Image Restoration Using Spatially Variant Innovative Inverse Filter with Edge Sharpening

Abstract  A technique is described for restoring digital images using an innovative spatially variant inverse filter that corrects for the image blurring caused when the point spread function (PSF) of a real imaging system with a lens changes its size or shape. The PSFs are estimated for each small local area using a recursive method, and an inverse filter is constructed for each area. The cost function for determining the PSFs is a function of the thickness of the phase-only-synthesis image-edge line. To obtain high-quality images, a new image sharpening technique is applied to the restored images. To reduce noise in the images, the perturbation method is introduced in our work. Simulation showed that this technique restores digital images better than conventional techniques.

Key words: Image restoration, Spatially variant degradation model, Edge sharpening

1. Introduction

Recently digital image is used in a variety of fields such as astronomy, medical electronics, and internet-communication etc. Such fields often need a processing of these images. Especially restoring and edge sharpening of a blurred image are required so frequently. Here we propose a new technique for restoration and sharpening. The traditional image restorations seldom consider the alteration in the size and the shape of point spread function for one frame of the image. Many real imaging systems, however, are actually spatially variant). A.J.Patti, A.M.Tekalp, et.al. proposed a space-varying image restoration filter). They used a Kalman filter with a reduced-order model. But the imaging system is restricted by only motion(rotation) blurring. And hence it cannot be applied to a general two-dimensional(2-D) space-variant PSF image system directly. Another technique) includes projections onto convex sets(POCS). The observation model, however, is idealized and seems not to fit to the actual case so well. Actually, although POCS method has an approach based on local Fourier transform(LFT) analysis), the iteration can only identify the one parameter radius of the PSF. This means that the all space variant PSF has the same figure(circular PSF) with only different radius. And furthermore a priori information about the space-varying degradation process, the noise static, and the ideal image itself is needed. Hence in a realistic blurring case under consideration here, it looks hard to use the POCS method. Other linear space-variant(LSV) restoration filters are given by the). They applied the 2-D reduced update Kalman filtering (RUKF), and 2-D reduced order model Kalman filtering(ROMKF). For the case of Kalman filtering, propagating and updating the whole state needs an extraordinarily large amount of processing. In order to reduce these computational requirements, the RUKF propagates the entire state, but limits updates to only within a neighborhood of the present pixel by employing a sub-optimal state propagation equation. Again, the filtering problem of the actual observation model appears because of their usage of the causal auto-regressive(AR) model. The realistic spatially variant filter has been proposed by Trussell. These are sectioned method but the PSF of each section has been assumed known. We also take the sectioned method from the viewpoint of filtering efficiency but the PSF of each section is estimated.

In this paper, first the typical standard single focal lenses and zoom lenses are inspected for identifying the PSF figure. And the experiment shows that both the size and the shape of the PSF are different between an
image center and the side areas. In this main stream a spatial variant inverse filter is constructed through the estimated PSF. Section 2 represents a typical PSF of a lens together with its Fourier-Bessel pattern. Section 3 shows an image restoration filter presented in this work. Here a recursive technique for estimating the PSF on each area is utilized. The cost function of this recursive system is made from the sharpness of the phase-only synthesis (POS) and in this section the traditional restoration filter is also introduced briefly. Section 4 reveals in detail how to determine the PSF on each area. Finally a new image edge sharpening is described in section 5. Concluding remarks are followed to section 5.

### 2. Point spread function of the lens system

PSF of an imaging system is almost always required in an image restoration problem. Generally the shape of PSF in an original (non-degraded) image is considered to be a real point light source with radius half pixel, and its Fourier transform is constant, so nothing will be changed from the original image through the imaging system. But in an out of focus case the size of PSF will become large to 2 or 3 pixels, or more. Since the Fourier transform of such PSF is just like the shape of Mexican hat with plus and minus area, both the amplitude and the phase of the observation image are different from those of the original image respectively. Here we have checked the PSF figures of a typical single focal and zoom lenses.

