1. Introduction

The surface quality of continuously cast material is mainly affected by the phenomena occurring during the solidification of the first shell layer. Surface cracks are closely related to the development of strain and stress caused by the shrinkage of the solidifying steel. Non-homogeneous cooling conditions due to gap formation between strand and mold are leading to cross-sectional distortion of blooms and billets. The gap is minimized by using molds with empirically determined taper. Linear tapers are applied frequently. But the optimum taper should take the non-linear shrinkage of the strand into account.

The shrinkage of steel during solidification is not understood quantitatively and, consequently, there are no data available for the optimization of the mold geometry. Direct measurements in a mold under continuous casting conditions are not feasible. But studies on metals cast in ingot molds have been performed using measurement of external dimensions of the casting at room temperature, and of gap formation during solidification applying non-contact and contact sensors. Investigations have been carried out on lead,1,2) aluminum,3) copper,4) cast iron5) and steel.6–8)

It is well known that apart from the casting conditions the chemical analysis has a considerable influence on the phenomena occurring in the mold. Certain steel grades with about 0.1 mass% carbon have been found to be more sensitive to surface defects, surface roughness, geometrical defects and breakouts than grades with elevated carbon contents. Several investigations have demonstrated that the heat flux in the mold passes through a minimum at about 0.1 mass% carbon and other studies show a reduced friction force for these grades. Generally, such observations are attributed to the increased shrinkage brought about by the δ’/γ transformation.

In fact, the thermal contraction of iron–carbon alloys depends strongly on carbon content. Figure 1 shows the thermal contraction of iron–carbon alloys at various temperature intervals ΔT=T_s−T below solidus.9) When the same phase exists at T_s and T (at low carbon content above 1392°C and at carbon contents right from point I) the contraction is small and practically constant. But an increase (of absolute value) occurs if γ phase is present at T and δ phase at T_s which is caused by the decrease of volume (in-
Fig. 2. Formation of $\delta$, $\delta + \gamma$, $\gamma$ bands during the first seconds of solidification of carbon steels. Semi-quantitative. (a) 0.03 mass% carbon, (b) 0.08 mass% carbon, (c) 0.11 mass% carbon, (d) 0.14 mass% carbon. The steels contain 0.195 mass% silicon, 0.960 mass% manganese, 0.014 mass% phosphorus, 0.014 mass% sulfur, 0.032 mass% aluminum. The lower diagrams demonstrate the change of thermal contraction in the different layers.
crease of density) associated with the δ/γ transformation. During solidification of an ingot (strand) the δ/γ transformation occurs layerwise, due to the temperature gradient, Fig. 2. Consequently, there is a δ+γ band in the solid shell with steep change of the thermal contraction value, moving inwards with time, and this effect will enhance the formation of thermal stress. Hence, the quantitative treatment of the shrinkage of steel during solidification and of the associated formation of stress is of high interest for various reasons.

We have performed extensive shrinkage measurements on iron–carbon alloys and developed a corresponding mathematical model for the computation of displacements, strains and stresses originating during solidification and subsequent cooling. The results are reported in the following.

2. Experimental Method for Shrinkage Measurements

As has already been mentioned shrinkage*1 measurements under continuous casting conditions are extremely difficult or even impossible, but they are feasible on standing ingots. The experimental setup used in the present study is shown in Figs. 3 and 4. It consists of an instrumented mold with feeding channel installed within the chamber of a vacuum induction furnace. The round mold is made of grey cast iron and has a height of 470 mm, an I.D. of 140 mm (at half height) and an O.D. of 240 mm. The mass of the cast steel melts is 50 kg.

The surface position of the solidifying ingot is determined with a contact mechanism, Fig. 4, using an alumina sheath as a sensor and an inductive transducer to record its position. In the early stages of solidification the shell is thin and soft, and the contact of the sensor must be gentle to avoid deformation of the surface. Therefore, the sensor is not pressed permanently against the ingot but is driven by a motor towards the surface and then withdrawn immediately after contact has been made. The contact is recorded electrically via a platinum bead located at the tip of the sheath and connected, with a platinum wire, to the control unit of the motor. So, the sensor moves towards the ingot and the platinum bead touches the ingot surface. The control unit receives a signal to change the polarity of the power supply of the motor and the motor rotates in reverse direction to remove the sensor, and then the cycle is repeated. The frequency and amplitude of the sensor movement can be adjusted in such a manner that the measurement is performed with high resolution in the beginning and with a reduced number of cycles after final solidification.

