A Digital Analysis of the "Digitally-Derived Geometric Design" of the Front Wall of St. Paul's Church in Macao

-A Study on the Architecture and City of Macao, No. 1-

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Abstract

With reference to related historical archives and literature, this study conducted research into the history, the designer and the design dimensions of St. Paul's Church in Macao. The remaining drawing of the facade merely reflects the correlation deduction of each controlling point in its geometric drawing. Through digital analysis, this study has further obtained the "digitally-derived geometric design" relations of the five-layer elevation design drawing and number-rounding adjustment of the constructing dimensions: the application of the law of equal-partition in the horizontal direction, the application of the law of upward successive-subtraction in stratification in the vertical direction, and the complicated combination of golden-rectangle controlling designs. The latter two applications are key geometrical features of Baroque Architecture.

Keywords: St. Paul's Church; facade; geometric drawing; the law of upward successive-subtraction in stratification; the golden ratio

1. Introduction

There are few remaining examples of Renaissance-style and Baroque-style architecture from the Age of Navigation, 16th-17th centuries, in East Asia. Of these, there remains only one Renaissance-style Guia Church (Igreja de nossa Senhora da Guia; its construction completed before 1622)\(^1\), and few examples of Baroque-style architecture, with St. Paul's Church (Igreja de S. Paulo, its construction completed from 1602 to 1637) as one of the masterpieces of Baroque-style architecture, in Macao. During the 1620s and 1680s, the Baroque style prevailed at its birthplace, Italy; its popularity spread to other countries later (Suzuki, 1998)\(^2\). St. Paul's Church was designed by Father Carolo Spinola, who was born in Italy. The Front Wall of St. Paul's Church was rebuilt at a new site during the construction of Guia Church. It is the most ancient remaining landmark building in Macao, and a masterpiece of the early-stage Baroque Architecture not only in East Asia, but throughout the world as well.

St. Paul was a foreign disciple chosen by Jesus Christ who had gone on at least three expeditionary missions. The Church was named after St. Paul, implying that the Society of Jesus carried on the cause of St. Paul by travelling across the world to the Far East and preaching Catholicism (Ljungstede, 1997)\(^3\). St. Paul's Church has experienced multiple reconstructions, damaging disasters, and architectural restoration; today, only the facade of the Front Wall survives. The original appearance of the whole church can only be imagined through viewing historical documents and paintings. Previous researchers have not conducted an exact digital analysis of the facade.

2. Research Objectives and Methods

Geometric composition has been a beloved method for architects of western classical architecture, while geometry is a derivation of "digits". With St. Paul's Church as the research object, this study synthetically applied historical document research, as well as field measurement and digital analyses of the elevation of the Front Wall. This study has obtained the digitally-derived geometric proportion-relationship and design patterns of the Front Wall of the Church, and analyzed features of the Church's design style.

This study applied the following research methods:

a. A Cross-reference Research of Historical Documents and Literature:

Through a detailed investigation, this study collected related historical documents and literature about remaining western classical architecture in Macao. It also conducted a cross-reference research on the useful information thus extracted, while verifying the results of digital analyses.

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b. Field Measurements, Digitization and Analyses of Related Drawings:

b.1 File 2901/90 from the Macao Cartography and Cadastre Bureau documented the design of the facade and the line of connection among the main controlling points (Fig.3). Section One of Chapter Five of this article contains the analysis of the drawing connection of the controlling points documented in this historical document.

b.2 After field measurement, the author scanned the design of the facade documented in File 2901/90 from the Macao Cartography and Cadastre Bureau and inputted it into the computer. This historical document merely recorded the line of connection among the controlling points, with no reflection of their proportion. Section Two of Chapter Five of this article contains further digital analysis of the aesthetics-mathematics patterns hidden behind the elevation of the Front Wall, including the Golden Ratio, the Law of Equal-partition and the Law of Successive-subtraction.