#### 2.1 Point Spread Function of Typical Lens

A point-spread function of a camera lens is a function of distance among the lens, an infocus plane (object), and outfocus plane (background). And furthermore the focal length of the lens and the figure of the aperture should be considered. Hence, strictly speaking, PSF's shape is highly complex and it is space variant. Fig. 1(a) and Fig. 1(b) are the typical PSF's figures of the standard single focal lens. Even for a zoom lens we have also gotten almost the same result when the degree of the out focus is not high. In the case of high degree out of focus the PSF figures become quite different ones for each lens because of the several lens aberrations and the lens aperture eclipse. But in a restoration problem the outfocus degree i.e., the blurring level is usually not so high because a serious singularity problem rises up in the highly blurred image and it is very hard to restore the image. Hence we utilize the property of PSF figures like shown in Fig. 1(b) in this paper. From these figures we see that the PSF has a same shape through all of the image area when it is infocused, but when it is out-focused the shape of PSF changes generally depending on the position. Especially from the Fig. 1(b) we can guess the PSF figure roughly at any point if we know the central PSF size (radius of the circle), the point angle and the distance from the center. We use these guesses about the PSF figures as initial values for our proposed recursive method.

#### 2.2 Fourier-Bessel pattern

Usually the figure of the PSF is assumed to be cylindrical (circle) pattern in a traditional image restoration problem. And the imaging system has been usually assumed invariant. But in fact our experiment shows us that the PSF figure changes slightly from position to position even in an excellent camera lens. This means that an imaging system should be variant. The cylindrical shape is shown in Fig. 2(a). The Fourier transform of this shape is also shown in Fig. 2(b) and is called Fourier-Bessel function. Fig. 3(a) is a typical PSF of a camera lens at the marginal position of image frame. The Fourier transform of Fig. 3(a) is shown in Fig. 3(b). It looks similar to that of Fig. 2(b) but actually it is not radial symmetry. Consequently the
magnitude and the phase (plus or minus sign) are both different from those of Fig. 2(3). In this paper we estimate the PSF at each point by using a recursive way described in section 4.

3. Image Restoration Filter

3.1 Image blurring model and restoration filter

In the case that an image \( f(x, y) \) is degraded by the spatially variant system with a point spread function \( h(x_1, y_1, x, y) \), we can express the blurring model\(^{12}\) as

\[
g(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y, \alpha, \beta) f(\alpha, \beta) d\alpha d\beta \quad (1)
\]

where \((x, y)\) is the point \((x, y)\) on the observation plane. The PSF \( h(x, y, \alpha, \beta) \), however, is not so sensitive function of \(x, y, \alpha\) and \(\beta\). That is to say, the PSF is almost the same for any point in a small region. Hence we can approximate the observation model by using succeeding locally space-invariant ones\(^8\). Usually PSF’s actual size is considered to be \(3 \times 3 \sim 9 \times 9\) depend upon the picture size for digital image restoration. If the size of PSF is much bigger, then the resulting degenerated image is too much blurred to restore because of its inherent singularity problem\(^{12}\). Consequently in the tiny region \(R_e\) Eq.1 can be approximated as

\[
g(x, y) = \int_{R_e} \int h_{R_e}(x - \alpha, y - \beta) f(\alpha, \beta) d\alpha d\beta \quad (2)
\]

where \(h_{R_e}(x, y)\) is a PSF in the region \(R_e\). Hence if we approximate \(g(x, y)\) for any \((x, y)\) by Eq.(2), then the original image must be sectioned into tiny blocks with the same area \(R_e\). Here the size of \(R_e\) is chosen to be much bigger than the size of the PSF for the restoration processing because of its edge effect of the Fourier transform. After processing, however, the inside smaller part is taken as a desired result. Hence the edge effect of area \(R_e\) is suppressed sufficiently. Eq.2 is called sectioned convolution integral\(^{19}\) known in linear spatially invariant system. In this paper \(R_e\) is taken as \(64 \times 64\) pixels with 48 pixels overlap to the next region. And the computation result is taken only for the central \(16 \times 16\) pixels. When the block size smaller than \(64 \times 64\) is taken the spatial frequency approximation of the PSF (Point Spread Function) becomes worse and consequently the proposed restoration filter gives a poor results. Conversely the bigger block size makes the filtering inefficient in the CPU (Central Processing Unit) time aspect with no more resulting high picture quality. That is, even if the block size bigger than \(64 \times 64\) pixel is employed the better results can not be obtained any more. On the other hand the computation time increase with the block size. From this point of view the size \(64 \times 64\) pixel seems to be optimum in this case. Then we select this block size \(64 \times 64\) through our many trails. Although this block size depends upon the original image size, we confirm that the size \(64 \times 64\) is sufficient for the original image sizes \(256 \times 256, 512 \times 512,\) and

