Study on Changes in Ancient City Agoras Using Fractal Analysis
— Using Shaded Image to Describe the Formation of Agora
in 300 B.C., 150 B.C., and 100 A.D. —

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Abstract
This paper examines the "complexity" of space in Agoras by analyzing their architecture using image-processing technology. Agoras were central to the lives of Western European people and can be called the starting point of Western European civilization. Image processing technology was used to perform the initial restoration of the architecture and city shape. Each building in the 3D model of the plaza was then constructed, centering on the formation of the Agora. The box-counting method was used to determine the relationship between the shadow image and the fractal analysis of the 3D plaza composition. This was applied in the fractal analysis of the changes in the arrangement of the facilities from generation to collapse.

Keywords: ancient city; fractal; Agora; shaded image; restoration model

1. Introduction
The ancient Greek city of Athens gave birth to Western architecture and civilization and its 2500-year history has been extensively studied by international researchers. Plato*1 and Aristotle*2 developed theories of the ideal city that considered the interrelation of different social classes as well as the physical layout and organization of the city. The concept of city planning was developed in the Hellenistic period by Pliny*3, Frontinus*4, Witolwius*5, and others. However, a substantial portion of the technological research and books 1) - 13) that can be referred to today concern only the ruins that remain as they relate to cadastral surveys and water supply.

This study analyzes Agora as a single assembly promoting harmony between indoor (within the temple's premises) and outdoor environments (ambience) instead of each temple as a separate entity. The Agora is believed to be the cradle of life and Western European civilization. A. Doxiadis*6 (1972), a Greek architect and city planner, wrote a paper, "Architectural Space In Ancient Greece," that described the system of site planning for temples in ancient Greece. His paper has had an enormous influence on current knowledge and modern technology. Doxiadis analyzed the disposition of buildings in precincts, basing his viewpoint on the precinct entrance (origin of the polar coordinate system). He measured the distance between this viewpoint and a particular point, e.g., the intersection of the axes of the building and the precinct). He then calculated the ratio of these distances, measured the angle between their sight lines as observed from the viewpoint, and studied the relationship between the distances and the angles. The above procedure helped Doxiadis reveal the geometric

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order of the architecture. Previous studies on the disposition of buildings included mainly an analysis of the geometric order of architecture on the basis of two-dimensional drawings, much in the same way adopted by Doxiadis. This paper, an improvement on previous papers, discusses two prominent features. Firstly, instead of the analysis of two-dimensional drawings, analysis of the urban Agora is represented as a three-dimensional space. In this study, a model based on the geography of the Agora was generated as a three-dimensional computer-aided design (references 14, Endo (2003) and 15, (2003a)). Further, shaded images impart a three-dimensional structure to the model of the Agora. The model is illuminated by a light source for better visibility (references 16, Negoro (2004) and 17 (2004)). This research, which analyzes the human perceptions of visible space, is related to ISOVISTA (references 18, Alasdair Turner (2001) and 19, Michael Batty (2001), and differs from ISOVIST based on the three-dimensional information of the internal shadow image instead of the two-dimensional. In addition, if we divided the analysis that used the mesh method, it would enable us to abstract information from the subsequent fractal dimension analysis. Thus, this research made use of the light source method to provide detailed morphological information from the image. In addition, this study aims to analyze the Agora as an urban space by placing the viewpoint at the entrance of the Agora, which serves as an intersection of roads, rather than the entrance of an individual edifice such as a temple.

Secondly, the complexity of the geography of the Agora is analyzed on the basis of the perception of complex human vision. Apparently, human vision cannot accurately perceive architectural geometry from a stationary position. A theory of complex systems or a fractal theory is used for analysis in this study to quantify the complex fractal dimensions not visible to the naked eye (references 20, Chonabayashi (2005) 21, Kuroiwa (2004) and reference 22 Borvil (1997). Fractal dimension quantifies self-similarity of the part and the whole. The fractal dimensions, as perceived by human vision in all age groups (shaded image), from the entrance of the Agora are analyzed.

Thus, by analyzing the changes in perception of the fractal dimensions in the case of all age groups (shaded image) from the entrance of an urban space, the complexity of the disposition through a transition of the Agora is determined offering new insight into the geometric order of architecture discussed in the previous studies.

2. Transition of Ancient Agora (Fig.2.)

Ancient Agora at the Acropolis in the center of Athens, Greece.

Agora about 120km northwest of Athens at the ancient Oracle at Delphi on Mount Parnassus.

