New Time-Cost Trade-Off Model Considering the Sequence of Alternatives Between Activities

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Abstract

Many studies related to discrete Time-Cost Trade-Off models have been conducted with the goal of optimizing the total construction cost. However, since the existing discrete Time-Cost Trade-Off models do not consider the sequence of the alternatives between activities, there are limitations concerning their practical applicability. Thus, this research develops a new Time-Cost Trade-Off model that considers the sequence of alternatives between activities. In addition, all of the possible relationships (FS, FF, SS, and SF) between activities are considered in the model to enhance its practicability. This new Time-Cost Trade-Off model is expected to make initial planning as efficient as possible.

Keywords: Time-Cost-Trade-Off; critical path method; optimization models; linear/integer programming; construction scheduling

1. Introduction

1.1 Background and Purpose of the Research

With the significantly increased size and complexity of today's construction projects, the need has emerged for advanced technology to satisfy their intricate requirements. In addition to these demands, budgets and time have also been limited. Therefore, the efficiency of construction planning has become a main factor in a project's success or failure (Shin et al. 2006). In order to achieve the systematic planning and management of a project, a network-based planning technique such as the Critical Path Method (CPM) is currently being used in the construction industry (Kim 2012). Proper selection of crews, equipment, and construction methods must be reviewed and determined to utilize CPM more efficiently. However, determining the optimal size of crews, selecting equipment, and choosing the construction method in a way that minimizes the total project cost is not a simple task due to the nature of the problem. It is classified as a combinatorial problem, which is difficult to solve without an advanced model.

For this reason, many studies have been conducted in an attempt to find an optimal scheduling solution to minimize the total project cost, which is composed of both the direct cost and indirect cost (Feng et al. 1997; Demeulemeester 1998; Hegazy and Ayed 1999; Leu et al. 1999; Zheng et al. 2004; Chen and Weng 2009; Eshtehardian et al. 2009).

However, previous models do not consider the sequence of the alternatives between activities. It is difficult to apply those models to practical planning since the technical and logical relationship cannot be expressed without the sequence. The sequence must be included in the TCTO model to obtain a realistic solution. Thus, the suggested model includes the sequence of alternatives between activities, which enables the model to represent the realistic project scenario. In addition, all of the possible relationships (Finish to Start, Finish to Finish, Start to Start, and Start to Finish) between activities are considered in the proposed model. A real planning scenario can be represented with the proposed model. This enhances the practicability of the model.

1.2 Scope and Methodology of the Research

The scope of this research is to develop a new scheduling model based on the discrete Time-Cost Trade-Off (TCTO) Model. The discrete TCTO model assumes that each activity in the project can have several alternatives (Feng et al. 1997), and that furthermore, each alternative has its corresponding duration and cost. This assumption is justified because an activity can have alternatives finished with different methods, crews, equipment, etc. The discrete TCTO model identifies the optimal combination of the construction methods for each activity in order to minimize the total construction cost. Fig.1. shows a typical example of a discrete relationship between time and cost.
The research process is shown as follows: The basic formulation of the discrete TCTO problem has been studied through a literature review of the existing TCTO models. Formulating the sequence of the alternatives and various relationships between activities has been completed. A new TCTO model is completed with the newly developed formulation. This new and improved model is applied to project scenarios with the sequence of the alternatives between activities. The result obtained from the model is compared to the result obtained from the existing discrete TCTO models. The proposed model is validated through the iteration of finding all of the possible schedules and calculating the corresponding total cost.

Fig. 2 shows the flow of the research process. In addition, a template based on the Excel program is used to enhance the user interface of the model. This enables a user to input the variables and constraints of the TCTO problem with simple requirements. The Solver™, which is an Excel-based linear programming optimizer, was used to obtain the optimal solution.

