \]

\[ = r_p - \frac{D}{h_0} \left[ 1 - \cos \left( \frac{\Delta h}{R} (1 - x) \right) \right] = r_p - X \] ..............(4)

where \( r_p = \Delta h/h_0 \) is the total reduction in the roll pass, \( \Delta h \) is the absolute draft of the pass, and \( h_0 \) is the initial workpiece thickness. Here, all reductions are expressed in decimal fractions. The current true strain in the roll bite \( \varepsilon(x) \) is then

\[ \varepsilon(x) = - \ln[1 - (r_p - X)] \] ........................................(5)

With the aid of the above relations the variation in strain rate along the roll bite can be expressed as

\[ \dot{\varepsilon}(x) = \frac{2v}{\sqrt{R h_0}} \frac{(4 - 3r_p)(1 - r_p) \sqrt{r_p - r(x)}}{\sqrt{2 - r_p}^2 (1 - r(x))^2} \] ..............(6)

where \( v \) is the circumferential velocity of the roll.

To compare the strain rate variations for different pass reductions and rolling speeds, it is convenient to normalize the strain rate as \( \dot{\varepsilon}(x) = \dot{\varepsilon}(x)/\dot{\varepsilon}_0 \), where \( \dot{\varepsilon}_0 = \dot{\varepsilon}(x=0) = 2v/ \)

Fig. 1. Schematic representation of the roll bite in flat rolling.
\[ \sqrt{Rb_0} \] is the initial strain rate at the entry side of the roll bite. With this normalization, the quantity \( \xi \) describes the variations in strain rate induced purely by the geometry of the roll bite.

As shown in Fig. 2, the variations in \( \xi \) along \( x \) differ dramatically depending only on the bite geometry through the pass reduction \( r_p \). At small reductions (\( r_p < 30\% \)), the strain rate decreases continuously to zero from entry to exit from the roll bite. At higher reductions, the strain rate first increases to a maximum but then drops sharply to zero at the exit from the roll bite. The larger the pass reduction, the higher the maximum strain rate and the corresponding strain, i.e. the closer the maximum strain rate is to the exit side of the roll bite.

With reference to Fig. 2, constant strain rate deformation can be viewed as a fair approximation of the greater part of a rolling pass when \( r_p = 30\% \). Constant ram speed compression, which is also frequently employed in hot deformation studies, gives a good approximation for the greater part of a rolling pass when \( r_p = 65\% \).

### 3.2. Effect of Strain Rate Variations in the Roll Bite on the Flow Stress

Due to the high rate sensitivity of the stress at elevated temperatures, the flow stress tends to follow the variations in strain rate during a roll pass, as illustrated by the flow curves of Figs. 3 and 4. All of the curves exhibit stress peaks that, as in constant strain rate compression, reflect the balance between hardening and softening. This balance, however, is more complicated under variable \( \dot{\varepsilon} \) conditions. At low simulated reductions (\( r_p < 30\% \)), strain hardening is balanced not only by dynamic restoration, which can involve DRX, but also by the progressive strain rate softening brought about by the continuously decreasing strain rate. At high simulated pass reductions, strain hardening is initially complemented by the strain rate hardening due to the increasing strain rate; by contrast, during the later stages of deformation, the strain rate softening caused by the decreasing strain rate takes over instead. This softening can, of course, be enhanced by dynamic restoration including DRX. Nevertheless, the interactions between strain and strain rate hardening and softening in a single flat rolling pass can induce stress peaks in the flow curves even in the absence of DRX. For this reason, variable \( \dot{\varepsilon} \) flow curves for the Nb steels also exhibit pronounced stress peaks, a behavior that contrasts with the constant \( \dot{\varepsilon} \) curves.

It is worth noting that the variations in strain rate at the beginning of a roll pass are quite small (Fig. 2). The strain hardening behavior in the beginning of the pass does not therefore differ much from that observed in constant deformation. The effects of strain rate variations become especially significant as the end of a pass is approached at higher reductions, where the effect of dynamic restoration, enhanced by the strain, also becomes substantial.