Wycherley (1980) describes the Agora as an assembly resembling a market thronged with people engaged in various intensive activities as compared to any other public spaces in other cities. The Agora serves as the city’s heart rather than as a public plaza and its initial shape was rather simple. Development of the Agora aimed at optimizing architecture. Toward this aim, a number of buildings such as city council office, meeting place, temple, altar, fountain field office, records office, and government headquarters were constructed. The capacity of the Agora to accommodate altars, temples, and the practice of various rituals surpassed even that of the Acropolis.

In addition, the increasing role of the market as a pivot of civilization fostered the construction of many
administrative offices and courts. Late Hellenism witnessed the construction of an auditorium and a library; in particular, the store buzzed with many foreign merchants. Each architectural addition promoted the elements necessary for the prosperity of the democratic city of Athens.

The prosperity and decline of the Agora over the centuries has been illustrated above and each of the gradual architectural transitions has been mentioned in references 1-13.

However, the illustrations do not reveal the complex changes of layout in the urban Agora. This study focuses on these complex changes; it elucidates the changes in the urban space transition and uses the fractal theory to determine the complexity from the part-whole relationships of the Agora.

3. Study Method

Firstly, we conducted a survey across all age groups and used the results of previous studies on architecture to investigate the shape of urban constructions in the Agora and to analyze the gradual changes in the architecture. Next, the authors created a recovery model and used image-processing techniques to analyze its fractal dimensions. This revealed the changes in the functional and part-whole relationships of urban spaces throughout the history of the Agora.

1. Basic materials and techniques used in restoration of the previous hypothesis (traditional pattern of construction, processing, intensity of rock, and proportion). The layout material, foundation, and substructure were obtained by field surveys.

Period of investigation

Athens: July, 2001; Delphi: July, 2001

Subjects of investigation

Foundation and cornerstone, size of foundation substructures, building arrangement and style, and method of processing the stone.

2. To reveal changes in the distribution of administrative, judicial, etc. functions and increase and decrease in the number of buildings during the redevelopment of the Agora and to study the distribution of buildings at each age, as illustrated in the figure provided below.

3. Each building was constructed using a 3D plaza model centering on the formation of the Agora.

4. The box-counting method was used to determine the relationship between the shadow image and the fractal analysis of the 3D plaza model.

4. Process of Plaza Formation

4.1 Analytical method

Using the site plan from references 1-13, the formation processes were classified by age into three categories.

1. Architecture that had existed in each age: existing architecture

2. Architecture that was constructed during the age: constructed architecture

3. Architecture that was extinct in each age: extinct architecture

Architecture in the plaza was classified by building use into five types separated by age using the site plan from the reference.

1. Governmental
2. Public
3. Commercial
4. Religious precincts
5. Housing
6. Cultural

The categorization is made for five ages (500 B.C., 400 B.C., 300 B.C., 150 B.C., and 100 A.D.), and the functions and distribution of the buildings are analyzed. During these five ages, Athens was the most powerful city in Greece. 500 BC: Many of the early important structures such as The Royal Stoa and Heliaia, clearly designed for the needs of civic life, seem to date to this period. 400 BC: In the last thirty years of the third century BC, a burst of activity ensued. The Stoa of Zeus, South Stoa One, and the Mint all appear to date from this time even while Athens was dealing with the Peloponnesian War and its aftermath. 300 BC: The period of early third century BC, when Lykourgos controlled the city's finances, witnessed many more improvements in the facilities designed for civic life. 150 BC: Remodeling in the second century BC fixed the main lines of the Agoras for the rest of antiquity. The area of the enclosed square and that of the surrounding buildings was now about 50,000 m². 100 BC: The Hellenistic building served the same dual purpose. This remodeling of the 2nd century B.C. fixed the main lines of the Agora for the rest of antiquity (reference 23).

4.2 Analysis of facility arrangement

Table 1., Fig.3. shows the changes in the functions, distribution, and increase and decrease in the number of buildings. The transition of the Agora, from construction to the disappearance of facilities, and its usage, are described below. Many small naos, as well as mesoscale commercial, and cultural facilities, were constructed on the west side in 500 B.C.

In 400 B.C., a new road was constructed, enclosing the central portion and new facilities were constructed alongside it. Most of the facilities were naos, while commercial facilities were also constructed alongside the road.

In 300 B.C., small-scale naos gave way to new public facilities. In particular, commercial facilities and naos on the east side were replaced by large-scale public facilities. The road on the south side, which did not pass by the facilities, also disappeared.

In 150 B.C., the construction of large-scale commercial facilities was notable. Roads on the south and east that had disappeared by 300 B.C. were
restored and large-scale commercial facilities were constructed along them. Small- and medium-scale naos were located in the west, while medium-scale naos and large-scale commercial facilities were located in the south and east.