3. Historical Research

St. Paul's Church was built based on Santa Maria Maggiore, which was first built at the beginning of 1565 and its construction was officially finished on December 21st, 1565. It is located near St. Anthony's Church. At first, it was built with straw and a wooden frame. Later, with a donation from the Portuguese businessman, Antonio de Vilhena, this church was rebuilt with an additional clay-built sanctuary in 1573. It was extended in 1575; by then, the sanctuary already included 10 ample rooms, as well as a sitting room. Alessandro Valignano praised it as the best sanctuary of the Society of Jesus in the Far East. By 1580, the rooftop of the sanctuary had been rebuilt with tiles and the floor recovered with wood (Ajuda Library, 1994). In the City Map of Macao in 1590, St. Maria Maggiore (St. Paul's Church) had become a complex of one two-storey building, one three-storey building and one three-storey bell tower (Xue, 2012) (Fig.1.).

The origin of St. Paul's Church dates back to the early years of the opening of Macao port. According to Padre Manuel Teixeira:

Upon the establishment of the Macao diocese in 1576, Pope Gregory XIII said, “there is a Santa Maria Maggiore in here” (Teixeira, 1987). In the Macao Chronicle, Beatriz Bastao da Silva also wrote:

He (Bishop Carneiro) passed away in Macao on August 19th, 1583. He was buried in St. Paul's Church in the center of the city...On October 17th, 1591, Father Antonio de Almeyda passed away in Shaozhou of Guangdong Province (the original translation version made a mistake by translating it as Chaozhou). His remains were carried to Macao and buried in St. Paul's Church (Silva, 1995).

In 1592, the sanctuary in its previous position could no longer satisfy people's needs and it was rebuilt in the current position. It came into service upon the completion of its rebuilding in 1594 (Li, 2001). In 1595, the new church was destroyed in a catastrophic fire. Though it was restored, another fire burned it down again in 1601. So, in the same year, Portuguese Businessmen and citizens in Macao decided to rebuild it for the third time. According to historical records, the donation received at that time reached 3130 Macao gold coins. The reconstruction officially started in 1602. The church was finished around the year 1603; the remaining Front Wall was built later in 1637. The reconstruction lasted for over 30 years; the construction of the Front Wall alone cost 30 thousand Liang of silver coins (Guerreiro, 1997). The reconstruction of St. Paul's Church was led and supervised by Father Valentim de Carvalho. It is said that the designer of the Church was Father Carlo Spinola. When the Japanese Shogunate enacted a prohibition on preaching western religions, a great number of Japanese Catholics took refuge in Macao in 1614 and they participated in the construction and decoration of the Church. The elevation of St. Paul's Church is 24 meters high and 23 meters wide (Valente, 1993). The grandness of its construction, the magnificence of its decoration and the
The fineness of its equipment all made it the crown jewel among churches in the Far East.

Peter Mundy, who traveled to Macao in 1637, said:
The ceiling of this Church is the most exquisite construction that I've ever seen. It is built so ingeniously. The woodcarving comes from the hand of Chinese craftsmen. The colors of the golden outline include vermilion and sky-blue, etc. The ceiling is divided into multiple checks and on each junction of the checks, there is a large rose. The petals of the roses overlap with one another, and gradually shrink into a small ball; those roses are hung one yard below the ceiling. Moreover, there is an elegant decorated archway in front of the Church, which could be reached through climbing the wide stair (Mundy, 1967).

St. Paul's Church is not only a church of Catholicism; it is also a construction complex of large scale (Fig.2.). According to A Brief History of Macao, "the Church includes over 100 monk's cells," (Ying and Zhang, 1992) from which we can imagine the grandness of its scale. Upon its completion, St. Paul's Church immediately became the symbol of the city of Macao.

