Physical Models and Algorithms for Device Simulation for High-Speed Image Sensors

(高速イメージセンサのためのデバイス・シミュレーションの物理モデル)

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Abstract Physical models and algorithms for device simulation for high-speed image sensors are proposed. To obtain consistent basic equations for both the device and wave optical simulations, the electromagnetic scalar field is newly introduced to realize current injection and absorption at electrode surfaces and carrier generation-recombination in semiconductors. The propagation of an electromagnetic field induced by electrodes can be considered by using the proposed equations and similar algorithms as used in the FDTD method in device simulation. The 1D field propagations in silicon and Si₃N₄ are calculated by ignoring magnetic field. It is shown that the proposed equations and algorithms can realize device simulation for high-speed image sensors by considering field propagation in semiconductors.

Key words: Maxwell's equations, electromagnetic field, scalar field, charge conservation, high-speed image sensor, field propagation

1. Introduction

The frame rate of high-speed image sensors has been drastically enhanced in the last decade.^{1)~3)} The device simulation of such devices is extremely important to realize high sensitivity and high-speed perfect readout of signal charge from photodiodes.^{1)~3)} In near future, the unification of device and wave optical simulations will be needed by the demand for high frame rate as the streak cameras,⁴⁾ whose time resolution is less than 1 ps, because the propagation time of electric field induced by electrodes cannot be ignored in the future devices. The transit time t_{tr} of an electron for the distance L from the photodiode edge to the readout gate edge is given by $t_{tr} = L^2/(\mu_n V)$, where μ_n is mobility of electron and V is the potential difference between the photodiode and the readout gate edges. On the other hand, the propagation time t_{pr} of electric field for the substrate thickness d is given by $t_{pr} = n_{si}d/c$, where n_{si} is refractive index of silicon and c is speed of light in vacuum. By assuming $n_{si} = 3.0$ and $c = 3 \times 10^{10}$ cm/s, we obtain $t_{pr} = 0.1$ ps for $d = 10$ μm and $t_{pr} = 1$ ps for $d = 100$ μm . Since several times of t_{pr} is necessary to stabilize the potential in silicon as shown later, $t_{tr} > 10t_{pr}$ should

be satisfied to ignore the propagation delay of electric field. Therefore, the propagation delay of electric field cannot be ignored in the case of $t_{tr} < 1$ ps for $d = 10$ μm and $t_{tr} < 10$ ps for $d = 100$ μm . If we assume $V = 3.3$ V and $\mu_n = 1000$ cm²/(Vs), L must be less than 0.6 μm for $t_{tr} < 1$ ps and 1.8 μm for $t_{tr} < 10$ ps. These values of L and t_{tr} as the photodiode width and the driving pulse width of the readout gate seem realistic for the future high-speed image sensors including line type. Therefore, we should consider the propagation delay of the electric field induced by the gate electrodes for the transient simulation as the photodiode charge readout characteristics analysis in the future devices. But there is the serious problem that the basic equations for device simulations are not always consistent with Maxwell's equations on which wave optical simulations are based. In general, wave optical simulation is based on Maxwell's equations,^{12)~14)} written as

$$\mathbf{J} = \nabla \times \mathbf{H} - \varepsilon \frac{\partial \mathbf{E}}{\partial t}, \quad (1)$$

$$\rho = \varepsilon \nabla \cdot \mathbf{E}, \quad (2)$$

$$\nabla \times \mathbf{E} + \mu \frac{\partial \mathbf{H}}{\partial t} = 0, \quad (3)$$

$$\nabla \cdot \mathbf{H} = 0, \quad (4)$$

where ρ is charge density, $\mathbf{J}$ is current density, $\mathbf{H}$ is magnetic field vector, $\mathbf{E}$ is electric field vector, ε is permittivity, and μ is permeability. ε and μ satisfy

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Fig. 1 Pinned photodiode under illumination.

