共振周波数を超える広帯域磁気ヘッドサーボシステムの設計
ーサーボ系設計における同相性の重要さー

斎藤 耕治†，大日方 五郎†，大内 一弘†
†AIT (秋田県高度技術研究所) 〒010-1623 冬利市新屋町字砂巻寄4-21
Tel.018-866-5800 Fax.018-866-5803 E-mail: saiko@ait.pref.akita.jp
†秋田大学工学資源学部 〒010-8502 冬利市手形学園町1-1
E-mail: obinata@control.mech.akita-u.ac.jp

あらまし
磁気記録密度の急速向上に伴い，磁気ヘッドサーボシステムの高速高精度化が必要とされている。高精度化には，サーボシステムの広帯域化が必要だが，機械系の共振がそれを困難にしていると考えられた。本報告では，機械系の振動モードが同相と言える性質を持つことによって，サーボ系の性能が向上することと，さらには同相の場合共振周波数を超えるサーボ帯域実現の可能性について述べる。

キーワード 磁気ヘッドサーボシステム，同相性，共振モード，広帯域化，同相化設計

Wide Range Servo System Surpassing Resonance Mode for Magnetic Head Actuator
ー Importance of In-Phase Property in Optimal Servo System Designー

Koji SAITO†, Goro OBINATA† and Kazuhiro OUCHI†
†AIT 4-21 Sanuki, Araya, Akita 010-1623, Japan
Tel.+81-18-866-5800 Fax.+81-18-866-5803 E-mail: saiko@ait.pref.akita.jp
†Akita Univ. 1-1 Tegata-Gakuen Cho, Akita 010-8502, Japan
E-mail: obinata@control.mech.akita-u.ac.jp

Abstract
As recording density is getting higher, the higher speed and more accurate magnetic head servo system is required. The servo band has to be expanded for the requirement. However a wide range servo system has a risk to deteriorate the close-loop system performance by exciting the mechanical resonance mode of the magnetic head carriage. Then servo systems have been designed to prevent the servo band from the natural frequency of the mechanical system. In this paper, the feasibility of the wide range servo system to surpass the mechanical resonance mode frequency without the deterioration is investigated in terms of the in-phase-ness of the controlled system. These results show that to make a controlled system in-phase is necessary and sufficient for the quasi-optimal closed-loop performance. In consequence of this, The potential of a conventional magnetic head carriage system to be in-phase is discussed.

key words magnetic head servo system, in-phase, resonance mode, wide range servo, in-phase design
1 Introduction

As recording density is getting higher, the higher speed and more accurate magnetic head servo system is required; allowable tracking error is now less than 50 nm, average access time has currently been reduced to 7 msec. The servo band of the system has to be expanded for higher frequency region to satisfy the requirement. However, the wide range servo system has the risk to excite the mechanical resonance modes of the magnetic head carriage. Commonly, the mechanical resonance is recognized as a factor to deteriorate the system performance, because the oscillation by the resonance is usually hard to settle. Then servo systems have been designed to prevent the servo band from the natural frequency of the mechanical system.

Nowadays, the necessary servo range is reaching to the natural frequency. The natural frequency must be raised up, if we continue to employ the conventional servo system design. There are two ways to raise the natural frequency. The one is reducing the size of the mechanics, i.e., adopting 2.5 inch disks as the standard. The other one is that the mechanics is made from lighter materials with keeping the stiffness. The first way is disadvantaged for the recording capacity, and the second way brings higher cost. Hence, the new design methodology for servo system which includes the dynamics of mechanical resonance is demanded.

The concept of in-phase is an idea that focuses the sign of each mode of the system. Then in-phase-ness is defined for the system represented by the sum of a rigid body mode and flexible modes. The in-phase means that every initial derivative of its impulse response of the modes takes the same sign. It is well known that in-phase systems are able to be well-controlled [1, 2, 3]. We also studied role of in-phase-ness in a servo system, especially addressed whether the wide range servo system over the natural frequency of mechanics is available when the mechanics is in-phase. Furthermore, a design method for a mechanical system that realize the in-phase property on the system has been discussed [4].

In this paper, results of any works about the in-phase-ness on closed-loop performances will be reviewed, and in-phase analysis of a conventional magnetic head carriage will be demonstrated.