Before the first experiments the accuracy of the measuring system was checked using a setup in which the ingot surface was simulated by a micrometer gauge. The position of the tip of the gauge was changed in steps of 0.5 mm and the movement of the sensor was recorded. From the measuring trace the maximum error of the system was determined to be 0.05 mm.

As shown in Fig. 3 the measuring system is mounted on a horizontal segment of the mold at half height of the mold.

*1 The term “shrinkage” is used to denote the decrease of the outer radius $r_o$ of the cylindrical ingot or strand. It is equal, in absolute value, to the displacement $u(r_o)$ of the surface of the ingot, but the “displacement” is negative whereas the “shrinkage” is taken positive.
parable experiments. This phenomenon is caused by a lateral ingot movement in the mold during the solidification which is superimposed on the shrinkage. Therefore, it was not possible to obtain reproducible results using a single sensor. The subsequent experiments have been performed with three identical sensors mounted on the mold with angles of 120 degrees apart. Assuming that the ingot cross section stays circular the exact momentary position of the ingot axis and the shrinkage of the ingot (relative to the mold) can be deduced from the signals of the three probes.

2.2. Expansion of Mold

Since the sensors were mounted on the mold the sum of mold expansion and movement of the ingot surface are recorded in the experiments, and, in order to obtain the real shrinkage values, the mold expansion must be known. Several experiments were carried out to determine the mold expansion. For this purpose a water cooled copper ring was fixed to the mold on one side. On the other side a sensor was mounted on the ring with which the outer mold surface was measured relative to the non-expanding ring.

Although the ingot shrinkage could now be evaluated directly from the measured data the measurements became very complex and difficult. Another more simple way of determining the mold expansion was used by computing the mold expansion from the temperature profile in the mold which was measured with six thermocouples in all the experiments.

If the coefficient of thermal expansion of the mold material is known, the increase of the outer radius of the mold resulting from the temperature increase can be calculated, to a close approximation, with Eq. (1)

\[ u_m = \frac{2r_m}{r_m^2 - r_i^2} \int_{r_i}^{r_m} (\hat{\varepsilon}_m^\text{th}(T) - \hat{\varepsilon}_m^\text{th}(T_o))r dr \]

The coefficient of thermal expansion \( \hat{\varepsilon}_m^\text{th} \) for grey cast iron GGG40 is known from the literature. It can also be deduced from the measured expansion of the mold. For this purpose \( \hat{\varepsilon}_m^\text{th} \) is represented in the form of the polynomial \( \hat{\varepsilon}_m^\text{th} = a + bT + cT^2 \) so that the RHS of Eq. (1) can be integrated. The constants \( a, b \) and \( c \) were determined by a least error fit of Eq. (1) to the measured \( u_m(t) \) values. The equation obtained is

\[ \hat{\varepsilon}_m^\text{th} = -2.730 \cdot 10^{-4} + 1.074 \cdot 10^{-5}T + 7.130 \cdot 10^{-9}T^2 \]

with \( T \) in °C which is in good agreement with the literature data.

2.3. Expansion of Sensor

The thermal expansion of the alumina rod with respect to an initial temperature gradient in the rod has been calculated from Eq. (3) using the literature data for the coefficient of thermal expansion.

\[ \Delta l_i = \int_0^l (\hat{\varepsilon}_s^\text{th}(T) - \hat{\varepsilon}_s^\text{th}(T_o))dl \]

It was assumed that the local temperature in the rod was the same as that in the mold. For the part of the rod which is outside of the mold a constant gradient to the ambient temperature was taken.