![Fig. 2 (a) PSF's figure at the image center](image1)

![Fig. 2 (b) Fourier transform of Fig.2(a)](image2)

![Fig. 3 (a) PSF at the image marginal area](image3)

![Fig. 3 (b) Fourier transform of Fig.3(a)](image4)
\[ G(u, v) = H_{R_e}(u, v) F(u, v) \]  
\[ G(u, v), H_{R_e}(u, v) \] and \( F(u, v) \) are the 64 \times 64 Fourier transforms of \( g(x, y) \), \( h_{R_e}(x, y) \) and \( f(x, y) \) respectively. From Eq.(3) the Fourier transform \( F(u, v) \) of the original image \( f(x, y) \) is obtained as

\[ F(u, v) = G(u, v) / H_{R_e}(u, v) \]  

Taking inverse Fourier transform we obtain the original image \( f(x, y) \). This is called a sectioned inverse filter. But in fact inverse filter can not usually give us a satisfying result because of its singularity nature. In order to solve this problem, Wiener introduced a minimum mean square error filter (MMSE filter) known as an optimum filter:

\[ F(u, v) = G(u, v) \frac{H^*(u, v)}{|H(u, v)|^2 + p_n(u, v) / p_f(u, v)} \]  
\( (H(u, v)^* : \text{complex conjugate of } H(u, v)) \)  
\( (p_n(u, v) : \text{Power spectrum of noise}) \)  
\( (p_f(u, v) : \text{power spectrum of original image}) \)

Although Wiener filter is excellent it is sometimes troublesome to use practically because the power spectra of the signal and the noise are required. We may not be able to get such information in some situation. Hunt propose a simpler filter with a constant \( \Gamma \) instead of the term \( p_n / p_f \) as shown in Eq.6. The value of \( \Gamma \) is determined experimentally. This is very convenient for practical use and effective. We utilize this filter for local block area filtering in this paper. In Eq.(6) we need the PSF estimation in each block. It is shown in Section 4.

\[ F(u, v) = G(u, v) \frac{H^*(u, v)}{|H(u, v)|^2 + \Gamma} \]  

3.2 Traditional method

The above introduced filter is traditional one utilized for a spatial invariant image restoration problem. Especially Wiener filter is a well-known optimum one and utilized as a typical standard image restoration filter for the comparison of the filter performance. Such frequency domain filters are called classical ones and historically they are much reliable and stable. In those cases the PSF is assumed to be the same figure at every position on the image plane. Even in a space-variant case the figure of the PSF has usually been assumed to be cylindrical with only different radius depending upon the position on the image plane. But from our experiments it is clear that this condition is true only for the limited center area of the image. Actually PSF changes not only in the shape but also in the size depending on that position. Consequently unless we employ such a spatially variant PSF for an image restoration problem, we will not be able to get a satisfactory result. In this work we estimate such spatially variant PSF by using a new recursive method. As a space variant traditional method we choose a sectional one [Trussell9] for comparison of the results in our simulations. The Trussell's method is basically MAP (maximum a posteriori) estimation one with an iterative solution. And the PSF of each block is assumed to be known Gaussian function. Here we employ the Trussell's method with known circular (cylindrical) PSFs for comparison with our present method.