2. Literature Review

2.1 Literature Review of Time-Cost Trade-Off

There are two typical TCTO model scenarios: the continuous and the discrete (Son et al. 2013). The continuous TCTO model assumes that each activity has a normal and a crash duration. The normal and crash durations also have corresponding costs, which are the normal cost and the crash cost. The model assumes that cost and time have a continuous linear relationship within the normal and crash duration. Sometimes, the assumption of a linear relationship makes the model rather unpractical. On the other hand, the discrete TCTO model assumes that each activity in the project has several alternative construction methods, and that each method has its own duration and cost. Since each alternative is expressed with a point in the time-cost graph (See Fig. 1.), even a nonlinear relationship can be represented with the discrete TCTO model. Moreover, in practice, there is a limited number of ways to accelerate an activity, and thus only a finite number of discrete points can exist (Ammar 2011). From a practical point of view, the number of discrete points is finite and usually ranges from one to four (Eldosouky et al. 1991). Consequently, the number of variables required for the discrete TCTO model is manageable even for large projects. Thus, this research mainly focuses on developing a new scheduling model based on the discrete TCTO model. The traditional discrete TCTO model will be adopted in this study. The following section summarizes the research pertaining to previous discrete TCTO models. Burns et al. (1996) presented an algorithm using linear and integer programming to efficiently select alternatives in order to optimize the TCTO problem of a construction project. Feng and Burns (1997) proposed a model using genetic algorithm and pareto front approach to efficiently solve TCTO problems of large-scale CPM networks. Leu and Yang (1999) proposed an optimal scheduling model based on multi-criteria, which integrates the TCTO model, resource-limited model, and resource-leveling model. Feng et al. (2000) suggested a hybrid approach that combines simulation techniques and genetic algorithms to solve the TCTO problem under uncertainty. Hegazy et al. (2001) proposed an integrated scheduling model with TCTO analysis, resource allocation, resource leveling, and cash flow management. Moussourakis et al. (2004) presented a solution to the TCTO problem, which has a project deadline and resource constraints. Zheng et al. (2004) proposed a TCTO model with multi-objective functions by using genetic algorithm. Chassiakos et al. (2005) suggested a linear and integer programming model to solve the TCTO problem considering incentives and penalties for early and delayed project completion. Chen and Weng (2009) proposed a genetic algorithm based on the TCTO model to resolve a resource-constrained situation, which includes the interruption and overlapping of
the resources. Senouci et al. (2009) presented a multi-objective optimization model that generates and evaluates efficient construction resource utilization and scheduling plans. It simultaneously minimizes the time and maximizes the profit of construction projects. Ammar (2011) provided a TCTO model for minimum project cost and took into account the cash flows of the net present value.

A literature review is summarized in Table 1. to easily compare the models' capabilities.

<table>
<thead>
<tr>
<th>TCTO Models</th>
<th>Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burns et al.</td>
<td>Linear and Integer Programming</td>
</tr>
<tr>
<td>Feng and Burns</td>
<td>Genetic Algorithm (GA)</td>
</tr>
<tr>
<td>Leu and Yang</td>
<td>Multi criteria including resource</td>
</tr>
<tr>
<td>Feng et al.</td>
<td>Hybrid approach that combines simulation techniques and GA</td>
</tr>
<tr>
<td>Hegazy et al.</td>
<td>Integrated scheduling model with TCTO, resource, and cash flow</td>
</tr>
<tr>
<td>Moussourakis et al.</td>
<td>Project deadline and resource constraints</td>
</tr>
<tr>
<td>Zheng et al.</td>
<td>Multi-objective function using GA</td>
</tr>
<tr>
<td>Chassagnoux et al.</td>
<td>Incentive and Penalty</td>
</tr>
<tr>
<td>Chen and Weng</td>
<td>Interruption and overlapping of the resource</td>
</tr>
<tr>
<td>Senouci et al.</td>
<td>Resource utilization</td>
</tr>
<tr>
<td>Ammar</td>
<td>Cash flow of Net Present Value</td>
</tr>
</tbody>
</table>

As shown above, various studies pertaining to TCTO models have been conducted using diverse optimization techniques. Also, many attempts have been made to reflect practical constraints. However, none of the studies have considered the sequence of the alternatives between activities. In addition, many studies only use the Finish to Start (FS) relationship for the relationship between activities. For these reasons, there are practical limitations to the previously developed TCTO models. Therefore, this research develops a new TCTO model that considers the sequence of the alternatives between activities. In addition, all of the possible relationships (FS, FF, SS, and SF) between activities are considered in the model to enhance its practicability. This prototype model can be applied to those previously developed discrete TCTO models to take advantage of their unique features.

2.2 Discrete TCTO Model Formulation

The scope of this research is to develop a new scheduling model based on the discrete TCTO model. In general, CPM has a network in which many activities are connected to other activities. In the planning stage, one alternative of an activity can affect the whole project. Thus, the construction scheduler needs a construction plan that considers every possible combination of alternatives for activities. However, examining all of the possible combinations of alternatives is impossible without using a systematic model. For example, if there are 20 activities in the CPM network and each activity has 3 alternatives, then the number of possible combinations with those alternatives is 3,486,784,401 (320). Since there are many more activities in a construction project, many systematic models have been developed to find an optimal solution for the TCTO problem. In this paper, a typical mathematical formulation for the TCTO problem is introduced, which will later be adopted to develop a new discrete TCTO.