2.4. Evaluation of Real Ingot Shrinkage and Typical Experiment

Finally, the real ingot shrinkage was evaluated from the measured shrinkage by subtracting the expansion of the mold and by adding the expansion of the sensor. Figure 5 shows a typical example for the data registered in the experiments. The carbon content of the steel is 0.34 mass%. The upper diagram gives the temperature development in the mold and the lower diagram the signals \( s_1(t), s_2(t) \) and \( s_3(t) \) of the three sensors. The difference between the three displacement curves is caused by the mentioned lateral motion of the ingot. The three curves were processed, using a computer program, to obtain the “apparent shrinkage” which is shown in Fig. 6 by the curve \( s(t) \). The displacement of the mold periphery and the expansion of the sensor

![Fig. 5. Typical example for the data registered in the experiments.](image)

![Fig. 6. Evaluation of ingot shrinkage from measured apparent shrinkage, displacement of mold surface and expansion of sensor.](image)
are given by the curves \( u_m(t) \) and \( \Delta l_s(t) \), respectively, and the final shrinkage of the ingot is obtained via Eq. (4)
\[
\text{shrinkage} = -u(r_0, t) = s(t) - u_m(t) + \Delta l_s(t) \quad \ldots (4)
\]

3. Effect of Carbon Content on Ingot Shrinkage

Pure iron–carbon alloys were used to avoid the effect of other elements. The starting material was electrolytic iron and graphite. The deoxidation was with carbon only. Oxygen was excluded by working under vacuum (during deoxidation) or argon (5 mmHg). The superheat was maintained constant at approximately 50°C above liquidus temperature.

Figure 7 represents the ingot shrinkage evaluated according to the described procedure for various time steps as a function of carbon content. Although there is a scatter in the data a maximum in shrinkage can be observed clearly at about 0.1 mass% carbon. Comparing the peak value at about 0.1 mass% carbon with the average shrinkage level it can be seen that the peak is developed very early and that the difference between the ground shrinkage value and the maximum is about 0.3 mm and stays rather constant during the subsequent solidification. It is mentioned that the shrinkage values reported by the authors some time ago\(^7\) are of similar magnitude and show the peak at about 0.1 mass% carbon. Also Emi et al.\(^8\) find the shrinkage peak at the same carbon content.

The measured shrinkage curves as shown in Fig. 6 have the three characteristic ranges separated by the points \( P_1 \), \( P_2 \) and \( P_3 \). At \( P_1 \) the solidification process has terminated and the core temperature drops rapidly. Consequently, also the shrinkage rate increases. The times of final solidification evaluated from the measured shrinkage curves are shown in the lower diagram of Fig. 8. The scatter at the lowest carbon contents can be explained by the roughness of the ingot surface leading to variable heat transfer conditions and to higher error in the measurements.

Similarly as in dilatometer experiments, \( P_2 \) and \( P_3 \) are associated with the begin and the end of the phase transformation of austenite to ferrite. Due to the lower density of ferrite compared to austenite, an expansion of the material occurs which is superimposed on the overall thermal shrinkage and causes a maximum on the shrinkage curve. Finally, at \( P_3 \) the transformation to ferrite has ended and the ingot shrinks corresponding to the temperature decrease. The time coordinate of these points was determined as intersection of the two tangents drawn to the parts of the curve before and after change of slope, respectively. (This procedure is somewhat subjective.) The effects of carbon content on the begin and the end of phase change are shown in the upper diagram of Fig. 8. Up to carbon contents of about 0.2 mass% the begin and the time interval of the phase transformation is influenced by the carbon content which is qualitatively in accord with the iron–carbon phase diagram. But for higher carbon contents there is not much influence of carbon content on these transformation times.

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![Fig. 7](image-url)  
**Fig. 7.** Influence of carbon content on shrinkage of ingot at various times. The initial radius of the ingot (=internal radius of mold) is 70 mm.

![Fig. 8](image-url)  
**Fig. 8.** Influence of carbon content on the times of the end of solidification and of the begin and end of the austenite transformation.
4. Numerical Simulation of Shrinkage Behavior

The measured shrinkage data have been interpreted theoretically using a numerical model which consists of algorithms for the solution of the heat flow equation and of the various equations describing the mechanical behavior of the ingot.