### 3.3. Critical Conditions for the Initiation of DRX in Variable Strain Rate Deformation

When a flow curve has a stress peak, the plot of strain hardening rate \( q \) versus stress \( s \) always shows an inflection. In constant \( \dot{\varepsilon} \) deformation, both the stress peaks and the inflections in the \( q-s \) plots are attributed to DRX. However, the stress peaks in variable \( \dot{\varepsilon} \) flow curves are the consequence of the high rate sensitivity of the stress and are not indicative of the occurrence of DRX. Consequently, if the

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**Fig. 2.** Strain rate variations along the length of the roll bite.

**Fig. 3.** Flow curves for the AK steel at \( \dot{\varepsilon}_0 = 1 \text{ s}^{-1} \); 1 - constant \( \dot{\varepsilon} \); 2–6 - variable \( \dot{\varepsilon} \), 2 - \( r_p = 10\% \), 3 - \( r_p = 20\% \), 4 - \( r_p = 30\% \), 5 - \( r_p = 50\% \), 6 - \( r_p = 70\% \).

**Fig. 4.** Flow curves for the Nb steel at \( \dot{\varepsilon}_0 = 1 \text{ s}^{-1} \); 1 - constant \( \dot{\varepsilon} \); 2 - variable \( \dot{\varepsilon} \), \( r_p = 30\% \), 3 - variable strain rate, \( r_p = 50\% \).
latter type of flow curve is numerically differentiated with respect to strain, the resultant plot will always have a point of inflection that, however, does not necessarily indicate that DRX has been initiated. This ambiguity can be resolved in two ways. The first is to determine the true strain hardening rate, which, by definition, is the partial derivative of stress with respect to strain taken at constant strain rate. If the flow stress at constant temperature is solely a function of strain and strain rate, the total differential can be expressed as:

$$d\sigma = \left(\frac{\partial \sigma}{\partial e}\right)_{e} de + \left(\frac{\partial \sigma}{\partial \dot{e}}\right)_{e} d\dot{e}$$

so that the true strain hardening rate $\theta$ can be obtained as the difference between the total derivatives:

$$\theta = \left[\frac{\partial \sigma(e)}{\partial e}\right]_{e} = \frac{d\sigma}{de} - \frac{\sigma(e)}{e} \frac{d\dot{e}(e)}{de} \quad \text{(7)}$$

where $m = (\ln \sigma / \partial \ln \dot{e})_{e}$ is the strain rate sensitivity of the flow stress. Assuming that $m$ is not affected by the strain rate variation and can be obtained from the constant flow curves, the total derivatives related to the flow curve and the strain rate profile (Eq. (6)) can be computed numerically.

The alternative and much easier way involves the conversion of variable $\dot{e}$ flow curves into the unified form described in detail in the companion paper applicable to constant $\dot{e}$ deformation. In general, plastic deformation can be viewed as the evolution of a material towards its stationary, strain independent steady state. This state corresponds to zero strain hardening rate, a description that applies to both peak and steady state stresses. In constant strain rate deformation, $\sigma_{p}$ and $\sigma_{s}$ are known to be solely functions of the Zener–Hollomon parameter $Z$, through the dependence that is generally referred to as the hyperbolic sine law,

$$Z = \dot{e} \exp \left(\frac{Q_{\text{lat}}}{RT}\right) = A \sinh (\alpha \sigma_{s})^{m}, \quad \text{(8)}$$

$$\sigma_{s} = \frac{1}{\alpha} \text{Arsh} \left( \left(\frac{Z}{A}\right)^{m} \right) = \varphi(Z)$$

where $A$, $\alpha$, $n = 1/m$, and $Q_{\text{lat}}$ are material dependent coefficients that are slightly different for the peak and steady state stresses but remain constant within certain ranges of temperature and strain rate. The strain at which the stationary ($\theta = 0$) state is attained in constant $\dot{e}$ deformation (i.e., the peak strain $\epsilon_{p}$ or the steady state strain $\epsilon_{s}$) also depends on $Z$ as

$$\epsilon_{s}(Z) = \psi(Z) = BD\epsilon_{s}^{2} \quad \text{(9)}$$

where $D_{0}$ is the initial grain size**, and $B$, $p$, and $k$ are material constants.