$$\varepsilon\mu = \frac{1}{c^2}, \quad (5)$$

where c denotes speed of light in the material. (1) and (2) result the equation of charge conservation,

$$\nabla \cdot \mathbf{J} + \frac{\partial \rho}{\partial t} = 0. \quad (6)$$

On the other hand, device simulations are ordinarily based on Poisson's and current continuity equations ^{9)~11)}

$$\varepsilon \nabla^2 \psi = -q(N_D - N_A + p - n), \quad (7)$$

$$\nabla \cdot \mathbf{J}_p + q \frac{\partial p}{\partial t} = GR, \quad (8)$$

$$\nabla \cdot \mathbf{J}_n - q \frac{\partial n}{\partial t} = -GR, \quad (9)$$

where ψ is potential, p and n are hole and electron concentration, N_D and N_A are donor and acceptor ion concentration, $\mathbf{J}_p$ and $\mathbf{J}_n$ are hole and electron current density, q is magnitude of electronic charge, and GR is the carrier generation-recombination rate.

For example, we think the current analysis of a pinned photodiode⁷⁾ as shown in **Fig. 1**. Electrodes exist in the n-type photodiode region and the backside of the p-type substrate, where the substrate electrode is connected to the ground and the applied voltage of photodiode electrode V_{pd} is positive value. Under illumination, electron and hole pairs are generated by light. Then the electron current is absorbed by the photodiode electrode and the hole current is absorbed by the substrate electrode. If light intensity and V_{pd} do not depend on time, current and charge distribution in the device are constant. Then, the electrode region does not satisfy equation (6), because charge density is independent on time and divergence of $\mathbf{J}$ is not zero at electrode surfaces. In addition to the electrode region, the light current generating region does not satisfy (1), (2) and (6). The total current $\mathbf{J}$ and total charge density ρ are given by the summation of current and charge

of electrons, holes, and ions,

$$\mathbf{J} = \mathbf{J}_p + \mathbf{J}_n, \quad (10)$$

$$\rho = \rho_p + \rho_n + \rho_{ion}, \quad (11)$$

where $\rho_p = q\rho$, $\rho_n = -qn$, and $\rho_{ion} = q(N_D - N_A)$. Since Maxwell's equations satisfy the principle of superposition,¹³⁾¹⁴⁾ the total electric and magnetic fields can be also given by the summation of those for electrons, holes, and ions,

$$\mathbf{E} = \mathbf{E}_p + \mathbf{E}_n + \mathbf{E}_{ion}, \quad (12)$$

$$\mathbf{H} = \mathbf{H}_p + \mathbf{H}_n, \quad (13)$$

where $\mathbf{E}_p$, $\mathbf{E}_n$, and $\mathbf{E}_{ion}$ are electric fields induced by holes, electrons, and ions, and $\mathbf{H}_p$ and $\mathbf{H}_n$ are magnetic field induced by holes and electrons, respectively. Although $\mathbf{J}$, ρ , $\mathbf{E}$, and $\mathbf{H}$ satisfy (1)-(4), the hole current and its induced field $\mathbf{J}_p$, ρ_p , $\mathbf{E}_p$, $\mathbf{H}_p$ and electron current and its induced field $\mathbf{J}_n$, ρ_n , $\mathbf{E}_n$, $\mathbf{H}_n$ do not satisfy (1) and (2). Because (8) and (9) need additional terms of vector $\vec{\alpha}$ and scalar β as

$$\mathbf{J}_p = \nabla \times \mathbf{H}_p - \varepsilon \frac{\partial \mathbf{E}_p}{\partial t} + \vec{\alpha}, \quad (14)$$

$$\rho_p = \varepsilon \nabla \cdot \mathbf{E}_p + \beta, \quad (15)$$

$$\mathbf{J}_n = \nabla \times \mathbf{H}_n - \varepsilon \frac{\partial \mathbf{E}_n}{\partial t} - \vec{\alpha}, \quad (16)$$

$$\rho_n = \varepsilon \nabla \cdot \mathbf{E}_n - \beta, \quad (17)$$

where $\vec{\alpha}$ and β are differences from Maxwell's equations for current and charge density, respectively, and satisfy

$$\nabla \cdot \vec{\alpha} + \frac{\partial \beta}{\partial t} = GR. \quad (18)$$

Consequently, the charge conservation equation (6) is too strict for device simulation to calculate current injection and absorption at electrode surfaces and carrier generation and recombination in semiconductor.