4. Flow chart

Cannon developed the blind deconvolution method in a spatially invariant imaging system. We utilize his idea but the method is quite different. Fig.4 shows a basic flow chart of this procedure including PSF pattern detection. The object image is sectioned into small blocks with 64 \times 64 pixels with a 48 pixel overlap. The overlap area is needed to suppress an edge (or marginal) effect of the Fourier transform. The Fourier transform makes often a curious result near the margins of the image where we call it marginal effect. The next step is initial setting of the PSF. We estimate the center area PSF i.e. cylindrical shape. The marginal area PSFs can be roughly estimated from its position and the center area PSF because we know the nature of the PSF shape from our PSF checking test. In fact the marginal PSFs appear as a concentric circle pattern and it is a function of the length from the center of the image. Then we can guess the PSF shape roughly at any position if we know the position coordinate and the image center block PSF.
Consequently we utilize these rough estimates as initial values of the marginal PSFs in our recursive estimation. The unknown parameters are the scales of the long and short axes of the marginal PSF.

For the image center block PSF the unknown parameter is only radius of the PSF circle. Hence in this paper the restoration process is performed from the image center block to the marginal blocks in clockwise manner. We approximate the marginal block PSF as an ellipse. Actually in the initial parameter setting of PSF in Fig. 4 we make the PSF in the spatial frequency domain from the first because the Fourier transform of a circle is known as a Fourier-Bessel function with parameter radius \( \rho \).

The frequency domain radius \( \rho \) is usually much greater than the space domain radius \( r \) of the PSF because these parameter \( \rho \) and \( r \) are inversely proportional each other and \( r \) is generally small. Hence it is just preferable to choose the radius \( \rho \) as a detailed changeable parameter because of the smooth approximation. Even the marginal PSFs can be expressed approximately in the frequency domain as a modified Fourier-Bessel function.

We make the modified Fourier-Bessel function as a separable function with long and short axes. The cross section pattern in each axis has a same figure(Fourier-Bessel) with different scale. Fig. 3(b) is the Fourier transform of the marginal PSF shown in Fig. 3(a). We confirm that our approximation of the marginal PSF is fairly good for our purpose in the simulation. In our recursive way we can control easily the long axis and short axis of the elliptic shape by using an integer dynamic programming. The unstable situation may often occur if the space domain radius \( r \) is chosen as a changeable parameter because of its small size. It is hard to control the situation delicately.

### 4.1 PSF pattern detection

Next step in the flow chart is for an observed block phase correction. Here we utilize only the phase terms \( \exp(j\theta_{PSF}) \) and \( \exp(j\theta_{obs}) \) where \( \theta_{PSF} \) and \( \theta_{obs} \) are the Fourier phases of the PSF and the observation image of that block respectively. The observation image has a degraded Fourier phase through the imaging system PSF. In this paper we use a phase-only synthesis (POS) for PSF estimation\(^{(15)}\). POS is defined as the inverse Fourier transform of the only phase term of the signal Fourier transform\(^{(16)}\). The examples of POS are shown in section 5.1.

The POS gives a clear image when the image is in-focused, i.e., when the observed phase is that of the non-blurred image. But when the image is blurred by some PSF with nonzero phase the POS is not able to give a clear image because of the phase change from the original image. Hence after the phase correction the POS will give a clear image if the corrected phase is close to the true value. Changing the PSF scales in the frequency domain by a small step we can apply an iterative procedure for the phase correction. Since the radius \( \rho \) of the Fourier-Bessel function is generally pretty big (about 20 pixel or more) we can easily employ an integer dynamic programming for the best choice of each \( \rho \) for long or short axis. The cost function for determining PSF here is the sharpness of the POS image. The sharpness is measured by digital gradient of the POS image for that block. The digital gradient is defined as

\[
\phi(i,j) = (\Delta_h^2(i,j) + \Delta_v^2(i,j))^{1/2}
\]

\[
\Delta_h(i,j) = f(i,j) - f(i - 1,j)
\]

\[
\Delta_v(i,j) = f(i,j) - f(i,j - 1)
\]

and the cost function for PSF determination is

\[
\max : \Psi = \sum_{i,j \in R_i} \phi^2(i,j)
\]
where $R_l$ is the central small area $16 \times 16$ pixel region for each $64 \times 64$ pixel block. In this work, first the best value for short axis is searched for fixed long axis value and then the best long axis value is searched for the determined short axis value. Actually the length of long axis is about the same as that of the center block PSF without depending the position. Hence as an initial value the radius of the center block PSF is used as a long axis of the marginal PSF. The long axis line is perpendicular to the line between that position and the center of the image. The short axis is determined by repeating to change the length of it by step ($\pm 0.01$) from the center PSF radius $\rho$ to $\rho/2$ in the Fourier-Bessel domain until satisfying Eq.10. The average repeating time for these detection is about 23. After determination of the short axis the long axis is adjusted and fixed again like a short axis.