If the flow stress is represented as a function of the fraction of the total strain $\epsilon_{s}$ required to attain the stationary state, then the flow curve can be described by a constitutive equation of the type

$$\sigma(Z, w) = \varphi(Z)f(w) \quad \text{(10)}$$

where the values of the “normalized” plastic strain, $w = e^{\epsilon}/e_{s}$, and of the ratio

$$u = \frac{\sigma(Z, w)}{\sigma_{s}} = \frac{\sigma(Z, w)}{\varphi(Z)} = f(w) \quad \text{(11)}$$

run from 0 to 1 and are independent of $Z$. Thus, the flow stress at any given normalized strain $w$ is viewed as the degree to which the material has approached its stationary state $\sigma_{s} = \varphi(Z)$ in the course of deformation.

Equation (10) suggests that, during deformation, the evolution of the material towards its strain independent stationary state $\varphi(Z)$ proceeds along the same path within the range of $Z$ for which the function $f(w)$ does not change. Then $u$–$w$ plots obtained from the initial stress-strain curves for different $Z$ values should fall on a single curve.

In constant strain rate deformation, the $u$–$w$ plot can be differentiated to obtain the $Z$-independent evolution rate $\Theta$ and the onset of DRX identified from the point of inflection of the $\Theta$–$u$ plot. For a given steel, the inflection points in $\Theta$–$u$ plots correspond to approximately the same $u$-values, i.e., to approximately constant critical stress ratios $u_{c} = \sigma_{c}/\sigma_{s}$. Consequently, the ratio of the critical stress to the peak stress, $e_{c}/e_{p} = w_{c}$, is also approximately constant for a given steel deformed within a given temperature and strain rate range. Moreover, if no stress peaks can be seen in the constant $\dot{e}$ flow curves, the onset of DRX can still be quantified in terms of the ratio of the DRX critical stress to the steady state stress, $\sigma_{c}/\sigma_{s}$, and of the DRX critical stress to the steady state strain, $e_{c}/e_{s}$.

For variable strain rate deformation, the stresses and strains must be normalized by a series of virtual peak stresses $\sigma_{v}(x)$ and peak strains $\epsilon_{v}(x)$; i.e., by those that would be observed if the deformation were being performed at a series of constant strain rates equal to those at successive points $x$ along the roll bite. Some $u$–$w$ plots obtained in this way for the rolling simulations are presented in Fig. 5, where the plots for constant strain rate compression are also shown for comparison purposes. The “instantaneous” values of $\epsilon_{v}(x)$ and $\sigma_{v}(x)$ were computed from Eqs. (8) and (9) using the data from the constant strain rate tests.

As is evident from Fig. 5, the $u$–$w$ plots that correspond to various strain rate profiles can nevertheless be superimposed. The minor deviations that can be seen are probably attributable to experimental scatter. The coincidence of the plots in Fig. 5 indicates that the evolution function for a given material is same for both constant and variable strain rate deformation. Clearly athermal hardening, as discussed above, does not depend on strain rate and its variations. Consequently, if DRX is initiated at constant strain rate, it will also occur under variable strain rate conditions in tests carried out with the same initial strain rate.

Because of the coincidence of the $u$–$w$ plots for various $Z$ and strain rate profiles, the evolution rate $\Theta$ for variable strain rate deformation can be determined from these plots.

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**In the present work, the dependence on $D_{0}$ was not taken into account because the experiments were designed to keep the initial grain size constant.
in the same way as under constant strain rate conditions and without complicated differentiation. The resultant evolution rate $\dot{\Theta}$ must be identical to the one that applies to constant strain rate deformation. The $\Theta - u$ dependencies shown in Fig. 6 for variable strain rates are in keeping with this assumption. They inflect, indicating the initiation of DRX during rolling. The inflections correspond to approximately the same values of stress ratio $u_c$ and hence to the same values of strain ratio $w_c$ as in constant $\dot{e}$ tests: $u_c \approx 0.77$ and $w_c \approx 0.5$ for the AK steel and $u_c \approx 0.84$ and $w_c \approx 0.5$ for the Nb steel. Thus, values of the critical ratios $\sigma_c/\sigma_p$ and $\varepsilon_c/\varepsilon_p$ for DRX obtained in constant strain rate deformation can be applied to variable strain rate deformation as well.

