A modified kinetics model of ferrite and pearlite transformations under applied stress is proposed based on the Johnson–Mehl–Avrami (J–M–A) equation and is shown as
\[ f = 1 - \exp(-b(\sigma))^{n(\sigma)} \]
where \( f \) is the transformation fraction of ferrite or pearlite, \( b(\sigma) \) is the transformation function of ferrite or pearlite, \( n(\sigma) \) is the stress function and the derivation of the suggested equation was not shown in their paper. Moreover, they did not concern the ferrite transformation under stress.

The present article attempts to propose a modified kinetics model of isothermal ferrite and pearlite transformations under applied stress based on J–M–A equation and the parameters in the suggested model were determined by regression of the data from experiments. Furthermore, the onset and the end of ferrite and pearlite transformations under applied stress in a 0.38C–Cr–Mo steel and a eutectoid carbon steel were calculated from the suggested modified model and compared with experimental data. Literature has shown that ferrite transformation is inhibited by hydrostatic stress and can be accelerated by monoaxial tension and compression stresses even the former is more effective. To study the origin of the effect of stresses on ferrite and pearlite transformations is an important topic. One of the present author (Hsu) considers that the work done by applied stress will be beneficial for transformation as the chemical driving force for ferrite or pearlite transformation is rather small (than that for bainitic transformation) and will discuss this problem in detail elsewhere.

2. Kinetic Models under Applied Stress

The Johnson–Mehl–Avrami (J–M–A) equation for diffusional transformation without applied stress is expressed as
\[ f = 1 - \exp[-b(0)^{n(0)}] \]
where \( f \) is the transformation fraction of ferrite or pearlite, \( b(0) \) and \( n(0) \) are constants in J–M–A equation without applied stress and \( r \) is isothermal time.

We propose a kinetics model of ferrite and pearlite transformations under applied stress as
\[ f = 1 - \exp[-b(\overline{\sigma})^{n(\overline{\sigma})}] \]
where \( \overline{\sigma} \) is equivalent stress, \( b(\overline{\sigma}) \) and \( n(\overline{\sigma}) \) are constants in modified J–M–A equation under applied stress. Referencing the previous experimental results of ferrite and pearlite transformation is remarkable and the constants \( A \) and \( B \) in the suggested model vary with the different steels and different transformations.

KEY WORDS: dilatometric curve; ferrite /pearlite transformation; J–M–A equation; kinetics under stress.
pearlite transformations as Refs. 4)–6), we suggest that
\( b(\bar{\sigma}) \) and \( n(\bar{\sigma}) \) may be expressed as
\[
\begin{align*}
    b(\bar{\sigma}) &= b(0)(1 + A\bar{\sigma}^B) \quad \cdots \cdots \cdots \cdots \cdots \cdots (3) \\
    n(\bar{\sigma}) &= n(0) \quad \cdots \cdots \cdots \cdots \cdots \cdots (4)
\end{align*}
\]
where \( b(0) \) and \( n(0) \) are constants in J–M–A equation without applied stress, which vary with isothermal temperature, \( t \) is isothermal time, \( A \) and \( B \) are constants, which can be obtained from experiments and are independent on the isothermal temperature.

3. Kinetics of Ferrite and Pearlite Transformations in a 0.38C–Cr–Mo Steel

In order to obtain the parameters \( A \) and \( B \) in the suggested model for a 0.38C–Cr–Mo steel, we measured the expansion ratio during the ferrite and pearlite transformations in this steel without applied stress and under applied stress and derived the relationship between the transformation fraction and expansion ratio based on the elastic-plasticity theory and the transformation plasticity theory. Finally, we can obtain the \( b(\bar{\sigma}), b(0), n(\bar{\sigma}) \) and \( n(0) \) by regression of the experimental results with the J–M–A equation.

3.1. Experimental Method

The chemical composition of the investigated 0.38C–Cr–Mo steel is shown in Table 1. The specimen is 8 mm in diameter and 15 mm in length and the thermal simulation experiments were conducted in a Gleeble 3500 apparatus at isothermal temperatures of 690, 675, 660 and 645°C, under various compression stresses of 0, 15, 30 and 45 MPa, being lower than the yield stress. The schematic illustration of thermal simulation experiment is shown in Fig. 1.