2. Modification of Maxwell's Equations

Using vector potential $\mathbf{A}$ and scalar potential ψ , electric field vector $\mathbf{E}$ and magnetic field vector $\mathbf{H}$ are written as

$$\mathbf{E} = -\nabla \psi - \frac{\partial \mathbf{A}}{\partial t}, \quad (19)$$

$$\mathbf{H} = \frac{1}{\mu} \nabla \times \mathbf{A}. \quad (20)$$

In order to realize current injection and absorption at electrode surfaces and carrier generation and recombination in semiconductor, we newly introduce the electromagnetic scalar field γ defined by

$$\gamma = \nabla \mathbf{A} + \frac{1}{c^2} \frac{\partial \psi}{\partial t}. \quad (21)$$

In Lorenz gauge condition⁽¹³⁾⁽¹⁴⁾ of $\gamma = 0, \mathbf{J}, \rho, \mathbf{A}$, and ψ satisfy

$$\mu \mathbf{J} = -\square \mathbf{A}, \quad (22)$$

$$\rho = -\varepsilon \square \psi, \quad (23)$$

where $\square$ is d'Alembertian defined by

$$\square \equiv \nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}. \quad (24)$$

If we assume (22) and (23) even in the case of $\gamma \neq 0$, current and charge density equations (1) and (2) are rewritten by

$$\mathbf{J} = \nabla \times \mathbf{H} - \varepsilon \frac{\partial \mathbf{E}}{\partial t} - \frac{1}{\mu} \nabla \gamma, \quad (25)$$

$$\rho = \varepsilon \nabla \mathbf{E} + \varepsilon \frac{\partial \gamma}{\partial t}. \quad (26)$$

Therefore, the carrier generation-recombination rate is given by

$$GR = \nabla \mathbf{J} + \frac{\partial \rho}{\partial t} = -\frac{1}{\mu} \square \gamma. \quad (27)$$

The above relation permits current injection and absorption at electrode surfaces and carrier generation and recombination in semiconductor. It should be noticed that $GR = 0$ needs not $\gamma = 0$ but $\square \gamma = 0$. When we consider about Lorentz covariance,⁽¹³⁾⁽¹⁴⁾ $\mathbf{E}, \mathbf{H}, \mathbf{A}, \psi, \mathbf{J}$, and ρ have same transformation as original Maxwell's equations. Because the relations among $\mathbf{E}, \mathbf{H}, \mathbf{A}$, and ψ are same as original Maxwell's equations, although γ, GR , and d'Alembertian operator $\square$ are not changed by Lorentz transformation. In addition to Lorentz covariance, (19)-(21) satisfy the gauge invariance⁽¹³⁾⁽¹⁴⁾ by the transformation of

$$\mathbf{A}' = \mathbf{A} + \nabla \chi, \quad (28)$$

$$\psi' = \psi - \frac{\partial \chi}{\partial t}, \quad (29)$$

for the scalar function χ which satisfies

$$\square \chi = 0. \quad (30)$$

3. Simulation Algorithm for Electromagnetic Field Propagation

When we introduce conductivity σ and assume $\mathbf{J} = \sigma \mathbf{E}$, (25) gives

$$\frac{\partial \mathbf{E}}{\partial t} = -\frac{\sigma}{\varepsilon} \mathbf{E} + \frac{1}{\varepsilon} \nabla \times \mathbf{H} - c^2 \nabla \gamma. \quad (31)$$

(3) and (26) give

Fig. 2 Relation among electromagnetic field parameters and grids.

$$\frac{\partial \mathbf{H}}{\partial t} = -\frac{1}{\mu} \nabla \times \mathbf{E}, \quad (32)$$

$$\frac{\partial \gamma}{\partial t} = \frac{\rho}{\varepsilon} - \nabla \mathbf{E}. \quad (33)$$

If we define ψ, ρ and γ on grid points, $\mathbf{E}$ and $\mathbf{J}$ on the center of neighboring grid points, and $\mathbf{H}$ at the center of the faces of the cell bricks as shown in **Fig. 2**, we obtain the equations for discretized field propagation by using similar algorithm as FDTD method,⁽¹⁵⁾⁽¹⁶⁾ as shown in Appendix. It is difficult to consider the propagation of magnetic field induced by gate electrodes, because of complexity of current calculation in gate electrodes. If we consider only electric and scalar fields, the simulation of the field propagation is much simpler than the above case, by assuming

$$|\nabla \times \mathbf{H}| \ll \left| \varepsilon \frac{\partial \mathbf{E}}{\partial t} + \frac{1}{\mu} \nabla \gamma \right|. \quad (34)$$