In 1759, the Portuguese Government ordered the Society of Jesus to be dismissed. On July 5th, 1762, St. Paul's Church was placed in the hands of the Dominican Order, while St. Joseph's Church and the monastery were placed in the hands of the Franciscan Order. On November 5th, 1762, Jesuits in Macao were repatriated to Portugal. In 1798, the Church became the garrison for the Portuguese Prince Cavalry Regiment. On January 26th, 1835, the mass of firewood that the military kept in the kitchen caused a fire that burned down the Church and the monastery, leaving behind only the Front Wall of the Church. On April 8th, 1835, the Macao Camara commissioned the bishop's agent, Father Candido Goncalves Franco, to transform the Church and monastery into a graveyard. In 1878, Viscount pacod' Acros took over the position of Macao military leader, and ordered the graveyard be removed to St. Miguel (Liu and Chen, 2005).

4. The Designer and the Basic Measurement

The designer of St. Paul's Church, Fr. Carlo Spinola, had worked in Goa, India before he came to Macao. Palmos is the traditional measurement in Goa and Fr. Carlo Spinola continued applying this measurement in his design of the Church (Fig.3.).

The elevation of the Church is 100 palmos wide, with bilateral symmetry. The central symmetrical axis divides the elevation into two parts, each 50 palmos wide. The figure 50 palmos has appeared many times in the design of the Church, such as the height of the church wall, the width of the bell tower, the width of the entry porch, and the width of the altar. The length of the four porches surrounding the Church is approximately 50 palmos as well.

5. The Patterns of the Elevation Design of the Front Wall

5.1 The Correlation Deduction of Each Controlling Point in the Elevation Design Drawing

File 2901/90 from the Macao Cartography and Cadastre Bureau documented the design of the facade and the line of connection among the main controlling points (Fig.3.). This Section contains an analysis of the drawing connection of the controlling points documented in this historical document.

Based on Fig.3., each axis interaction point in Fig.4. is marked by a character and number; the line segment between two points is marked by the mark of the two endpoints. This study analyzed Spinola's design drawing of controlling points in the elevation. The analyses process is given below:

1) Draw two separate lines, one starting from A1 and forming an angle of 45 degrees with the horizontal axis, crossing with Axis ③ and forming B3, the other line starting from A7 and forming an angle of 135 degrees with the horizontal axis, crossing with Axis ⑤ and forming B5. Draw a line between B3 and B5. Set the line of Axis Ⓑ as the ground line for the second floor. We can work out that the height of the first floor is 33.5 palmos, which equals the line segment A1-A3.

2) Draw Line A4-B3 and Line A4-B5 from Point A4, and extend Line A4-B3 to cross Axis ② on C2, and Line A4-B5 to cross Axis ⑥ on C6. Draw a line between C2 and C6. Set the line of Axis Ⓒ as the ground line for the third floor. We can work out that the height of the second floor is 29.5 palmos.

3) Extend Line A1-C6 to cross with Axis ⑦ on D7. Extend Line A7-C2 to cross Axis ① on D1. Draw Line D1-D7. Set the line of Axis Ⓐ as the ground line for the fourth floor. We can work out that the height of the second floor is 29.5 palmos.
third floor is 14 palmos.

4) Mark the midpoint of Line D1-D7 as D4. Draw Line A1-D4 and extend it to cross Axis ⑤ on F5. Draw Line A7-D4 and extend it to cross Axis ③ on F3. We can see that the midpoint of Line F3-F5, F4, is the peak of the fifth-floor pediment; the height of the fourth floor and third floor is the same, 14 palmos. The height of the fifth-floor pediment is 11 palmos.

5) The intersection point J4 of Line A1-D7 and Line A7-D1 determines Axis ①.

The elevation drawing in the file merely reflects the line of deduction among each of the controlling points in the geometric drawing.

5.2 The Analyses of the Digitally-derived Geometric Design

The elevation drawing in File 2901/90 from the Macao Cartography and Cadastre Bureau merely recorded the line of connection among the controlling points yet contained no reflection of their relationship of proportion, while the beauty of geometric figures derives from these very mathematical patterns. The author inputted the elevation drawing from the file document into the computer and conducted further digital analysis of the aesthetics-mathematics patterns hidden behind the elevation of the Front Wall, including the Golden Ratio, Equal-Partition and Successive-Subtraction.