4.1. Calculation of Temperature Field

The heat flow model is that used for the thermal tracking of continuously cast slabs and has been described elsewhere. However, some modifications had to be made. Firstly, it was changed to the cylindrical geometry. Hence, the equation applied is

\[ \frac{\partial (\rho H)}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left( \lambda r \frac{\partial T}{\partial r} \right) = \lambda \frac{\partial^2 T}{\partial r^2} + \left( \frac{\partial \lambda}{\partial r} + \frac{\lambda}{r} \right) \frac{\partial T}{\partial r} \]  \hspace{1cm} (5)

Secondly, the model was extended to comprise also the mold, that is Eq. (5) was solved both in the solidifying ingot and in the mold. The boundary conditions are

\[ r = 0; \quad - \left( \lambda \frac{\partial T}{\partial r} \right) = 0 \]  \hspace{1cm} (6a)

\[ r = r_m; \quad T = T_m \]  \hspace{1cm} (6b)

\[ r = r_0; \quad - \left( \lambda \frac{\partial T}{\partial r} \right) \bigg|_{\text{ingot}} = h(T_{\text{ingot}} - T_{\text{mold}}) = - \left( \lambda \frac{\partial T}{\partial r} \right) \bigg|_{\text{mold}} \]  \hspace{1cm} (6c)

Equation (6c) serves for coupling the temperature fields in the ingot and in the mold. The initial conditions are

\[ t = 0; \quad 0 < r < r_0; \quad T = T_c \]  \hspace{1cm} (7a)

\[ r_0 < r < r_m; \quad T = T_0 \]  \hspace{1cm} (7b)

Some special experiments have been carried out to check the model and to determine the unknown heat transfer coefficient \( h(t) \). For this purpose five Pt/PtRh 18 thermocouples have been placed in the interior of the mold (in addition to the six thermocouples in the mold) through bores in the mold wall, protected with Mo/Cr2O3 cermet sheaths, to measure the temperature of the ingot during solidification and subsequent cooling. The obtained temperature traces are presented, for a selected experiment, in Fig. 9 as dotted lines. Using the model in an inverse manner the heat transfer coefficient was determined by fitting the calculated temperatures in the mold to the measured data. The accuracy of the thermal model can be estimated from Fig. 10 which proves that the measured data agree well with the numerically determined temperatures. The obtained function of the heat transfer coefficient \( h \) is very large initially. It then decreases rapidly, due to the increase of the gap. For the initial time period to the end of solidification the \( h(t) \) function can be represented by the empirical relationship

\[ h = 1450(t+1)^{-0.40} \quad [W m^{-2} K^{-1}] \]  \hspace{1cm} (8)

with time \( t \) in s. Later the sum of radiation and conduction coefficients is used taking the heat flux by conduction as a constant (6 500 W m\(^{-2}\))
with the emissivities in radiation being 0.5. Although the inverse computation was based on the temperatures in the mold wall because of their higher precision, the recalculated temperature traces of the ingot also agree satisfactorily with the measurements.\(^2\) Fig. 9. Furthermore, the model was checked with the times of the points \(P_1, P_2, P_3\) on the shrinkage curves, Fig. 8, exhibiting again rather good agreement with the experimental results. In summary it may be stated that the heat flow model reproduces the cooling of the ingots satisfactorily and, consequently, it is well suited as a basis of the computations with the mechanical model.

### 4.2. Calculation of Ingot Shrinkage

The mechanical model is essentially the same as that described elsewhere.\(^3\) But in contrast to the previous model which is for constant Young’s modulus \(E\), the temperature dependence of \(E\) is taken into account. This has been achieved, in an elegant manner, by using reduced stress components \(\sigma_r/E\) rather than the absolute stresses \(\sigma_r\). All the equations are applied in a time differentiated form since this is the appropriate way for treating solidification problems.\(^3\) Although the model is capable of dealing with the complex elasto-viscoplastic material behavior the calculations have been performed, because of the shorter computer time, assuming elastic behavior.

The equation used is that for the displacement velocity in \(u(r, t)\) of the ingot surface \((r=r_0)\) which is given as

\[
u(r_0, t) = \frac{1 + v}{1 - v} \int_{r_0}^{r_1} \hat{\varepsilon}^{(1)} r dr - \frac{(1 + v)(1 - 2v)}{(1 - v)} \int_{r_0}^{r_1} \int_{r_0}^{r} \frac{\partial}{\partial t} \left( \sigma_r \frac{\partial \ln E}{\partial r} \right) dr ] dr + \frac{C_1 r}{2} + \frac{C_2}{r} \tag{10}
\]