The objective function of a discrete TCTO model is expressed as follows:

\[
\begin{align*}
\text{Min} & \sum_{i=1}^{n} \sum_{j=1}^{m} \text{Cost}_{ij} \times X_{ij} + \text{IDC} \times PD \\
\sum_{j=1}^{m} X_{ij} &= 1, X_{ij} \in \{0,1\}
\end{align*}
\]

Where \( n \) is the number of activities in the network; \( i \) is the activity indicator; \( m \) is the number of alternatives of activity \( i \); \( k \) is the alternative indicator; and \( X_{ij} \) is a binary variable to which 0 or 1 is assigned. The meaning of \( X_{ij} = 1 \) is that the construction method (alternative) \( k \) was chosen for activity \( i \) and \( \text{Cost}_{ij} \) is the budget cost when the alternative \( k \) is chosen for activity \( i \). \( PD \) is the total project duration and \( IDC \) is the indirect cost rate ($/day).

Modification of the objective function is not required when different relationships are used with the discrete TCTO scenario. However, the constraints have to be modified for each of the different relationships. The modifications of the constraints for four different relationships can be found in the author’s previous study (Son et al. 2013). When Activity B succeeds Activity A, the formulations of the constraints are summarized as follows:

1. Finish-Start

\[
S_B \geq S_A + \sum_{i=1}^{n} (D_A \times X_{ai}) + \text{Lag}_A B
\]

\[
\sum_{i=1}^{n} X_{ai} = 1
\]

2. Start-Start

\[
S_B \geq S_A + \text{Lag}_A B
\]

3. Finish-Finish

\[
S_B + \sum_{i=1}^{n} (D_B \times X_{bi}) \geq S_A + \sum_{i=1}^{n} (D_A \times X_{ai}) + \text{Lag}_A B
\]

\[
\sum_{i=1}^{n} X_{ai} = 1
\]

4. Start-Finish

\[
S_B + \sum_{i=1}^{n} (D_B \times X_{bi}) \geq S_A + \text{Lag}_A B
\]

\[
\sum_{i=1}^{n} X_{ai} = 1
\]

\( S_A \) and \( S_B \) are the starting times of activities A and B, respectively. \( m \) is the number of alternatives of activity A and \( X_{ai} \) is a binary variable. The meaning of \( X_{ai} = 1 \) is that the \( k \)th construction method (alternative) is chosen for activity A. \( \text{Lag}_A B \) is the lag (lead) time between activity A and B.

The meaning of \( \sum_{i=1}^{n} X_{ai} = 1 \) is that only one construction alternative is allowed for activity A. \( D_A k \) is the duration of activity A when the \( k \)th alternative is chosen.
All the variables for Activity B are the same as for Activity A.

2.3 Limitation of Existing Discrete TCTO Models

Since existing discrete TCTO models do not consider the sequence of the alternatives between activities, the solution from those models cannot be used in practical planning. This section shows one example of a schedule that must consider the sequence in order to plan a real construction task.

Fig. 3 shows two activities and their alternatives for the earthwork of a gymnasium project in Korea. Activity A is the selection of types for the earth retaining wall and Activity B is the method of supporting the retaining wall. Activity A has two alternatives. One is referred to as A1, and installs H-files with the lagging board for four sides. The other is referred to as A2. It is a combination of the slope excavation for 3 sides and H-file with lagging boards for one side. Activity A has two alternatives. One is referred to as B1, and it uses the Raker method for four sides. The other is referred to as B2. It is a combination of slope excavation for 3 sides and earth anchor for one side. The existing discrete TCTO models might choose the combination of A1 and B2 or A2 and B1 because the sequences of the alternatives have not been considered in their model. The first alternative of Activity A (Installation of H-File with Lagging boards) has a sequential relationship with the first alternative of Activity B (usage of Raker method for four sides). Likewise, A2 has a sequential relationship with B2. Due to this sequential relationship, if A1 is selected, B1 must be selected from the alternatives of Activity B.

3. Model Development

3.1 Development of a Discrete TCTO Model and its Sequential Function

In this section, the formulations for the sequence of alternatives between activities are described with a simple example. In the example, Activity A and B each have 3 alternatives. The relationship between Activity A and B is a Finish to Start (FS) Relationship.