In 100 A.D., many new facilities, particularly cultural facilities, joined the existing facilities. Large-scale cultural facilities, medium-scale naos, and small-scale government facilities appeared on unoccupied land. Naos were built to the west, and construction extended outside the unoccupied land. Naos were to the west, and government, commercial, and cultural facilities were scattered about, raising the overall density.

Table 1. Analysis by Facility Use (500 B.C. - 100 A.D.)

<table>
<thead>
<tr>
<th>Extinct architecture</th>
<th>500 B.C.</th>
<th>400 B.C.</th>
<th>300 B.C.</th>
<th>150 B.C.</th>
<th>100 A.D.</th>
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<tr>
<td>ROYAL STOA</td>
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<td>SHrine of Zeus</td>
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<td>S. E. FOUNTAIN HOUSE</td>
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<td>TEMPLE OF APOLLO</td>
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The naos in the southwest remained unchanged in position, scale and usage from 400 B.C. to 150 B.C. Thus, at each age, many naos were present along the Kolonos Agoraios located in the west. Between 150 B.C. and 500 B.C. unoccupied land in the Agora that was open to the public underwent development. Although the scale and usage had changed, naos with many features were constructed in the west and along the road in a manner that enclosed the unoccupied land. By 100 A.D., however, the density of buildings had risen. A great number of cultural facilities moved into the unoccupied land. The authors analyzed this movement into the unoccupied land.

Table 2. Analysis by Facility Use (500 B.C. - 100 A.D.)

<table>
<thead>
<tr>
<th>Constructed architecture</th>
<th>500 B.C.</th>
<th>400 B.C.</th>
<th>300 B.C.</th>
<th>150 B.C.</th>
<th>100 A.D.</th>
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Fig. 3. Analysis by Facility Use (500 B.C. - 100 A.D.)

<table>
<thead>
<tr>
<th>Existing architecture</th>
<th>Constructed architecture</th>
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<td>Agora formation process every sort</td>
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<tr>
<td>Government</td>
<td>Naios</td>
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<td>Agora formation process every sort</td>
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362 JAABE vol.10 no.2 November 2011 Hirotomo Ohuchi
5. Fractal Dimension Analysis
To determine changes in the fractal dimension as a complexity of urban space, quantify the complexity of a building’s layout with the relationship between the functional and part-whole deployment using the fractal theory.

5.1 Analytical method
1. Formulation of 3D data (Fig.4.)
   The floor plan and elevation plan were imported as graphic data into CAD and converted into vector data. The height of the architectural structures etc., were measured according to references 1-13 and the fact-finding data. The vector data was opened with CAD. A three-dimensional computer model was used to read and place the data on a site map drawn by a three-dimensional computer model.

2. Construction of 3D Plaza model (Fig.5.)
   A 3-dimensional plaza model, 300 m x 300 m was created and designated as Range 1 and a second 3-dimensional plaza model, 135 m x 135 m was created and designated as Range 2.

3. Light source (Fig.5.)
   1. For Range 1, the light sources were set at eye level (1.6 m) at each of the three path intersections.
   2. For Range 2, the light source was set at eye level (1.6 m) at the center (Point 4) of the 3D Plaza model.

4. Light decay
   The light intensity distribution for the point light source is shown in Fig.6. Doxiadis discussed the spatial recognition ability of the Greeks in his book "Architectural Space in Ancient Greece." The recognition ability of humans is described as polar, where their point of view is centered and buildings are placed in all the directions and at different distances from the center. Using this idea, he analyzed the layout of buildings in sanctuaries by placing the viewpoint at the entrance of the sanctuaries. This viewpoint was set as the origin of the polar space, and the relationship between the distance and angle from the precincts was examined. The placement of the precincts showed a proportional and geometric relationship with the distance of each point in the building from the viewpoint. The viewpoint angles were set by dividing the 360° space into ten or twelve parts. Accordingly, the viewpoint was set at the entrance of the Agora, and it was considered as an important viewpoint in this study. In the case of the starting point for the attenuation of light the limitation of movement was set as 135m, citing the distance of grade five which is that of an average size Mediterranean city square. The attenuation of light is set as inverse square attenuation. It is similar to human cognition based on Weber-Fechner’s law (references 16, Negoro (2004) and 17 (2005) and reference 24, Thiel (1961)).

The light intensity was perfectly attenuated at Points 1 to 4 on a circular boundary circumscribed on a 3D model.

Fig.4. 3D Model (300 B.C. - 100 A.D.)

Fig.5. Model and Light Setting

Fig.6. Setting of Light
6. Fractal Dimension Analysis (Fig.7. and Fig.8.)