5.2.1 The Partition Pattern of the Overall Dimension of the Elevation: Equal-Partition, Successive-Subtraction and Golden Ratio

The elevation of the Front Wall is 100 palmos high and 102 palmos wide. It can be divided into 3 parts in the horizontal direction: Axis ① - ③ and Axis ⑤ - ⑦ are two symmetric parts at the two sides of the Mid-Axis ④. In the vertical direction: the main part between Axis ③ - ⑤ could be stratified into five layers. Parts of the second layer overlap with the third layer. The fifth layer is the pediment. The two parts with bilateral symmetry could be divided into four layers. This study analyzed the overall dimension of the elevation and obtained the following pattern:

a. The Law of Equal-Partition in the Horizontal Direction: the 100-palmos overall width could be divided into two parts of bilateral symmetry by the middle axis, each one of 50 palmos wide. It could also be divided into three parts of approximately the same width. The main middle Axis ③ - ⑤ is 33 palmos wide; the Axis ① - ③ and Axis ⑤ - ⑦ on each side are 33.5 palmos wide. Such a result should be a number-rounding adjustment based on equal trisection of 100 palmos. The mathematical relation is given as follows (unit: palmos, the same below):

$$100 = 50 \times 2 = 33.333 \times 3 = 33.5 + 33 + 33.5$$

b. The Law of Successive-subtraction in Stratification in the Vertical Direction: in the height direction, the height of the first layer, Axis Ⓐ - Ⓑ, is 33.5 palmos high. The height of the second layer, Axis Ⓑ - Ⓒ, is 29.5 palmos high. The height of the third layer with the overlapping part, Axis Ⓒ - Ⓓ, is 18.5 palmos high; the height of the third layer without the overlapping part, Axis Ⓒ - Ⓓ, is 14 palmos high. The height of the fourth layer, Axis Ⓓ - Ⓔ, is 14 palmos high. The height of the fifth layer, Axis E-F, is 11 palmos high. The difference between the height of the first layer and the second one is 4 palmos; the difference between the height of the second layer and the third one with the overlapping part is 11 palmos; the difference between the height of the second layer and the third one without the overlapping part is 15.5 palmos; the difference between the height of the third layer with the overlapping part and the fourth one is 4.5 palmos; the difference between the height of the fourth layer and the fifth one is 3 palmos. In the height direction, the heights from the first layer to the fifth one follow the law of Successive-subtraction.

St. Dominic's Church (Igreja de São Domingos) in Macao is another example of Baroque-style architecture and its design also followed the Law of Successive-subtraction in Stratification in the overall height in the vertical direction. Meanwhile, the Renaissance-style Guia Church in Macao has two identical golden rectangles partially overlapping with one another in the vertical direction instead of successive-subtraction¹. Successive-subtraction in Stratification in the vertical direction, along with complicated decoration combination of arcs, swirl-roll, and pediment, are significant geometric features of Baroque-style architectures.
c. Golden Ratio in the Overall Height: The overall height is 102 palmos. The height between the first layer and the second, Axis \((\text{A} \cdot \text{C})\), is 63 palmos; and the height between the third layer and the fifth, Axis \((\text{C} \cdot \text{F})\) is 39 palmos. The division in the height direction follows the pattern of golden ratio: \(\frac{63}{39} = \frac{\sqrt{5} - 1}{2}\).