2 In-phase-ness on Control Performance

2.1 in-phase-ness

First of all, we will define the in-phase. Consider the transfer function of mechanics $H(s)$ given as follows.

$$H(s) = \frac{1}{s^2 + \sum_{i=1}^{n} k_i s^2 + 2\eta_i \omega_i s + \omega_i^2} \quad (1)$$

The first term represents the rigid-body mode and the second term represents the flexible modes with the damping coefficients $\eta_i$ and the natural frequency $\omega_i$. Throughout the paper, we assume $\eta_i > 0$ ($\forall i$) and $\omega_i > 0$ ($\forall i$).

Definition 2.1 (in-phase) The $i$th flexible mode in transfer function $H(s)$ in equation 1 is said to be in-phase if the gain $k_i$ has the same sign as rigid-body mode, i.e., $k_i > 0$. When $k_i > 0$ ($\forall i$) in $H(s)$, we say the system $H(s)$ is in-phase (IP) [2, 3].

When a system is not in-phase, we say the system is out-of-phase (OP) [2, 3].

This means that if a system is IP, then every flexible mode component of its impulse response has the initial derivative of the same sign as the rigid-body mode component.

2.2 Controlled System

Let us consider a magnetic head servo system that has a flexible mode. The transfer function of the system is given

$$P(s) = \frac{K_p}{T_p} \left( \frac{1}{s^2} + \frac{a}{s^2 + 2\eta \omega s + \omega^2} \right). \quad (2)$$

$$K_p = 7.00 \times 10^2 \text{ms}^2/\text{V} \quad T_p = 1.00 \times 10^{-6} \text{m/trk} \quad \eta = 0.001 \quad a = \pm 0.5$$

Where $K_p$, $T_p$, $\eta$ and $a$ are the static gain, track pitch, damping coefficient, the natural frequency and the coefficient of the ratio to the rigid-body mode, respectively.

Figure 1 shows the bode diagrams of IP and OP cases with $\omega = 4$ kHz. The both systems are the same property at the frequency under the natural frequency. However, the phase delay of IP system is decrease at the natural frequency. In contrast to
higher performance. The $\gamma_{\min}$ can be calculated without synthesizing the controller $K(s)$ by applying the normalized coprime factorization to $P(s)$, i.e., $P(s) = \tilde{M}(s)^{-1} \tilde{N}(s)[5]$,

$$\gamma_{\min} = \left\{1 - \|\tilde{N}(s), \tilde{M}(s)\|_H^2\right\}^{-\frac{1}{2}}. \quad (5)$$

Where $\| \cdot \|_H$ denote the Hankel norm. This means that the limit of robust performance $\gamma_{\min}$ is mainly depend on the controlled system $P(s)$.

First of the examinations, we examined the effect of $P(s)$'s parameter $a$ on the performance index $\gamma_{\min}$. The result is shown in figure 3

![Figure 3: EFFECT OF a IN P(s) FOR $\gamma_{\min}$](image)

When $P(s)$ is OP; $a < 0$, $\gamma_{\min}$ is drastically getting smaller as $\|a\|$ is smaller. In spite of this, the $\gamma_{\min}$ is slightly getting smaller corresponding to $a$ in the IP region.

This result implies two fundamentals that the closed-loop performance $\gamma_{\min}$ is quasi-optimized when $P(s)$ is IP; $a > 0$, and IP flexible modes don't deteriorate the robust performance of the closed-loop system. The latter brings us the hopeful prospect that if the controlled system is IP then the servo system surpassing resonance mode is feasible. To confirm the prospect, we examined how affect the in-phase-ness on the closed-loop performance $\gamma_{\min}$ with modifying the natural frequency $\omega$. Despite of varying the natural frequency $\omega$, the performance index $\gamma_{\min}$ is stable when the controlled plant $P(s)$ is IP; the $\gamma_{\min}$ is rather getting smaller as the $\omega$ is getting lower. On the other hand, when the $P(s)$ is OP, the $gamma_{\min}$ is getting larger corresponding to the $\omega$. This result means
that the closed-loop performance $\gamma_{min}$ is mostly depend on the in-phase-ness of the controlled system. In other word, when the controlled system is IP, the closed-loop system performance $\gamma_{min}$ is quasi-optimal regardless of the magnitude of flexible mode $\|a\|$ and the natural frequency $\omega$.

2.4 Controller Design Example

By basing on the previous results, we have obtained the new paradigm of servo systems design that the controlled system should be IP then the closed-loop will be quasi-optimal. However any techniques will be employed while designing a controller in practical, the IP system is still superior to OP systems on the closed-loop performance. Actually, we tried the practical controller design for IP and OP system and then confirmed the advantage of IP system. The trial is as follows.