The PSF is made as a separable function $P(\rho_x, \rho_y)$ as

$$P(\rho_x, \rho_y) = B(\rho_x)B(\rho_y)$$  \hspace{1cm} (11)

where

$$B(\rho) = \frac{J_1(2\pi \rho)}{\rho}$$  \hspace{1cm} (12)

($J_1$: Bessel function of the first kind, order one)
($\rho_x, \rho_y$: radius of long or short axis)

Repeating this procedure until Eq.10 is satisfied we can obtain the best PSF for each block. This procedure is an integer dynamic programming. Hence the repeating time is few and the CPU time is short, i.e., within 2 second for one block ($64 \times 64$ pixel) for our workstation (DELL precision610).

Fig. 5 shows an estimated PSF in our simulation example. The radius of the PSF, however, is shown in its ten times length for our easily understanding. After determining the PSF we can proceed to the restoration filtering. The restoration filtering (Eq.6) is performed with $64 \times 64$ pixel size with $48 \times 48$ pixel overlap. And only the central area ($16 \times 16$ pixel) of the filtering result is taken as the restored image. $64 \times 64$ pixel size is sufficiently enough for frequency analysis and also sufficiently small for the full picture size i.e., $512 \times 512$ pixel in the space-variant image analysis. Consequently it is considered that the fixed PSF for each block works well. The final stage is a sharpening of the restored image, which is shown in the next section. In an image restoration problem it is usually hard to get a satisfied filtering result for the case where the image is degraded by both blurring and noise because deblurring and denoising need the opposite nature operations respectively. Especially it is the case when the imaging system is spatially variant. The proposed restoration filter gives also an unsatisfied result because of the compromising parameter determination ($\Gamma$ in Eq.6). In this work, however, since the phase-only synthesis is utilized as a measure of the PSF estimation the Fourier phase of the restored image is well corrected to be close to those of the nondegraded image. Hence if we apply an edge sharpening filter with the POS to the restored image then we will be able to get a satisfactory result.

5. Edge sharpening

Image restoration(deblurring) filter cannot give sometimes a clear image because of the poor noise suppression. Usually suppressing noise works reversely with deblurring. Hence it is almost always better to take an edge sharpening technique for the restored image. There are several image sharpening methods\textsuperscript{17}-\textsuperscript{19}). Here we develop a perturbed phase-only synthesis into the sharpening. Since the restored image has a corrected Fourier phase the phase-only synthesis of it gives us a clear image. Thus the POS becomes one of the powerful tools here for image enhancement.

5.1 Perturbed phase-only synthesis

A phase-only synthesis is defined as an inverse Fourier transform of only the phase term of the signal Fourier transform. It is well known that a phase-only synthesis is sometimes more powerful than a Laplacian operator for image sharpening\textsuperscript{20}). This is true when the observation Fourier phase is used for the phase-only synthesis even if the PSF of the imaging system is a nonzero-phase function. If we get a true phase, however, a phase only synthesis is much more powerful than a Laplacian operator because Laplacian operator with the correct phase has almost no more additive effect. But still a Laplacian

![Fig. 5 Estimated PSF (size is magnified 10 times)](image-url)
operator is itself useful for image smooth sharpening. For example, Fig. 6 shows the effectiveness of the both Laplacian operator and phase-only synthesis.