Then we obtain

$$E_x^{n+1}(i + \frac{1}{2}, j, k) = \frac{1 - \frac{\sigma}{2\varepsilon} \Delta t}{1 + \frac{\sigma}{2\varepsilon} \Delta t} E_x^n(i + \frac{1}{2}, j, k) - \frac{c^2 \Delta t}{\Delta x (1 + \frac{\sigma}{2\varepsilon} \Delta t)} \{ \gamma^{n+\frac{1}{2}}(i+1, j, k) - \gamma^{n+\frac{1}{2}}(i, j, k) \}, \quad (35)$$

$$E_y^{n+1}(i, j + \frac{1}{2}, k) = \frac{1 - \frac{\sigma}{2\varepsilon} \Delta t}{1 + \frac{\sigma}{2\varepsilon} \Delta t} E_y^n(i, j + \frac{1}{2}, k) - \frac{c^2 \Delta t}{\Delta y (1 + \frac{\sigma}{2\varepsilon} \Delta t)} \{ \gamma^{n+\frac{1}{2}}(i, j+1, k) - \gamma^{n+\frac{1}{2}}(i, j, k) \}, \quad (36)$$

$$E_z^{n+1}(i, j, k + \frac{1}{2}) = \frac{1 - \frac{\sigma}{2\varepsilon} \Delta t}{1 + \frac{\sigma}{2\varepsilon} \Delta t} E_z^n(i, j, k + \frac{1}{2}) - \frac{c^2 \Delta t}{\Delta z (1 + \frac{\sigma}{2\varepsilon} \Delta t)} \{ \gamma^{n+\frac{1}{2}}(i, j, k+1) - \gamma^{n+\frac{1}{2}}(i, j, k) \}, \quad (37)$$

$$\gamma^{n+\frac{1}{2}}(i, j, k) = \gamma^{n-\frac{1}{2}}(i, j, k) + \frac{\Delta t}{\varepsilon} \rho(i, j, k)$$

$$\begin{aligned}
 & -\frac{\Delta t}{\Delta x} \{E_x^n(i + \frac{1}{2}, j, k) - E_x^n(i - \frac{1}{2}, j, k)\} \\
 & -\frac{\Delta t}{\Delta y} \{E_y^n(i, j + \frac{1}{2}, k) - E_y^n(i, j - \frac{1}{2}, k)\} \\
 & -\frac{\Delta t}{\Delta z} \{E_z^n(i, j, k + \frac{1}{2}) - E_z^n(i, j, k - \frac{1}{2})\}, \quad (38)
 \end{aligned}$$

i, j, k , and n denote the grid address along x, y, z , and time axis, respectively, and γ is defined on the middle time between the time grids on which ψ and $\mathbf{E}$ are defined. The boundary condition on the electrode surface is given by

$$\gamma^{n+\frac{1}{2}}(i_e, j_e, k_e) = \frac{\psi^{n+1}(i_e, j_e, k_e) - \psi^n(i_e, j_e, k_e)}{c^2 \Delta t}, \quad (39)$$

where i_e, j_e , and k_e denote the grid address of electrode surface and $\psi^n(i_e, j_e, k_e)$ and $\psi^{n+1}(i_e, j_e, k_e)$ are given by the electrode voltage condition. We can obtain γ and $\mathbf{E}$ by alternately substituting them to (35)-(38), after calculating $\gamma^{n+\frac{1}{2}}(i_e, j_e, k_e)$ in every time step. The transient analysis of device simulation including electromagnetic field propagation can be achieved by solving current continuity equations (8) and (9) at the appropriate time grids with obtaining the potential distribution by the above method.