The lower limit of integration \((r=r_0)\) is the radius of the solidus front during the solidification, and the center of the ingot \((r=0)\) after the end of solidification. In the latter case the terms for \(r=0\) containing \(1/r\) (and \(1/r^2\) in Eq. (11)) have to be determined with the help of L’Hospital’s rule. The equation for the radial stress component \((r=0)\) differentiated that is contained in expression (10) is given as

\[
\hat{\sigma}_r = -\frac{1}{1 - v} \int_{r_0}^{r_1} \hat{\varepsilon}^{(1)} r dr - \int_{r_0}^{r_1} \frac{\partial}{\partial t} \left( \sigma_r \frac{\partial \ln E}{\partial r} \right) dr + \frac{1 - 2v}{1 - v} \int_{r_0}^{r_1} \int_{r_0}^{r} \frac{\partial}{\partial t} \left( \sigma_r \frac{\partial \ln E}{\partial r} \right) dr ] dr + \frac{C_1}{2(1 + v)(1 - 2v)} + \frac{C_2}{(1 + v)^2} + \frac{\nu C_3}{(1 + v)(1 - 2v)}, \tag{11}
\]

The integration constants \(C_1(t)\) and \(C_2(t)\) and the function \(C_3(t)\) are determined from the two boundary conditions at \(r=r_0\) and \(r=r_0\) or \(r=0\), and from the force balance in axial direction. The procedure is described in previous works.\(^3,14\)

The shrinkage of the ingot is equal to the negative value of displacement of the surface which is obtained by integration of Eq. (10)

\[
u(r_0, t) = \int_{t_0}^{t} \hat{u}(r_0, t) dt \tag{12}
\]

The used material properties are given in the Appendix.

### 4.3. Results of Model

Based on the temperature curves evaluated as described in the preceding section, the ingot shrinkage has been calculated for the performed experiments. The comparison between the calculated and measured shrinkage–time curve is given in Fig. 11 for the selected experiment with 0.34 mass% carbon. Finally, in Fig. 12 the experimentally and theoretically determined shrinkage values are presented as a function of carbon content for various time steps.

### 5. Shrinkage Values for Room Temperature

The last measurement with the sensors was taken in each experiment after cooling of the ingot to room temperature. Then, after removal from the mold, the diameter of the ingot was measured with a slide gauge, for the comparison. The obtained data are given in Fig. 13. There is no difference between both sets of data within the scatter of the points, Fig. 13(a), confirming the reliability of the sensor measurements. In Fig. 13(b) the “free shrinkage”—\(\varepsilon^{(1)} r\) is plotted as determined from the thermal contraction \(\varepsilon^{(1)}\) at room temperature. The thermal contraction (right ordinate in Fig. 13(b)) is obtained from the difference of densities at solidus temperature and at room temperature using the density functions given previously.\(^9\)

It is evident that the free shrinkage (thermal contraction)
does not exhibit a peak at about 0.1 mass% carbon at room temperatures as it does in the high temperature range below solidus. At carbon contents below 0.09 mass% carbon the free shrinkage is a little higher than that above 0.164 mass% carbon. This is caused by the fact that the density difference is larger when the material is δ(α) phase at both temperatures than it is if γ phase (with higher density than δ phase) is present at solidus temperatures and α phase + cementite at room temperature.

The measured values are close to the free shrinkage values. This is surprising at first. However, it can be proved from Eq. (10), applying it for the simplified case of $e^\alpha = \alpha T$ with $\alpha = \text{constant}$ and $E = \text{constant}$, that the shrinkage (displacement of the ingot surface) is exactly equal to the free shrinkage if the rate of temperature change $T$ is independent of radial coordinate. If $T$ becomes less negative towards the surface (lower cooling rate at the surface) the shrinkage increases compared to the free shrinkage, and if $T$ is less negative in the interior (lower cooling rate in the interior than at the surface) it decreases. During solidification the cooling at the surface of the shell may occur faster initially than in the interior of the growing shell, and later slower. After completion of solidification the cooling occurs faster in the interior of the ingot than at its surface over some time. In total, however, the variation of $T$ with $r$ is not large, see also Fig. 9. Hence, it appears to be reasonable that the measured shrinkage is close to the free shrinkage.