The controller is designed using McFalane and Glover's method [5] that is based on the normalized left coprime factorization. In the design method, the closed-loop properties are tuned by the open-loop frequency shaping. It is suitable to examine whether the IP system is still superior on closed-loop performance when the effects by the in-phase-ness are suppressed by the open-loop frequency shaping. The design procedure is briefly shown as follows.

1. Weighting function $W_1(s), W_2(s)$ is multiplied to both side of a controlled system $P(s)$ for shaping the open-loop frequency response. Thus the shaped system $P_w(s) := W_2(s)P(s)W_1(s)$ is obtained.

2. Calculate $\gamma_{min}$ of $P_w(s)$ by Eq.5.

3. The controller $K_0(s)$ for $P_w(s)$ is given by

$$K_0 = \left[ \frac{A_w + \gamma^2 W^2_{s}^{-1}Z(s)(C_w + D_w F)}{B_w X} \right]$$

$$P_w(s) = \left[ \frac{A_w}{C_w} \right] \left[ \begin{array}{c} B_w \\ \end{array} \right]$$

$$F = -S^{-1} (D_w^* C_w + B_w^* X)$$

$$A^c = A_w + B_w F$$

$$W_\gamma = I + (XZ - \gamma^2 I)$$

$$S = I + D^* D$$

Where $\gamma$ must be $\gamma \geq \gamma_{min}$, $X, Z$ are the symmetric and positive definite solutions of the generalized control algebraic Riccati equations (GCARE) and the generalized filtering algebraic Riccati equation (GFARE).

4. Finally, the controller is obtained as $K(s) = W_2(s) K_0(s) W_1(s)$.

The controllers for $P(s)$ with $a = 0.5$ and $a = -0.5$ are respectively designed. Where the natural frequency $\omega_r$ is $4kHz$. The weighting functions for flattening the open-loop frequency responses is

$$W_n(s) = k_{W_n} \frac{s^2 + 2 \eta_{W_{na}} \omega_{W_{na}} s + \omega_{W_{na}}^2}{s^2 + 2 \eta_{W_{da}} \omega_{W_{da}} s + \omega_{W_{da}}^2}$$

$$\omega_{W_{na}}, \omega_{W_{da}}, \eta_{W_{na}}, \eta_{W_{da}} \text{ of } W_1, W_2 \text{ are defined for flattening the frequency response of IP and OP } P(s) \text{ respectively.}$$

The static gain $k_{W_n}$ is defined as large as satisfying the constrain equation

$$\|T_{ur}(s)\|_2 \leq T_{ur}.$$  \hspace{1cm} (7)

The constrain works to limit the actuation power. The $\gamma_{min}$ is getting smaller as the $k_{W_n}$ is bigger in either IP or OP case thus the dissipated power will be same in both cases. The $T_{ur}$ is defines $1.00 \times 10^3$ while considering a real system. Consequently, the coefficients of $W_n(s)$ are as the listed in table 1.

The bode diagrams of shaped systems $P_w(s)$ is depicted in figure 5. As the figure shows, the gain characteristics are flattened and the phase delay is reduced. Even this, we shall note that the gain of IP system is higher than OP system, and the phase delay of IP system is less than $2\pi$rad in contrast to OP system. The $\gamma_{min}$ of IP and OP $P_w(s)$ are $2.3723$ and $2.8701$, respectively.
Table 1: COEFFICIENTS OF WEIGHTING FUNCTION: $W(s)$ ($\omega_r = 4$kHz)

<table>
<thead>
<tr>
<th>$W_s$</th>
<th>$k_w$</th>
<th>$\eta W$</th>
<th>$\omega_{W}(\text{rad})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_1(s)$</td>
<td>$k_w_1 = n$</td>
<td>$1.00 \times 10^{-3}$</td>
<td>$2\pi \times 4000$</td>
</tr>
<tr>
<td>(a = 0.5)</td>
<td>0.99</td>
<td>d</td>
<td>$2\pi \times 4100$</td>
</tr>
<tr>
<td>$W_2(s)$</td>
<td>$k_w_2 = n$</td>
<td>1.00</td>
<td>$2\pi \times 3266$</td>
</tr>
<tr>
<td>(a = 0.5)</td>
<td>1.00</td>
<td>d</td>
<td>$2\pi \times 4100$</td>
</tr>
<tr>
<td>$W_1(s)$</td>
<td>$k_w_1 = n$</td>
<td>$1.00 \times 10^{-3}$</td>
<td>$2\pi \times 4000$</td>
</tr>
<tr>
<td>(a = -0.5)</td>
<td>0.22</td>
<td>d</td>
<td>$2\pi \times 4100$</td>
</tr>
<tr>
<td>$W_2(s)$</td>
<td>$k_w_2 = n$</td>
<td>1.00</td>
<td>$2\pi \times 5557$</td>
</tr>
<tr>
<td>(a = -0.5)</td>
<td>1.00</td>
<td>d</td>
<td>$2\pi \times 5657$</td>
</tr>
</tbody>
</table>