4. Propagation of Electric Field Induced by Electrodes

Using the above algorithm, time dependence of 1D potential distribution was calculated in the simple structure shown in **Fig. 3**. The gate and substrate electrodes exist at the top and bottom of the structure consisting of silicon or Si_3N_4 with thickness of $10 \mu\text{m}$. The substrate electrode connects to the ground and the gate electrode voltage V_g linearly changes from 5 V to 0 V in 0.1 ps as shown in **Fig. 4**. The depth grid space was set to 10 nm for both structures and the time grid space was set to 0.05 fs and 0.025 fs for silicon and Si_3N_4 structures, respectively. The refractive indices of silicon and Si_3N_4 are 3.42 and 2.05, and the absorption coefficients of silicon and Si_3N_4 are assumed to be $1 \times 10^5 \text{ m}^{-1}$ and zero, respectively. The conductivity σ is calculated by

$$\sigma = \varepsilon_0 c_0 n \alpha, \quad (40)$$

where ε_0, c_0, n , and α are permittivity of free space, speed of light in vacuum, refractive index, and absorption coefficient, respectively.

Fig. 5 shows the potential distribution at the time of 0, 0.04, 0.2, 0.4, 0.6, and 0.8 ps in silicon (solid line) and Si_3N_4 (broken line). Although the amplitude of the

Fig. 3 Structure for the calculation of potential dependence on time.

Fig. 4 V_g dependence on time.

potential vibration in silicon decreases with time, that in Si_3N_4 does not change after 0.2 ps. Because the field energy is conserved in Si_3N_4 by reflection of the gate and the substrate electrodes, although it is absorbed in silicon. The transient analysis as readout characteristics of signal charge from photodiodes in 2D or 3D structures can be executed by calculating electron current at the appropriate time grids with obtaining the potential distribution by the above method.

5. Conclusion

In order to realize current injection and absorption at electrode surfaces and carrier generation-recombination in semiconductor, electromagnetic scalar field was newly introduced. It gives the consistent basic equations for the both of device and wave optical simulations. The 1D field propagations in silicon and Si_3N_4 were calculated by the proposed equations and algorithms. It was shown that the proposed equations and algorithms can realize the device simulation for high-speed image sensors with considering field propagation in semiconductor.

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Fig. 5 Potential dependence on time. (Solid line) Silicon, (Broken line) Si_3N_4 , (a) $t = 0$, (b) $t = 0.04$ ps, (c) $t = 0.2$ ps, (d) $t = 0.4$ ps, (e) $t = 0.6$ ps, (f) $t = 0.8$ ps.

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[Appendix]

The propagation of the discretized electromagnetic fields $\mathbf{E}$, $\mathbf{H}$, and γ are given by

$$\begin{aligned}
 E_x^{n+1}(i + \frac{1}{2}, j, k) = & \frac{1 - \frac{\sigma}{2\epsilon}\Delta t}{1 + \frac{\sigma}{2\epsilon}\Delta t} E_x^n(i + \frac{1}{2}, j, k) \\
 & + \frac{\Delta t}{\epsilon\Delta y(1 + \frac{\sigma}{2\epsilon}\Delta t)} \{ H_z^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k) \\
 & - H_z^{n+\frac{1}{2}}(i + \frac{1}{2}, j - \frac{1}{2}, k) \} - \frac{\Delta t}{\epsilon\Delta z(1 + \frac{\sigma}{2\epsilon}\Delta t)} \\
 & \{ H_y^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) - H_y^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k - \frac{1}{2}) \} \\
 & - \frac{c^2\Delta t}{\Delta x(1 + \frac{\sigma}{2\epsilon}\Delta t)} \{ \gamma^{n+\frac{1}{2}}(i + 1, j, k) - \gamma^{n+\frac{1}{2}}(i, j, k) \},
 \end{aligned} \tag{app.1}$$

$$\begin{aligned}
 E_y^{n+1}(i, j + \frac{1}{2}, k) = & \frac{1 - \frac{\sigma}{2\epsilon}\Delta t}{1 + \frac{\sigma}{2\epsilon}\Delta t} E_y^n(i, j + \frac{1}{2}, k) \\
 & + \frac{\Delta t}{\epsilon\Delta z(1 + \frac{\sigma}{2\epsilon}\Delta t)} \{ H_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) \\
 & - H_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k - \frac{1}{2}) \} - \frac{\Delta t}{\epsilon\Delta x(1 + \frac{\sigma}{2\epsilon}\Delta t)} \\
 & \{ H_z^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k) - H_z^{n+\frac{1}{2}}(i - \frac{1}{2}, j + \frac{1}{2}, k) \} \\
 & - \frac{c^2\Delta t}{\Delta y(1 + \frac{\sigma}{2\epsilon}\Delta t)} \{ \gamma^{n+\frac{1}{2}}(i, j + 1, k) - \gamma^{n+\frac{1}{2}}(i, j, k) \},
 \end{aligned} \tag{app.2}$$