6. Conclusions

Hence, it is evident that the shrinkage peak at about 0.1 mass% carbon can be reproduced also with the theoretical model. It appears, both from the measurements and from the computations, that the peak develops very early and that the difference between the "ground" and the maximum is about 0.3 mm in a cylindrical ingot of 70 mm radius and stays rather constant. This is explained by the fact that only the thin solid layer present in the initial stage of solidification responds, by increased shrinkage, to the increased thermal contraction values in the δ/γ band, Fig. 2. In the later stages of solidification the outer solid layer which is transformed already to γ-iron is thick and stiffens the whole shell so that the δ/γ transformation occurring more to the interior cannot produce much contraction of the ingot surface.

The δ/γ transformation occurs over the concentration range of 0 to 0.164 mass% carbon (point I) in the iron–carbon system. But, its effect on the ingot (strand) shrinkage during solidification depends on the size of thermal contraction associated to it and on the temperature range in which it occurs, Fig. 1. In the range of 0 to 0.09 mass% carbon (point H) all the solid is first in the δ-phase and, consequently, the thermal contraction due to the transformation is about the same after complete conversion to γ-iron. But, at 0.09 mass% carbon (point I) the transformation occurs at the highest temperatures (start at peritectic temperature, 1,494.6°C in iron–carbon system). Hence, in ingot solidification of iron carbon steels the δ/γ band, Fig. 2, begins to develop very early when the surface temperature has dropped below the peritectic temperature, that is when the solid skin just starts its growth. With decreasing carbon content the δ/γ transformation is shifted to lower temperatures and, consequently, it will begin later at the ingot surface when the shell is already thicker. So, it is to be expected that the ingot shrinkage decreases from its peak value at
about 0.1 mass% carbon with decreasing carbon content. At zero carbon content the transformation temperature is 1392°C (in pure iron). It takes some seconds till the surface temperature has dropped so far and during this time period the shell has grown already to a thickness of about 2 mm or so. At carbon contents larger than about 0.1 mass% the \( \partial / \gamma \) transformation is still shifted to somewhat higher temperatures, but the size of thermal contraction which it produces decreases with increasing carbon content to reach zero at 0.164 mass% carbon (point I in iron–carbon system). Consequently, its contribution to the shrinkage also decreases to zero at 0.164 mass% carbon.

It is also of interest to note that at room temperature there is no shrinkage peak any more at about 0.1 mass% carbon, and that the shrinkage is close to the free shrinkage.

Nomenclature

- \( h \): Heat transfer coefficient between ingot and mold
- \( l \): Longitudinal coordinate in alumina rod of sensor
- \( l_s \): Length of alumina rod
- \( \Delta I_s \): Change of length of alumina rod
- \( q_s \): Heat flux density at surface of ingot due to radiation
- \( r \): Radial coordinate
- \( r_i \): Radius of ingot surface
- \( r_m \): Radius of solidus contour (after solidification \( r_s=0 \))
- \( r_o \): Radius of outer mold surface
- \( r_n \): Radius of inner mold surface (in heat flow model \( r_n=r_i \))
- \( u \): Radial displacement
- \( u_m \): Radial displacement of outer mold surface
- \( s_1, s_2, s_3, s \): Apparent shrinkage values of the three sensors, average shrinkage
- \( t \): Time (\( t=0 \) when steel melt reaches measuring plane)
- \( t_o \): Time at which solidus temperature is reached at the ingot surface (\( r=r_o \))
- \( E \): Young’s modulus
- \( H \): Enthalpy per mass unit
- \( T \): Temperature (always in °C)
- \( T_0 \): Temperature profile in the mold at \( t=0 \)
- \( T_i \): Temperature in the ingot at \( t=0 \)
- \( T_m \): Temperature at the outer surface of the mold (obtained by extrapolation from the measured temperature profile in the mold)
- \( T_l \): Liquidus temperature
- \( T_s \): Solidus temperature
- \( \varepsilon_s \): Thermal expansion of solid steel, related to solidus temperature, therefore negative (called “thermal contraction”)
- \( \varepsilon_m \): Thermal expansion of mold, related to 25°C
- \( \varepsilon_o \): Thermal expansion of alumina rod of sensor, related to 25°C
- \( \lambda \): Thermal conductivity
- \( v \): Poisson’s ratio
- \( \rho \): Density
- \( \sigma_i \): Stress components, \( i=r, \theta, z \)
- \( \sigma^* \): Reduced stress components (\( \sigma^*_i=\sigma_i/E \))

\[ \dot{f} = \frac{\partial f}{\partial t} \]