If different relationships exist between activity A and B, equations (1) and (10) can be modified easily using formulations introduced in the previous section. The first case example assumes that each alternative of Activity A has only one sequential relationship with an alternative of Activity B (See Fig.4).

$$X_{al} + 1000Y_{ia} \leq 1000$$  \hspace{1cm} (11)
$$X_{al} + 1000Y_{ia} \leq 1000$$  \hspace{1cm} (12)
$$X_{al} + 1000Y_{ia} \leq 1000$$  \hspace{1cm} (13)
$$X_{al} + X_{ai} - 1000Y_{ia} \leq 0$$  \hspace{1cm} (14)
$$X_{al} + X_{ai} - 1000Y_{ia} \leq 0$$  \hspace{1cm} (15)
$$X_{ai} + X_{ai} - 1000Y_{ia} \leq 0$$  \hspace{1cm} (16)
$$Y_{ia} = [0,1]$$  \hspace{1cm} (17)
$$\sum_{k} X_{ak} = 1, X_{ai} = [0,1]$$  \hspace{1cm} (18)
$$\sum_{k} X_{ak} = 1, X_{ai} = [0,1]$$  \hspace{1cm} (19)

Fig. 4 shows two activities and their alternatives for the earthwork of a gymnasium project in Korea. Activity A is the selection of types for the earth retaining wall and Activity B is the method of supporting the retaining wall. Activity A has only one sequential relationship with Activity B. All the variables for Activity B are the same as for Activity A.

As shown above, the sequence of alternatives between activities is an important factor when the alternative is selected in the discrete TCTO Model. However, since the alternative selection in the existing discrete TCTO models does not include the sequence function, B2 can be chosen with A1 regardless of the technical and logical relationship. However, this is not practical.

Therefore, in order to resolve the problem, this study develops an advanced discrete TCTO model including the formulation for the sequence of alternatives between the activities.

This developed model proposes an optimal solution while satisfying the practical constraints.

Equations (11)-(19) explain the first case of the example. $X_{al}$ and $Y_{ia}$ are the binary variables which indicate the selection of alternatives. If $X_{al}$ is 1, it means that the $k^{th}$ alternative of Activity A is selected. If $X_{al}$ is 0, it means that the $k^{th}$ alternative of Activity A is not selected. $Y_{ia}$ is a binary variable, which controls the sequence of alternatives between Activities A and B.

In Case 1, each alternative of Activity A has a sequence with one corresponding alternative of Activity B. If alternative A1 is selected, $X_{al}$ and $X_{al}$ become 0 by equation (18). Also, $Y_{ia}$ becomes 0 by equation (11). If $Y_{ia}$ is 0, then $X_{al}$ and $X_{al}$ are automatically 0 by equation (14). Thus, $X_{al}$ is equal to 1 by equation (19). Therefore, as a result of the above process, if A1 is selected, B1 is automatically determined. All other sequences (A2-B2 and A3-B3) can be explained with the same process as above.

In the second case example, A1 has a sequential...
relationship with B1; A2 has two potential sequential relationships with B1 and B2; and A3 has three potential sequential relationships with B1, B2, and B3. For example, if A3 is selected, the succeeding activity of A3 can be any activity among B1, B2, and B3. (See Fig. 5.)

\[
\begin{align*}
X_{a1} + 1000Y_{a1} &\leq 1000 \\
X_{a2} + 1000Y_{a2} &\leq 1000 \\
X_{a3} + 1000Y_{a3} &\leq 1000 \\
X_{a2} + X_{a3} - 1000Y_{a1} &\leq 0 \\
X_{a2} - 1000Y_{a2} &\leq 0 \\
-1000Y_{a3} &\leq 0
\end{align*}
\]

\(Y_{ai} = \{0,1\}\)

\[
\frac{X_{ai}}{X_{ii}}X_{ai} = 1, X_{at} = \{0,1\}
\]

\[
\frac{X_{ai}}{X_{ii}}X_{ai} = 1, X_{at} = \{0,1\}
\]

3.2 Network Example

In this section, a network example is introduced to test the developed model with its sequence function. The network example has 8 activities with 4 different relationships and lag time to reflect the practical situation (See Fig. 6. and Table 2.). Also, each activity has at least 3 alternatives. Each alternative of Activity A, B, D, and H has at least one sequential relationship with the alternatives of its succeeding activities (See Fig. 7. and Table 3.).