### 3.4. DRX Critical Strains in the Roll Bite

Once the critical ratios $\sigma_c/\sigma_p$ and $\varepsilon_c/\varepsilon_p$ are known, the critical stresses and strains for DRX, as well as the exact locations where DRX is initiated in the roll bite, can be readily determined in a variety of ways. For this, it is convenient to plot the $u$-ratio against the current true strain in the roll pass $\varepsilon(x)$, or current reduction $r(x)$, or simply against $x$. Examples of such plots are presented in Figs. 7 and 8. Evidently, the critical stress ratio $u_c \approx 0.77$ (or, equivalently, the critical strain ratio $w_c \approx 0.5$) cannot be attained and hence DRX cannot be initiated in the AK steel in a roll pass with 10% reduction at $1 \text{s}^{-1}$ and 950°C (Fig. 7). DRX can begin in a pass with 20% at a strain of $\varepsilon_c = 0.14$ ($r_c = 13\%$).

Of course, higher reductions are more favorable for the initiation of DRX.

In the Nb steel rolled at 950°C and $1 \text{s}^{-1}$, the critical ratio $u_c \approx 0.85$ is barely attained in a pass with 20% reduction. In this case, DRX is initiated at the very end of the pass at a strain of $\varepsilon_c = 0.2$ (Fig. 8). In the pass with 30% reduction, DRX begins at a higher strain ($\varepsilon_c = 0.25$) but much earlier in the roll bite. Knowledge of the initiation of DRX in the roll bite is of particular importance in the analysis of rolling as it indicates the point in the schedule when MDRX can take place in the next interpass interval and therefore when load drops can be expected in subsequent stands.

The accumulation of strain along $x$ is essentially non-linear (Eqs. (4) and (5)) purely due to the geometry of the roll bite (Fig. 9). Therefore, the DRX start position in the roll bite strongly depends on pass reduction, as exemplified in Fig. 10 for the AK steel deformed at an initial strain rate of $1 \text{s}^{-1}$. The critical position in the roll bite, $x_c$, that is the point in the roll bite where DRX begins, can be derived if the critical strain $\varepsilon_c$ (or, equivalently, the critical reduction $r_c = r(x_c)$) is determined independently, for example, from a variable strain rate test. From Eqs. (4) and (5)

\[
1 - \frac{h_0}{D} (r_p - r_c) = \cos \left( \frac{\Delta h}{R} (1 - x_c) \right)
\]

so that
The values of $x_c$ calculated from Eq. (12) are shown in Fig. 10(a). The horizontal solid line represents the DRX critical strain determined in a constant $\dot{e}$ test, while the dashed lines show the critical strains in variable $\dot{e}$ tests. The points where the solid line crosses the $\epsilon(x)$ curves give the positions in the roll bite where DRX would be initiated if rolling were performed at constant strain rate. As seen in Fig. 10(a), at low reductions ($r_p < 30\%$) the critical strains in rolling are always lower than in constant strain rate deformation. At high reductions (e.g., 50%), the reverse applies. For the same initial strain rate, the higher the reduc-

$$x_c = 1 - \sqrt[\gamma]{\frac{R}{h_0 r_p}} \arccos \left[ 1 - \frac{h_0}{D} (r_p - r_c) \right] \quad \text{(12)}$$

Fig. 7. Stress ratios $u = \sigma / \sigma_p$ plotted against (a) true strain and (b) pass reduction for the AK steel.

Fig. 8. Stress ratio $u = \sigma / \sigma_p$ plotted against true strain for the Nb steel.

Fig. 9. Effect of pass reduction on strain accumulation in the roll bite.

Fig. 10. Effect of pass reduction on locations in the roll bite where DRX begins in the AK steel at 950°C.
tion, the earlier in the roll bite (i.e., at lower x) is DRX initiated, although the critical strains are, of course, higher in passes with higher reductions.

The effects of variable strain rate are illustrated in more detail in Fig. 10(b), where the relative strain distribution in the roll bite is shown. In the AK steel, DRX cannot begin at 950°C when a 10% reduction is employed because the critical strains required to initiate DRX exceed the total pass strain at initial strain rates of both 1.0 and 10 s\(^{-1}\) (Fig. 10(a) and Fig. 11(a)). Similarly, DRX cannot occur in a pass with 20% reduction performed at an initial strain rate of 10 s\(^{-1}\).