$$\begin{aligned}
 E_z^{n+1}(i, j, k + \frac{1}{2}) = & \frac{1 - \frac{\sigma}{2\epsilon}\Delta t}{1 + \frac{\sigma}{2\epsilon}\Delta t} E_z^n(i, j, k + \frac{1}{2}) \\
 & + \frac{\Delta t}{\epsilon\Delta x(1 + \frac{\sigma}{2\epsilon}\Delta t)} \{ H_y^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) \\
 & - H_y^{n+\frac{1}{2}}(i - \frac{1}{2}, j, k + \frac{1}{2}) \} - \frac{\Delta t}{\epsilon\Delta y(1 + \frac{\sigma}{2\epsilon}\Delta t)}
 \end{aligned}$$

$$\begin{aligned} & \{H_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) - H_x^{n+\frac{1}{2}}(i, j - \frac{1}{2}, k + \frac{1}{2})\} \\ & - \frac{c^2 \Delta t}{\Delta z(1 + \frac{\sigma}{2\varepsilon} \Delta t)} \{\gamma^{n+\frac{1}{2}}(i, j, k + 1) - \gamma^{n+\frac{1}{2}}(i, j, k)\}, \end{aligned} \quad (\text{app.3})$$

$$\begin{aligned} & H_x^{n+\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) = H_x^{n-\frac{1}{2}}(i, j + \frac{1}{2}, k + \frac{1}{2}) \\ & - \frac{\Delta t}{\mu \Delta y} \{E_z^n(i, j + 1, k + \frac{1}{2}) - E_z^n(i, j, k + \frac{1}{2})\} \\ & + \frac{\Delta t}{\mu \Delta z} \{E_y^n(i, j + \frac{1}{2}, k + 1) - E_y^n(i, j + \frac{1}{2}, k)\}, \end{aligned} \quad (\text{app.4})$$

$$\begin{aligned} & H_y^{n+\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) = H_y^{n-\frac{1}{2}}(i + \frac{1}{2}, j, k + \frac{1}{2}) \\ & - \frac{\Delta t}{\mu \Delta z} \{E_x^n(i + \frac{1}{2}, j, k + 1) - E_x^n(i + \frac{1}{2}, j, k)\} \\ & + \frac{\Delta t}{\mu \Delta x} \{E_z^n(i + 1, j, k + \frac{1}{2}) - E_z^n(i, j, k + \frac{1}{2})\}, \end{aligned} \quad (\text{app.5})$$

$$\begin{aligned} & H_z^{n+\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k) = H_z^{n-\frac{1}{2}}(i + \frac{1}{2}, j + \frac{1}{2}, k) \\ & - \frac{\Delta t}{\mu \Delta x} \{E_y^n(i + 1, j + \frac{1}{2}, k) - E_y^n(i, j + \frac{1}{2}, k)\} \\ & + \frac{\Delta t}{\mu \Delta y} \{E_x^n(i + \frac{1}{2}, j + 1, k) - E_x^n(i + \frac{1}{2}, j, k)\}, \end{aligned} \quad (\text{app.6})$$

$$\begin{aligned} & \gamma^{n+\frac{1}{2}}(i, j, k) = \gamma^{n-\frac{1}{2}}(i, j, k) + \frac{\Delta t}{\varepsilon} \rho(i, j, k) \\ & - \frac{\Delta t}{\Delta x} \{E_x^n(i + \frac{1}{2}, j, k) - E_x^n(i - \frac{1}{2}, j, k)\} \\ & - \frac{\Delta t}{\Delta y} \{E_y^n(i, j + \frac{1}{2}, k) - E_y^n(i, j - \frac{1}{2}, k)\} \\ & - \frac{\Delta t}{\Delta z} \{E_z^n(i, j, k + \frac{1}{2}) - E_z^n(i, j, k - \frac{1}{2})\}. \end{aligned} \quad (\text{app.7})$$

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