The data in Tables 2. and 3. represent information on the sample network. Figs. 6. and 7. depict the network information. The total cost of the sample project has two components: direct and indirect costs. The direct cost of each activity can be calculated based on the network information shown in Table 2. The calculation of indirect cost for the project is based on the indirect cost rate. The indirect cost rate is generally represented as a single cost per time period (e.g., day or week). In this example, the indirect cost rate is assumed to be $2,000/day. This assumption is made since the indirect cost rate of the project varies according to the project’s size, type, region, etc. There is no fixed cost for the indirect cost rate. In this paper, the indirect cost rate of $2,000/day is chosen to experiment and prove the model with examples. Nevertheless, in the proposed model, the indirect cost rate can be easily modified according to the requirements of the project. An Excel-based TCTO model was developed to have the functions for sequential relationships. A detailed explanation using this network example will be provided in the following section.

Fig. 5. Two or Three Potential Sequential Relationships

The sequential relationship of Activity A1 with Activity B1 was explained previously in Case 1. If A2 is selected, X_{a1} is equal to 1. X_{a1} and X_{a2} are both equal to 0 under equation (27). Due to the value of X_{a2}, Y_{a2} becomes 0 under equation (21). If Y_{a2} is 0, then X_{a1} is equal to 0 under equation (24). One from X_{a2} or X_{a1} will be selected according to the equation (28). Therefore, as a result of the above process, if A2 is selected, a sequential relationship which can select B2 or B3 is validated.

If A3 is selected, X_{a3} is equal to 1. X_{a1} and X_{a3} are both equal to 0 under equation (21). Y_{a2} becomes 0 under equation (22). According to equations (25) and (28), any alternative of Activity B can be a succeeding activity of A3. One out of three alternatives of Activity B must be selected due to equation (28). Therefore, as a result of the above process, if A3 is selected, a sequential relationship which can select one from B1, B2 and B3 is validated.

Using the cases stated above, every possible combination of different relationships, different numbers of alternatives, and different numbers of sequential relationships can be formulated through the modification of equations (11) through (28). In the next section, the developed model is applied to and validated with one network example having various numbers of alternatives and various sequence scenarios.

Fig. 6. Network Diagram

Fig. 7. Network Diagram with Sequence
Using Excel, a template is made to enhance the user interface of the model. The template makes the input of the variables and constraints of the TCTO Problem easier. The Solver™, which is an Excel-based linear and integer programming optimizer, was used to obtain the optimal solution. The main components of the model are shown in Fig.8. If the basic information for the network project is given, the model is designed to produce the optimal solution easily. The main components of the model will be explained in further detail. Column A is for Activity Description; Column B shows the alternatives of the activity; Column C shows the duration of the alternatives; Column D is the corresponding cost of the alternatives; Column E shows the sequence of the alternatives. Column I shows the binary variables, which determines the sequence of the alternatives between activities. Columns K and L are constraints expressed by the equation (11)-(28). They ultimately control the sequence expressed by Column I.

Column F forces only one alternative to be chosen by Column J, which shows the constraints expressed by equation (18). If Column F selects one alternative, the corresponding duration and cost are listed in Column G and H.

Succeeding activities are listed in the cells of O22:O32 and relationships are listed in the cells of P22:P32. The lag times are input in the cells of Q22:Q32, and the earliest start times are updated in the cells of P11:19. The project duration is calculated in the cell of O8, and the total cost of the project is shown in the cell of O6.

3.3 Developed TCTO Model Based on Excel

Using Excel, a template is made to enhance the user interface of the model. The template makes the input of the variables and constraints of the TCTO Problem easier. The Solver™, which is an Excel-based linear and integer programming optimizer, was used to obtain the optimal solution. The main components of the model are shown in Fig.8. If the basic information for the network project is given, the model is designed to produce the optimal solution easily. The main components of the model will be explained in further detail. Column A is for Activity Description; Column B shows the alternatives of the activity; Column C shows the duration of the alternatives; Column D is the corresponding cost of the alternatives; Column E shows the sequence of the alternatives. Column I shows the binary variables, which determines the sequence of the alternatives between activities. Columns K and L are constraints expressed by the equation (11)-(28). They ultimately control the sequence expressed by Column I.