In the Nb steel, DRX cannot be initiated at 950°C in a pass with 10% reduction (Fig. 11(b)). In a pass with 20% reduction performed at 1 s\(^{-1}\), DRX can begin at the very end of the pass just prior to exiting from the roll bite. At higher strain rates, DRX cannot occur in a pass of 20% reduction because the critical strain exceeds the pass strain. Even with 30% reduction, DRX is barely initiated at 10 s\(^{-1}\).

The examples in Figs. 10 and 11 show that the commonly used criterion \(\varepsilon_{\text{pass}} > \varepsilon_c\) for the initiation of DRX during rolling is misleading because of the strain rate profile in the roll bite. It is of particular note that, as long as DRX is initiated within the roll bite, MDRX will replace SRX during the next interpass interval. As the former softening mechanism is about an order of magnitude faster than the latter,\(^{9,10}\) it can lead to interstand softening when there is no time for SRX. When DRX is not initiated within a particular rolling pass and no SRX takes place in the following interstand interval (as in Nb-modified and stainless steels), the strain is accumulated from pass to pass and DRX can still be initiated in later passes.

### 3.5. Extrapolation to Higher Strain Rates

The flow curves presented in \(u-\varepsilon\) (or \(u-\varepsilon\) or \(u-x\)) coordinate form can be extrapolated to any deformation temperature and strain rate using a single \(u-w\) plot and the \(Z\)-dependencies of the peak stresses and strains. If the stress ratio \(u\) for some strain rate \(\dot{\varepsilon}_1\) is known, say from an experiment, the stress ratio for another strain rate \(\dot{\varepsilon}_2\) can be calculated from the relationship

\[
\frac{u(\dot{\varepsilon}_2)}{u(\dot{\varepsilon}_1)} = \left(\frac{\dot{\varepsilon}_1}{\dot{\varepsilon}_2}\right)^w \tag{13}
\]

Note that if \(\dot{\varepsilon}_2 > \dot{\varepsilon}_1\), then \(u(\dot{\varepsilon}_2) < u(\dot{\varepsilon}_1)\) and the \(u-\varepsilon\) curve for the higher strain rate lies below the curve for the lower one. The flow stresses are naturally higher for higher strain rates, but at the same time the higher the strain rate, the lower the ratio of the flow stress at a given strain to the peak stress, i.e., the farther is the flow stress of interest from the strain-independent stationary value. This kind of extrapolation is exemplified in Fig. 12 for single pass rolling with a reduction of 30% at 950°C and a strain rate of 100 s\(^{-1}\), which is typical for the final finishing passes in a hot strip.

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**Fig. 11.** Dependence of the DRX critical strain on pass strain at 950°C; a - AK steel; b - Nb steel.

**Fig. 12.** Stress ratio \(u = \sigma/\sigma_p\) plots obtained from the variable \(\dot{\varepsilon}\) tests together with an extrapolation to 100 s\(^{-1}\); a - AK steel, b - Nb steel.
mill. Evidently, DRX cannot be initiated in either the AK or the Nb steel because the critical stress ratio cannot be attained. Even a single pass with 50% reduction is insufficient to initiate DRX under these conditions.

This example shows that it is difficult to initiate DRX in a single rolling pass at strain rates typical of the final finishing passes in industrial hot strip rolling. On the other hand, as long as both DRX and SRX are absent, strain accumulation occurs. This will lead to DRX after two or more passes, followed by MDRX and by load drops.

The approach outlined here can, in principle, be applied to any strain rate profile, not necessarily to that of flat rolling. It can also be used in FEM simulations of flat rolling.

3.6. Application of the Pass Mean Strain Rate

Within the framework of the present discussion some essential aspects should be emphasized. To account for the exact strain rate profile in comprehensive modeling and to calculate either the critical strain or the critical stress in variable strain rate deformation (and particularly in rolling), one has to face certain computational difficulties. As follows from the experimental results and their analysis as presented above, even at constant temperature, the critical strain obeys the relationship:

\[ e_c = \frac{w_c B D}{\dot{\varepsilon}_0} \exp \left[ \frac{k Q_{\text{def}}}{RT} \right] \left( \dot{\varepsilon}_c \right) \]  
\[ \dot{\varepsilon}_c \]  
\[ (14) \]

The above equation can only be solved numerically.