The curve of (a) in Fig. 6 is assumed to be the cross section of the observation image and (b) is the Laplacian result of (a). (c) shows the phase-only synthesis of (a). The curve of (d) is the final sharpened image. The edge of this processed image looks very sharp. This is the edge sharpening effect. Although the phase-only synthesis (POS) image is attractive for edge sharpening, the resulting image is sometimes noisy. Hence we develop a perturbed POS to suppress the noise in this paper. The POS noise appears randomly. Therefore if we shift the original image \( f(i, j) \) to \( f(i+l, j+m) \) as shown in Fig. 7 then the POS noise of \( f(i+l, j+m) \) is different from that of \( f(i, j) \). Consequently taking an average value of these several shifted version POSs we can get the clear line POS image with low noise. We call this average value the perturbed POS here. The perturbed POS \( \varphi^p(i, j) \) is given as

\[
\varphi^p(i, j) = \frac{1}{K} \sum_l \sum_m \varphi_{lm}(i+l, j+m) \tag{13}
\]

where \( \varphi_{lm}(i, j) \) is the \((l, m)\) shifted version POS with the \(N \times N\) pixel size. And \( K \) is the number of perturbations. Here \( K \) is 25 and \( l \) and \( m \) are 0, ±3, ±6.

The resulting perturbed POS \( \varphi^p(i, j) \) has \( M \times M \) pixel where \( M = N - 12 \) and \( N = 512 \). Fig. 8 and Fig. 9 are the usual POS and the perturbed POS respectively. These sizes are 500 × 500 pixels. Since the image itself has no shift it is easy found that Fig. 9 is much clearer with less noise than Fig. 9.

5.2 Sharpening

With \( \hat{f}(i, j) \) the sharpened image, \( f(i, j) \) the original image, and \( \varphi^p(i, j) \) the perturbed phase-only synthesis the image edge sharpening is done as follows:

\[
\hat{f}(i, j) = f(i, j) + \beta_{ij}(a_{ij} \cdot l(i, j) + b_{ij} \cdot \varphi^p(i, j)) \tag{14}
\]

where \( l(i, j) \) is called Laplacian image and it is defined as

\[
l(i, j) = f(i, j) \otimes L(i, j) \tag{15}
\]
$k(i,j)$: Laplacian operator, $\otimes$ convolution sum

$a_{ij}$ and $b_{ij}$ are coefficients depending on the value $l(i,j)$, which are shown in Fig. 10. If $b_{ij}$ is zero and $\beta_{ij}$ and $a_{ij}$ are constant, then $f(i,j)$ is a traditional (Laplacian) edge sharpening image.

The horizontal axis of Fig. 10 is measured by the magnitude of the Laplacian image normalized with the maximum value 1. $M_h$ and $\sigma_h^2$ are the mean and variance respectively of the absolute value of the Laplacian image $l(i,j)$ except the flat area of the image. These parameters are determined experimentally. In the domain below the level 'a' in Fig. 10 the magnitude of the coefficient $a_{ij}$ is reduced because of the noise suppression on the flat area of the image. The coefficient $a_{ij}$ and $b_{ij}$ curves in Fig. 10 are called $\pi$-function and s-function respectively. $\beta_{ij}$ in Eq. (12) is a parameter which controls the enhancement degree and shows in Fig. 11. This is derived from the derivative of our visual response characteristic curve of the brightness level$^{22-24}$.

Actually on the dark level area(54x422) and bright level area the image enhancing is almost no effect for our human eyes because of its low sensitivity. For example, in the very dark or very bright area even if the gray level difference is enhanced we cannot find it so clearly because our human discriminative power for the gray level difference is very low in such area. But still the dynamic range of the enhanced image is expanded. Such dynamic range expansion is sometimes not preferable because all digital images have a fixed dynamic range. This means that the contrast of the image is reduced after normalizing the gray level. Hence from such an aspect $\beta_{ij}$ curve has a reasonable characteristic.

6. Simulation

Fig. 12 shows the simulation example.