The box-counting method (reference 20) Chonabayashi (2005), (reference 21), Kuroiwa (2004), (reference) 22) C. Borvil (1997) was used to binarize the shading image at the applicable threshold level. In this study, the threshold level was 2.3% (256×0.236; more than 6 shades of gray are considered white). The number of pixels is 1024.

Fractal dimension analysis is performed by box counting. When a binary image is covered by squares with each side measuring (r) pixels, if the number of squares contained in the target number of pixels is \( N(r) \) for each pixel interval (r), the following equation is obtained:

\[
N(r) = C \cdot r^{-D}
\]

where \( C \) is a constant and \( D \) is the fractal dimension.

If equation (1) is transformed, the following is obtained:

\[
\log(N(r)) = \log(C) - D \log(r)
\]

Here fractal dimension \( D \) indicates the slope of the log \( r \) and log\( N(r) \) line and can be estimated using the least-squares method. When the regression line that is obtained shows good linearity (in other words, when the value of the decision coefficient \( R^2 \) is high), the image being analyzed can be confirmed to possess a fractal nature.

6.1 Form analysis using fractal dimensions (Fig.9.)

The changes in the Agora in every age and at every point were examined using fractal analysis.

1. Analysis of every age (Fig.10.)

As a general tendency, fractal analysis decreased as the age progressed. Fractal analysis decreased rapidly at Light 3 in 150 B.C. and at Light 4 in 100 A.D.

As the fractal analysis proceeded, Light 5 and Light 4 showed approximate contrasts.

2. Analysis of every point (Fig.11.)

Fractal analysis showed little contrast at Light 1 in every age. Fractal analysis at Light 2 and Light 3 decreased rapidly in 150 B.C. At 100 A.D., the contrast was small. Fractal analysis of the contrast at Light 4 and Light 5 decreased in a phased manner. Fractal analysis at Light 4 decreased the most.

7. Conclusion

In this study, the authors used fractal analysis to examine the transitions in the arrangement of facilities from construction to collapse. Agora with small-scale naos in 300 B.C. had high fractal analysis. In 150 B.C., fractal analyses showed drastic decreases at both Light 3, where very large-scale commercial facilities were constructed along the south side of the road, and at Light 2, which was located between the commercial facilities on the south and east sides. Fractal dimension was kept relatively high in the case of Light 1 where there were no large commercial facilities along the road. In 100 A.D., however, construction of very large-
scale public facilities at the center of the Agora reduced the fractal analysis of the whole Agora. It is believed that the above process caused the changes in the Agora.

In this paper, the authors 1) restored the Agora using image processing technology, 2) constructed a three-dimensional analytical method for site planning using fractal dimensional analyses, and 3) examined the "complexity" of space on changes in ancient city Agora from the viewpoint of complex systems theory. In general, along with a change in the increase in Agora buildings, it is presumed that there was an increase in the complexity of the city. However, if we consider the image of visible space from the entrance of the square in Doxiadis's "Architectural Space in Ancient Greece," it is clearly evident that complexity has decreased. These are the new insights obtained from this study.

Notes
1) Plato
[427 – 347 B.C.] Plato was an ancient Greek philosopher and a disciple of Socrates. He founded the Academy in a suburb of Athens, and advocated body-soul dualism and spiritual superiority as the ideal philosophy.

2) Aristotle
[384 – 322 B.C.] Aristotle was a rhetorician, a Greek philosopher, Plato's pupil, and a tutor of Alexander the Great. Later, he founded a school in Athens that promoted positive and synthetic academic culture. Important subjects offered at the school were political science, metaphysics, natural Science, and poetics.

3) Plinius
[A.D. 79] Gaius Plinius Secundus, better known as Plinius was a high-ranking official of ancient Rome and a naturalist. He compiled 37 volumes of the "Journal of Natural History" and was a first-class intellectual.

4) Frontinus

5) Vitruvius
He was a Roman architect who looked after the water supply in Rome in the 1st century B.C. He designed civil and military machinery and authored the earliest written architectural theory called "Book of Architecture" and 10 other volumes, which influenced architecture during the Renaissance.

6) Doxiadis
[1913-1975] He was a Greek urban planner and founder of the worlds urban planning. He put forth the concept of an ecumenopolis. He was the Minister of Construction involved in rebuilding war-ravaged landscapes after World War II, and proposed the concept of human settlements. He proposed Ekistics: the science of human settlements. Human nature and the social function of living space in harmonious-space were defined in units of 14 steps from the house to the big city. The United Nations has used this concept widely in the sector and regional planning policies. Human nature refers to total awareness of one's right to live as a civilized human being.

References
A part of the present study was reported in the following research. References 14), 15), 25), 26).