These computational complexities can be simplified if a rolling pass is simulated by a constant \( \dot{\varepsilon} \) test with a strain rate \( \dot{\varepsilon} \) equal to the mean strain rate of the pass. Then the critical strain can be calculated as

\[ e_c = w_c B D \dot{\varepsilon}_c \exp \left[ \frac{k Q_{\text{def}}}{RT} \right] \]  
\[ \dot{\varepsilon}_c \]  
\[ (15) \]

This equation gives reasonable estimates for the critical strains, their dependences on pass reduction, and the location at which DRX is initiated in the roll bite. For this purpose, the mean strain rate should be expressed as a function of pass reduction using the approximation:

\[ \dot{\varepsilon}_{\text{test}} = \dot{\varepsilon}_0 \left( 1 - \frac{30}{2 - r_p} \right) \]  
\[ r_p \]  
\[ (16) \]

However, such an approach overlooks an important effect of variable strain rate. As noted above, the largest portion of the deformation in rolling is imposed relatively rapidly in the beginning of the pass (Fig. 9), whereas the strain rate decreases significantly towards the exit (Fig. 2) and the rate of accumulation of strain also slows down. Although an increase in pass reduction induces the onset of DRX at progressively higher strains, DRX is initiated progressively earlier in the roll bite (i.e. at smaller \( x \)). That is, the higher the pass reduction at a given initial strain rate, the wider is the strain range available for DRX to proceed and the more completely is DRX propagated prior to exit from the roll bite (Fig. 12). Moreover, due to the decreasing strain rate at the end of the roll pass, the larger strain range is covered in a longer time, which further favors the propagation of DRX. As a result, after DRX has been initiated, more time is available for its development in rolling than can be predicted from a constant \( \dot{\varepsilon} \) test, even with a strain rate equal to the pass mean. Thus, during rolling, DRX is closer to completion than in a constant \( \dot{\varepsilon} \) simulation at the same temperature and mean pass strain rate. Accordingly, MDRX will play a greater role in rolling than in constant \( \dot{\varepsilon} \) simulations.

Many hot strip mills employ variable rolling speeds along the length of a bar. Commonly, the rolling speed is initially low at the head end, but once the tension is established, the mill is accelerated to its maximum speed. At the end of the bar, the rolling speed is decreased towards its initial level. Special care should be taken in setting up the pass reductions if DRX is anticipated in this type of strip rolling. DRX is more likely to occur at the low initial speed. However, when the mill is ramped up to the maximum speed at the preset reductions, the DRX critical strains are increased and the position of initiation of DRX in the roll bite will shift closer towards the exit side. Even if these events do not prevent recrystallization completely, they impede DRX by shortening the strain range available for its development. With less time at the higher rolling speeds, the progress of DRX can be significantly retarded for the body of the strip compared to the head and tail ends. This may lead to considerably higher rolling forces than would be expected merely from the mill acceleration and to large variations in the as-hot rolled microstructure and properties of the strip.

If DRX controlled rolling is considered for this type of mill, the extrapolation procedure described above will help to evaluate the maximum rolling speed at which DRX can proceed to a sufficiently great extent with a given setup of pass reductions.

4. Conclusions

(1) For a given material, the ratios of the DRX critical stress to the peak stress (or to the steady state stress) and of the DRX critical strain to the peak strain (or to the steady state strain) are approximately independent of strain rate variations.

(2) Variations in strain rate along the roll bite significantly influence the critical strain for DRX. For low pass reductions (<30%), the DRX critical strain is lower than in constant strain rate deformation with the same initial strain rate. By contrast, at high pass reductions (>30%), the DRX critical strain is higher than in constant strain rate deformation. These effects stem from the coupling of strain rate softening and hardening, respectively, with strain hardening and dynamic restoration at elevated deformation temperatures.

(3) With increasing pass reduction, DRX starts earlier in the roll bite, i.e. closer to the entry end. Also, with increasing pass reduction, longer time intervals and wider strain ranges in the roll bite are available for the propagation of DRX.

(4) The normalized description of flow curves (and of the critical strain for DRX) developed for constant \( \dot{\varepsilon} \) defor-
mation can be extended to variable $\dot{\varepsilon}$ deformation.

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