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Fig.9. is a screen-shot of options and specifications of the Solver™, an Excel Add-on linear and integer programming software. The objective function is set to minimize the total project cost in the cell of O6. Variables in the model are as follows:

- Column F (selection of alternatives): binary variables
- Column I (control of sequence): binary variables
- P10:P19 (ES of activities): integer variables

Constraints of the model are expressed using the values on the excel sheet. Constraints regarding selection, sequence, network logic, etc. are set in the
The project duration ranges from a minimum of 27 days to a maximum of 46 days. Several project durations (day 34, 37, 40, 42, 45) are omitted from the range since there are no possible combinations of alternatives that would produce those project durations. Firstly, the authors identify all possible project schedules that generate the maximum duration (i.e. 46 days), and manually calculate the total cost of each schedule. The total costs are then compared, and the schedule with the minimal cost is marked. Authors iterate the procedure until the specific project duration reaches the minimum of 27 days. Table 5. summarizes the marked (least-cost) schedules for each project duration day.

The 2nd column in Table 5. shows each selected alternative of activities. As seen in Table 5., the optimal total cost is $126,500 when the project length is 36 days. The corresponding schedule scenario is A3, B3, C2, D3, E3, F4, G2, H3, and I1, which is identical to the optimal plan from the proposed model. Since the minimum cost schedule from Table 5. is the same as the optimal schedule from the proposed model, the proposed model is validated.

The purpose of using a simplified sample network is to clearly show all the formulations for possible sequence scenarios in regards to the Time-Cost tradeoff problem. Since all of the potential modular formulations have been established, the model can easily be applied to the expanded project example.

Table 5. Least-Cost Schedule for Each Project Duration Day

<table>
<thead>
<tr>
<th>Duration</th>
<th>Activity (selected alternatives)</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>27</td>
<td>A3, B3, C2, D3, E3, F4, G2, H3, I2</td>
<td>$126,000</td>
</tr>
<tr>
<td>28</td>
<td>A3, B3, C2, D3, E3, F4, G2, H3, I2</td>
<td>$130,500</td>
</tr>
<tr>
<td>29</td>
<td>A3, B3, C2, D3, E3, F4, G2, H3, I2</td>
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</tr>
<tr>
<td>30</td>
<td>A3, B3, C2, D3, E3, F4, G2, H3, I2</td>
<td>$129,500</td>
</tr>
<tr>
<td>31</td>
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<td>33</td>
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<tr>
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<td>35</td>
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<tr>
<td>46</td>
<td>A3, B3, C2, D3, E3, F4, G2, H3, I2</td>
<td>$128,500</td>
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</tbody>
</table>

6. Conclusion

This paper presented a new discrete TCTO model that considers the logical sequence of alternatives between activities. This feature enables the proposed model to represent the project more realistically. The main contribution of this study is formulating the
sequence and developing an advanced practical TCTO Model. Excel is used to enhance the user interface of the model. The Excel template makes the input of the variables and constraints of the TCTO Problem easier. The Solver™, which is an-Excel based optimization software, was used to obtain the optimal solution. If the basic information of the project network is given, the developed Excel template is utilized to input the network information into the model. Once network information was input into the template, the solver was used to produce the optimal solution. This model can solve various TCTO problems that consist of practical TCTO scenarios. In addition, the commercial Solver™ has the capacity to handle 32,000 variables and 32,000 constraints by using the Solver large-scale engine (standard). With that capacity, projects of various sizes can be suitable for this model. Fortunately, since computing time and memory capacity have been significantly enhanced and techniques for decomposing the project network have been studied (Taha 2006), the proposed model would be able to handle projects of any size or complexity.

Using an example, the model formulation and the analysis of the resulting solution were presented. The output can be easily interpreted in the form of the precedence diagram for scheduling, and can help a project manager make a viable decision for the success of a project. Sensitivity analysis was implemented to find the possible range of the project duration, which includes the longest and shortest durations of the project. This information is also important for project managers to be able to effectively manage a project. Iteration of finding possible schedules and calculating the corresponding total costs was implemented to validate the proposed model. The schedule with the minimum cost from the iteration is the same as the optimal one from the proposed model.

The Excel-based optimizer named Solver™ was investigated and shown to enable a user to generate a lengthy formula with simple input requirements.

Building up the database, which contains alternatives and their sequence in the activity level for the project, is very crucial to the implementation of the proposed model. It requires extra effort for the scheduler to collect such information. However, like other advanced models required, constructing the database is the first step to ensure the success of the suggested system. This might be the main challenging task as well as the limitation of utilizing this suggested model.

Future research could involve the non-linear relationships between the cost and the time in the proposed TCTO model. The proposed model can be integrated with BIM technology for 4D simulation. In addition, a resource assignment and leveling function could be added to the proposed model for multi-objective function for future research. This could make the model more versatile.

Acknowledgement
This work was supported by the 2014 Hongik University Research Fund.

References