3.2. Measured Dilatometric Curves

The measured dilatometric curves are shown in Fig. 2. It can be seen that the expansion ratio (\( \Delta d/d_0 \), where \( d_0 \) is original diameter of specimen and \( \Delta d \) is the variation of diameter) increases with the isothermal time and applied stress, but also varies with the temperature. Under higher temperature and higher stress, e.g. in Fig. 2(a), as stress \( \leq 45 \) and 30 MPa, the austenite may receive a certain amount of creep deformation during transformation.

| Table 1. The chemical composition of 0.38C–Cr–Mo steel (wt%). |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| C               | P               | S               | Si              | Mn              | Cr              | Mo              |
| 0.38            | 0.013           | 0.01            | 0.28            | 0.55            | 0.92            | 0.2             |

Fig. 1. Schematic illustration of thermal simulation experiment.

Fig. 2. Isothermal dilatometric curves of 0.38C–Cr–Mo steel under a constant compressive stress ranging from 0 to 45 MPa at isothermal holding temperature: (a) 690°C, (b) 675°C, (c) 660°C, and d) 645°C.
most of the measured data may be available for kinetics study.

3.3. Relationship between Expansion Ratio and Transformation Fraction

The schematic illustration of expansion ratio versus isothermal holding time \( t \) during ferrite and pearlite transformations under applied stress or without applied stress is shown in Fig. 3, in which point A denotes the start of ferrite transformation, point B the finish of ferrite transformation or start of pearlite transformation, point C the finish of pearlite transformation, and point D the expansion ratio at isothermal holding time \( t_0 \). Figure 3 shows that the expansion ratio increases with the increment of fraction of ferrite and pearlite transformations and point B indicating abrupt change of slope means that the ferrite transformation finished and the pearlite transformation starts.

The transformation fraction of ferrite or pearlite in Eq. (2) can be expressed as

\[
f = V_i / V_{i\text{max}}
\]

where \( f \) is the transformation fraction of ferrite or pearlite, \( V_{i\text{max}} \) is the final fraction of ferrite or pearlite at isothermal holding temperature, \( V_i \) the fraction of ferrite or pearlite transformation at isothermal holding time \( t \).

Substituting Eq. (5) into Eq. (2), Eq. (2) can be changed into:

\[
\ln \left( 1 - \frac{V_i}{V_{i\text{max}}} \right) = \ln(b(\sigma)) + n(\sigma) \ln t
\]

3.3.1. Relationship between Expansion Ratio and Transformation Fraction without Applied Stress

In Fig. 3, \( (\Delta d/d_0)_i \) denotes the expansion ratio at isothermal holding time \( t_i \). \( V_i^P \), the ferrite fraction, and \( V_i^F \), the pearlite fraction at isothermal holding time \( t_i \) can be obtained as

\[
V_i^P = V_{i\text{max}}^P \left( \frac{\Delta d_i}{d_0} \right)_\text{max} \left( t_i \leq t \leq t_i^F \right)
\]

\[
V_i^F = V_{i\text{max}}^F \left[ \left( \frac{\Delta d_i}{d_0} \right)_\text{max} - \left( \frac{\Delta d_i}{d_0} \right)_\text{min} \right] \left( \frac{\Delta d_i}{d_0} \right)_\text{max} \left( t_i < t < t_i^F \right)
\]

where \( (\Delta d/d_0)_i^P \) and \( (\Delta d/d_0)_i^F \) are the maximum expansion ratio of ferrite transformation and pearlite transformation respectively which can be obtained from the dilatometric curves of isothermal ferrite and pearlitic transformations.

Substituting Eq. (7) or Eq. (8) into Eq. (6) yields the constants \( b(0) \) and \( n(0) \) by linear regression for ferrite or pearlite transformations.

3.3.2. Relationship between Expansion Ratio and Transformation Fraction under Applied Stress

The expansion ratio of ferrite transformation under stress \((\Delta d/d_0)_i^P \), includes the expansion of ferrite transformation, \( (\Delta d/d_0)_i^P \), transformation plasticity of ferrite \((\Delta d/d_0)_i^p \) and the increment of elasticity caused by the variation of elasticity modulus during ferrite transformation under stress \((\Delta d/d_0)_i^E \), i.e.

\[
\frac{\Delta d_i}{d_0}^E = \frac{\Delta d_i}{d_0}^F + \frac{\Delta d_i}{d_0}^p + \frac{\Delta d_i}{d_0}^e
\]

According to the dilatation accompanying phase transformation, the expansion of ferrite transformation can be obtained

\[
\frac{\Delta d_i}{d_0}^E = \beta_i V_i^E
\]

where \( \beta_i \) is the expansion coefficient of ferrite at temperature \( T \) and \( V_i^E \) is the fraction of ferrite at isothermal holding time \( t \).