Figure 6: GAIN DIAGRAMS OF $T_{zw}$ ($\omega_r = 4$kHz) IP:SOLID, OP:DOTTED

the point is located at lower frequency than resonance point when $P(s)$ is IP. That is inverse to when $P(s)$ is OP. The other one reason is the phase delay. As mentioned before, the phase delay of the OP system is less than $2\pi$ rad. Such the system is a typical well-controlled system, even if a static controller is applied the system.

3 In-Phase System Design

3.1 In-Phase-ness Analysis

We will consider the IP system design. At first, examine the in-phase-ability on the model that is the simplified conventional magnetic head carriage as shown in figure 7.

Figure 7: SCHEMA OF HEAD ARM

Where the black circle is the center of gravity and the double circle is the pivot of the arm and we assume that only the pivot has the elastics. $u$ and $y$ are the control input and the output of this system. $l_u$, $l_y$ and $l_\phi$ are the distances from the pivot to the points of $u$, $y$ and the center of gravity. The signs
of these lengths are defined by referring the pivot as the origin. \( m, J, d, k \) are the mass and the moment of this system, the dumping and spring constant of the pivot, respectively.

The transfer function of the system \( P(s) \) is derived as

\[
P(s) = \frac{l_uy}{J + l_2y} \left( \frac{1}{s^2 + \frac{(J + l_2(y - l_2)m)(J + l_2(y - l_y)m)}{Jl_2ym}} \right).
\]

Since the denominator of equation 8 is always positive, when the numerator of the equation is positive the system \( P(s) \) is obviously in-phase.

\[
\frac{(J + l_2(y - l_2)m)(J + l_2(y - l_y)m)}{Jl_2ym} > 0
\]

3.2 Numerical Example:

The region that \( P(s) \) is in-phase of \( l_u, l_2 \) is calculated. Where

\[
\begin{align*}
m &= 0.033\text{kg} & J &= 1.7 \times 10^{-5}\text{kgm}^2, \\
l_y &= 0.06\text{m}
\end{align*}
\]

the ranges of \( l_u, l_2 \) are

\[
\begin{align*}
-0.06 &\leq l_u \leq 0.06 \\
-0.06 &\leq l_2 \leq 0.06
\end{align*}
\]

The result is depicted in figure 8.

Figure 8: IP REGION (WHITE) IN TERMS OF \( l_u \) AND \( l_2 \)

The modal deformations corresponding to each region in figure 8 are depicted in figure 9. Since the area between the magnetic head point \( y \) and the pivot is occupied by magnetic disks, the region (2) is seemed infeasible to assemble. Though the region (3) is available formation in a conventional hard disk case, the offset of the gravity center from the pivot will make the carriage statically imbalance. The inevitable imbalance for making the carriage in-phase will require the stiffer pivot than present or cause some problems in dynamic motions, for example, the unstable latitude of magnetic heads by ricketiness of the pivot.

Therefore, our analysis clarified the potential of a conventional magnetic head carriage system to be in-phase. However, the result showed that both in-phase-ness and static balance are not simultaneously realized in the conventional system. The result exhibits us the needs of the new carriage system.

4 Conclusion

The effect of the in-phase-ness in the servo system was confirmed by the numerical simulation in a magnetic head servo system. The result showed the feasibility of the wide range servo system surpassing the mechanical resonance mode frequency if the controlled system is in-phase. Thus the practical closed-loop designs for both in-phase and out-of-phase systems were confirmed regarding the feasibility. These results brought us the new paradigm
of servo systems design that the making controlled system in-phase is necessary and sufficient for the quasi-optimal closed-loop performance.

The study of the in-phase-ness led us to examine the in-phase-ability of conventional magnetic head carriage systems. Our analysis showed that the conventional systems are difficult to be in-phase by minor modifications. In consequence of this, we have been trying to develop the new in-phase magnetic head carriage system.

References