Partial derivative of function \( f \) with respect to time

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Appendix. Materials Properties

Enthalpy

An algorithm for the enthalpy of iron–carbon alloys is contained in the thermal tracking software developed at the institute. It is based on the known enthalpy–temperature function for pure iron, graphite and cementite, and on the enthalpy of mixing in the melt and in austenite.

The mold consists of grey cast iron and has a carbon content of 3.8 mass%. It was assumed that all the carbon is present as graphite. Consequently, the enthalpy of the mold was also computed with the available algorithm for iron–carbon alloys.

Density and Thermal Contraction

The density and thermal contraction of iron–carbon alloys was taken from a previous paper. The density of the mold material was computed from the value 6920 kg m⁻³ for GGG40 at 20°C for GGG40 at 20°C for GGG40 at 20°C and the measured thermal expansion given by Eq. (2).

Thermal Conductivity

An algorithm for the thermal conductivity of iron–carbon alloys has been developed by evaluating the literature data. The following equations were deduced

\[ \lambda_{\infty}=80.91 - 9.9269 \cdot 10^{-2} T + 4.613 \cdot 10^{-5} T^2 \quad [W m^{-1} K^{-1}] \]

\[ \lambda_{\infty} \] (A1)
\[ \lambda_f = 20.14 + 9.319 \cdot 10^{-3} T \quad [\text{W m}^{-1} \text{K}^{-1}] \quad \text{(A2)} \]
\[ \lambda_d = 21.60 + 8.35 \cdot 10^{-3} T \quad [\text{W m}^{-1} \text{K}^{-1}] \quad \text{(A3)} \]
\[ \lambda_i = 39 \quad [\text{W m}^{-1} \text{K}^{-1}] \quad \text{(A4)} \]

with \( T \) in °C. For the domain of the homogeneous melt \( \lambda_i = 39 \text{ W m}^{-1} \text{K}^{-1} \) was multiplied with a factor of five to take account of convection.

The influence of carbon on the thermal conductivity of austenite and \( \delta \)-ferrite is probably small,\(^{14}\) and no reliable data exist for iron–carbon melts. Hence, the values for the pure phases are taken for the carbon containing austenite, \( \delta \)-ferrite and melt.

In the heterogeneous zones \( l + \delta, l + \gamma, \gamma + \delta \) and \( \gamma + \alpha \) the conductivities were composed of the conductivities of the phases using the simple mixing law and computing the phase fractions from the Fe–C phase diagram, or from the \( A_\gamma \) and \( A_\alpha \) lines, respectively,\(^{9}\) applying the level rule. The conductivity of the melt in the mushy region is taken to be

\[ \lambda_{l, \text{mushy}} = 39(1 + 4f_l^{100}) \quad [\text{W m}^{-1} \text{K}^{-1}] \quad \text{(A5)} \]

where \( f_l \) is the mass fraction of the liquid. By the use of this equation a transition is attained from the value of 39 W m\(^{-1}\) K\(^{-1}\) in most of the mushy zone to 5 \( \cdot \) 39 W m\(^{-1}\) K\(^{-1}\) in the homogenous melt.

Below \( A_\gamma \) the conductivity depends on carbon content with the influence of carbon increasing with decreasing temperature. The following equations represent the thermal conductivity of \( \alpha + \text{Fe}_3\text{C} \) mixtures\(^{14}\)

\[ \lambda_{\alpha + \text{Fe}_3\text{C}} = \lambda_{\alpha}(1 - k_1C^k) \quad \text{(A6a)} \]
\[ k_1 = 0.425 - 4.385 \cdot 10^{-4} T \quad \text{(A6b)} \]
\[ k_2 = 0.209 + 1.09 \cdot 10^{-3} T \quad \text{(A6c)} \]

with \( T \) in °C.

**Young’s Modulus**

The temperature function for Young’s modulus was taken from previous papers.\(^{16,17}\)