The increment of elasticity caused by the variation of elasticity modulus during ferrite transformation under stress by the Hook principle is shown as

\[
\left( \frac{\Delta d_i}{d_0}^E \right)_F = 0.3\sigma \left( \frac{1}{E_A} - \frac{1}{E_F} \right)
\]

where \( E_A \) and \( E_F \) are the elasticity modulus of austenite and ferrite respectively, \((\Delta d/d_0)_i^F \) only contributes about 7% to kinetics or \( (\Delta d/d_0)_i^F = 0.07(\Delta d/d_0)_i^E \) for 0.38C–Cr–Mo steel at 645°C under stress of 30 MPa.

Transformation plasticity of ferrite is shown as

\[
\left( \frac{\Delta d_i}{d_0}^p \right)_F = -0.5K_p \sigma V_i^E (2.V_i^E - 1)
\]

where \( \sigma \) is the stress and \( K_p \) is the constant of transformation plasticity of ferrite.

Substituting Eqs. (10), (11) and (12) in which the \( V_i^E \) is replaced by \( V_{i\text{max}}^E \), representing the final fraction of ferrite, into Eq. (9), yields the constant of transformation plasticity of ferrite.
3.4.1. Parameters

3.4. Calculation Results

1.45

and can also obtain the parameters following expression for pearlite transformation:

Substituting Eqs. (10), (11) and (12) into Eq. (9), we obtain the expression of expansion ratio versus the transformation fraction of ferrite under applied stress at an indicated isothermal holding time:

where

Substituting Eqs. (10), (11) and (12) into Eq. (9), we obtain the modified J–M–A equation for the pearlite transformation under applied stress, but changes significantly with the isothermal temperature.

and can also obtain the parameters \( b(\bar{\sigma}) \) and \( n(\bar{\sigma}) \) in the modified J–M–A equation for the pearlite transformation under applied stress.

The contribution of transformation plasticity to kinetics is about 13.5% in 0.38C–Cr–Mo steel at 645°C under stress of 30MPa.

In Eqs. (14) and (15), the linear expansion coefficients of austenite, ferrite and pearlite are taken as 2.35×10⁻⁵/°C, 1.45×10⁻⁵/°C and 1.328×10⁻⁵/°C respectively. The modulus of elasticity for austenite, \( E_A \), and ferrite, \( E_F \), are taken as \( E_A = 197.115.38 \) and \( E_F = 192.30767 \) respectively.

3.4. Calculation Results

3.4.1. Parameters \( b(\bar{\sigma}) \) and \( n(\bar{\sigma}) \) in Eq. (2) for Ferrite Transformation

\( V^f_i \) can be calculated by solving Eq. (14) according to the measured dilatometric curves of 0.38C–Cr–Mo steel under a constant compressive stress and temperature. Substituting various values of \( V^f_i \) into Eq. (6), we can obtain the parameters \( b(\bar{\sigma}) \) and \( n(\bar{\sigma}) \) in Eq. (4) for the ferrite transformation under applied stress by linear regression. The \( b(\bar{\sigma}) \) and \( n(\bar{\sigma}) \) in Eq. (2) for ferrite transformation are shown in Figs. 4 and 5, which indicate that the value \( b(\bar{\sigma}) \) increases with the increment of compression stress and the value \( n(\bar{\sigma}) \) changes very little with the increment of compression stress, but changes significantly with the isothermal temperature.

Through regression of the data of ferrite transformation under different isothermal temperature and applied stresses in Fig. 4 with the form of Eq. (3), we obtain:

where \( b(\bar{\sigma}) \) is constant for ferrite transformation in the modified J–M–A equation under applied stress, \( b(0) \) is the constant for ferrite transformation in J–M–A equation without applied stress.

By comparison of Eq. (3) with Eq. (16), the parameters \( A \) and \( B \) in Eq. (3) can be obtained as 0.063 and 1.05 respectively for ferrite transformation.

3.4.2. Parameters \( b(\bar{\sigma}) \) and \( n(\bar{\sigma}) \) in Eq. (2) for Pearlite Transformation

\( V^p_i \) also can be calculated by solving Eq. (15) according
to the measured dilatometric curves of 0.38C–Cr–Mo steel under a constant compressive stress and temperature. Substituting various values of \( V_i \) into Eq. (6), we can obtain the parameters \( b(\bar{s}) \) and \( n(\bar{s}) \) in Eq. (2) for the pearlite transformation under applied stress by linear regression, as shown in Figs. 6 and 7. It can be seen that the value \( b(\bar{s}) \) increases with the increment of compression stress and the value \( n(\bar{s}) \) also changes very weakly with the compression stress and isothermal temperature.