Fig. 12(a) is the original image with the size $512 \times 512 \times 8$bit. Fig. 12(b) is the defocus image with a film grain noise through a spatially variant defocusing system (TAMRON 90mm F2.5). The point spread functions of this system look like those shown in Fig. 1(b). Fig. 12(c) is the restored image through the proposed filtering method with the estimated PSFs. From this figure it is found that the PSF has been estimated well in each sub-block. Hence the Fourier phase of the sub-block image is also corrected well. This makes the edge sharpening effective because the POS of Fig. 12(c) is close to that of Fig. 12(a) (original image). Fig. 12(d) is the image sharpened by the proposed perturbation POS method. It is clear that Fig. 12(d) is much preferable to others and has a better SNR. Fig. 12(e) is also shown for comparison, which is the restored image by traditional sectioned spatial variant filter$^{[Trussell8]}$. In this traditional case the block size is the same ($64 \times 64$ pixels) as that of our case and the spatial variant PSFs are assumed known (equivalent circular PSF for each sub-block). The proposed filtering result (Fig. 12(c)) is clearly better than the traditional one (Fig. 12(e)). Fig. 12(f) is also an edge sharpening image of Fig. 12(e) by traditional Laplacian-only sharpening filter. This figure looks sharp but a little bit strong ringing appears. Fig. 12(g) is the sharpened image of Fig. 12(e) by our proposed sharpening method. But no more improvement can be gotten due to the no retrieval of the phase. Comparing it with Fig. 12(d) we can find easily that Fig. 12(d) has a better quality. This shows us the POS is effective.

Fig. 13 show the examples of the SNR characteristic curves with different aspect ratios of the PSF. Fig. 13(a) is for 5:4 aspect ratio and the Fig. 13(b) is for 5:2 ratio. From these figures we see that the proposed method gives us the best result. For the traditional filter the bigger aspect ratio makes the result worse. This is because the equivalent circular PSF is used for the traditional one. In the sharpening of the traditional filter result, the improvement of the SNR does not come up. On the other hand the presented method gives almost the same SNR curves without regarding to the image and the aspect ratio because at each place on the image the PSF is estimated optimally. The SNR here is defined
Fig. 12 (a) Original image

Fig. 12 (b) Blurred image [SNR=26.7dB]

Fig. 12 (c) Restored image [SNR=29.5dB]

Fig. 12 (d) Edge sharpened image [SNR=30.5dB]

Fig. 12 (e) Traditional Restored image [SNR=26.7dB]

Fig. 12 (f) Edge sharpened image (Laplacian) [SNR=26.3dB]
as a peak signal to noise ratio (PSNR).

Fig. 14 is shown for the evaluation of our proposed restoration-only filter. Each plotted points of the curve A (proposed method) are mean values of the processed results of three different aspect ratios 5:4, 5:3 and 5:2 for the flower image. The other three curves B, C and D show the traditional restored results for the same image as the aspect ratio 5:4, 5:3 and 5:2 respectively.

The dead end of the aspect ratio is set to be 5:1 in our algorithm. It seems sufficient because even for a popular zoom lens the aspect ratio 5:2 for the edge (corner) area of the image. For single focal lens it is about 5:3 or more. From these figures we see that the proposed filtering method is much better than the traditional one.

When the aspect ratio is 1:1, the PSF’s shape estimated by the proposed method is same with that by the traditional one theoretically. But in our proposed restoration filter since the optimum PSF for the prescribed cost function is estimated for each aspect ratio, the difference of the restoration result for each aspect ratio is very small (within ±0.5dB).

7. Conclusions

One spatially variant image restoration filter with edge sharpening has been presented. The proposed filter is well applied for an actual camera lens system. Usually it is very difficult to get a fine restored image from the degraded observation image blurred by a spatially variant imaging system. In such a case, however, using the retrieved Fourier phase we can make an image sharpening. Especially a perturbed phase-only synthesis is very powerful for image edge sharpening. The recursive estimation of the point-spread function (PSF) at each sub-block on the image is the major topic of the proposed restoration filter. Here, the phase-only synthesis is utilized again for the evaluation. The pro-
cessing time has been 2 minutes for our workstation (OS: FreeBSD3.2 PLATFORM: DELL Precision 610). On the other hand the time of the traditional method has been about one minute. The main reason of this time difference comes from the recursive way of our purposed method for searching each block PSF. The quick computation is a left problem to be solved in near future.

[References]