Through regression of the data of pearlite transformation under different isothermal temperatures and applied stresses in Fig. 6 with the form of Eq. (3), we obtain:

\[
b(\bar{s}) = b(0)(1+0.0028\bar{s}^{0.5}) \quad (17)
\]

where \( b(\bar{s}) \) is a constant for pearlite transformation in the modified J–M–A equation under stress, \( b(0) \) is a constant for pearlite transformation in J–M–A equation without applied stress at a certain temperature.

By comparison of Eq. (3) with Eq. (17), the parameters \( A \) and \( B \) can be obtained as 0.0028 and 0.5 respectively for pearlite transformation.

### 3.5. Comparison of the Calculated and Measured Results in a 0.38C–Cr–Mo Steel

The beginning of ferrite transformation (1%) and end of pearlite transformation (99%) under various applied stress of 0, 15, 30 and 45 MPa at isothermal temperatures 690, 675, 660 and 645°C in 0.38C–Cr–Mo steel are calculated with Eqs. (2), (4), (16) and (17) shown in Figs. 8 and 9 in which the experimental data are also presented.

The metallographic measurement of the ferrite fraction after isothermal transformation at different temperatures...
and various stresses in a 0.38C–Cr–Mo steel is carried out and the comparison with the calculated results from kinetics equations is made as shown in Fig. 10, which implies that nearly linear relation between the dilatometric change and transformed volume fraction still exists. Figures 8, 9 and 10 show that the calculated results from the suggested model are in agreement with experimental data from measured dilatometric curves. The effect of applied stress on both ferrite and pearlite transformations is remarkable and the time required to achieve a particular volume fraction (1% and 99%) of transformation has been shortened by increasing the applied stress. The acceleration of uniaxial compressive stress on the ferrite transformation increases linearly, but on the pearlite transformation exponentially \(\exp(-\sigma^{0.5})\). This may result from the fact that expansion in ferrite transformation is more significant than that in pearlite transformation which includes a contraction process, the precipitation of cementite.

4. Kinetics of Pearlite Transformation in a Eutectoid Carbon Steel

The investigated material in the experiments of Denis et al. is a eutectoid carbon steel with composition shown in Table 2. The parameters \(b(0)\) and \(n(0)\) at isothermal temperature 663°C and 673°C are calculated from the time-temperature-transformation diagram of 0.89-carbon steel\cite{16} whose chemical composition is approximately the same as the eutectoid carbon steel. The parameters \(A\) and \(B\) are obtained by fitting the experimental results of Denis et al.\cite{7} with Eqs. (2)–(4) and the parameters \(b(0), n(0), A\) and \(B\) are listed in Table 3.

Table 2. The chemical composition of eutectoid plain carbon steel (wt%).\cite{7}

<table>
<thead>
<tr>
<th>C</th>
<th>P</th>
<th>S</th>
<th>Si</th>
<th>Mn</th>
<th>Cr</th>
<th>Ni</th>
<th>Al</th>
<th>Cu</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.82</td>
<td>0.02</td>
<td>0.007</td>
<td>0.27</td>
<td>0.73</td>
<td>0.02</td>
<td>0.04</td>
<td>0.05</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Table 3. The parameters used in Eqs. (2)–(4).

<table>
<thead>
<tr>
<th>T (°C)</th>
<th>(b(0))</th>
<th>(n(0))</th>
<th>(A)</th>
<th>(B)</th>
</tr>
</thead>
<tbody>
<tr>
<td>663</td>
<td>3.084×10^4</td>
<td>3.84</td>
<td>0.02</td>
<td>1.55</td>
</tr>
<tr>
<td>673</td>
<td>1.147×10^5</td>
<td>3.66</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

5. Conclusions

(1) The kinetics model of ferrite and pearlite transformation under applied stress can be expressed as:

\[
f = 1 - \exp\left[-\left(\bar{\sigma}\right)^n\right]
\]

\[
b(\bar{\sigma}) = b(0)(1 + A\bar{\sigma}^B)
\]

\[
n(\bar{\sigma}) = n(0)
\]

where \(\bar{\sigma}\) is the equivalent stress, \(A\) and \(B\) are constants which vary with the different steels and different transformations.

(2) The effect of stress on ferrite or pearlite transformations is remarkable, but the effect decreases slightly with the increment of applied stress. The acceleration of applied stress increases linearly for ferrite transformation but exponentially \(\exp(-\sigma^{0.5})\) for pearlite transformation in the

![Fig. 11](image-url) Measured and calculated results of beginning and finishing time of pearlite transformation versus applied stress in a eutectoid carbon steel at isothermal temperature (a) 663°C and (b) 673°C (experimental data from Ref. 7).
0.38C–Cr–Mo steel.

Acknowledgement

The authors are grateful for financial support from the National Natural Science Foundation of China (Grant No. 50075053).

REFERENCES