The mathematical relation is given as follows:

\[
102(\text{overall height}) = 63 (\text{A} \cdot \text{C}) + 39 (\text{C} \cdot \text{F}) = 102 \times \sqrt{5} \times \frac{\sqrt{5} - 1}{2} + 102 \times \sqrt{5} \times \frac{\sqrt{5} - 1}{2} + 14 (\text{the 1st layer} \text{B} \cdot \text{C} + \text{A} \cdot \text{D}) + 14 (\text{the 2nd layer} \text{B} \cdot \text{C} + \text{A} \cdot \text{D}) + 14 (\text{the 3rd layer} \text{B} \cdot \text{C} + \text{A} \cdot \text{D}) + 11 (\text{the 4th layer} \text{B} \cdot \text{C} + \text{A} \cdot \text{D}) + 11 (\text{the 5th layer} \text{B} \cdot \text{C} + \text{A} \cdot \text{D})
\]

5.2.2 Digital Analyses of the Elevation: The Complicated Combination of Golden-Rectangle Control Designs

Besides the overall dimension of the elevation, Baroque-style architectures also features a complicated elevation combination. Through digital analyses of the details of each elevation, we can obtain more measurement patterns, especially the complicated combination of golden-rectangle control designs (Fig. 4, Table 1).

Table 1. The Golden-Rectangles and Their Formation

<table>
<thead>
<tr>
<th>The Golden-Rectangles and their Formation</th>
<th>Height</th>
<th>Width</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main Rectangle 1</td>
<td>A2-A6-F6-F2</td>
<td>102</td>
</tr>
<tr>
<td>Sub-Rectangle 1</td>
<td>C2-C6-F6-F2</td>
<td>39</td>
</tr>
<tr>
<td>Square 1</td>
<td>A2-A6-C6-C2</td>
<td>63</td>
</tr>
<tr>
<td>Main Rectangle 2</td>
<td>A1-A3-K3-K1</td>
<td>54</td>
</tr>
<tr>
<td>Sub-Rectangle 2</td>
<td>B1-B3-K3-K1</td>
<td>20.5</td>
</tr>
<tr>
<td>Square 2</td>
<td>A1-A3-B3-B1</td>
<td>33.5</td>
</tr>
<tr>
<td>Main Rectangle 2</td>
<td>A3-A5-K5-K3</td>
<td>54</td>
</tr>
<tr>
<td>Sub-Rectangle 2</td>
<td>B3-B5-K5-K3</td>
<td>20.5</td>
</tr>
<tr>
<td>Square 2</td>
<td>A3-A5-B5-B3</td>
<td>33.5</td>
</tr>
<tr>
<td>Main Rectangle 3</td>
<td>A3-A8-C8-C3</td>
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</tr>
<tr>
<td>Sub-Rectangle 3</td>
<td>J3-J8-C8-C3</td>
<td>24</td>
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<tr>
<td>Sub-Rectangle 3.1</td>
<td>J3-J4-C4-C4</td>
<td>24</td>
</tr>
<tr>
<td>Square 3.1</td>
<td>J4-J8-C4-C4</td>
<td>24</td>
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<tr>
<td>Square 3</td>
<td>A3-AK-JK-J3</td>
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</tr>
<tr>
<td>Main Rectangle 4</td>
<td>K4-K8-E8-E4</td>
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<tr>
<td>Sub-Rectangle 4</td>
<td>D4-D8-E8-E4</td>
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</tr>
<tr>
<td>Square 4</td>
<td>K4-K8-D8-D4</td>
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<td>Main Rectangle 5</td>
<td>J2-J3-C3-C2</td>
<td>24</td>
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<tr>
<td>Sub-Rectangle 5</td>
<td>K2-K3-C3-C2</td>
<td>9</td>
</tr>
<tr>
<td>Square 5</td>
<td>J2-J3-K3-K2</td>
<td>15</td>
</tr>
</tbody>
</table>

(unit: palmos)

1) Main Rectangle 1 = Sub-Rectangle 1 + Square 1 + Main Rectangle 1: in the width direction Line segment A2-A6 is 63 palmos long; in the height direction Line segment A2-F2 is 102 palmos long. The ratio of width to height is in accordance with the golden ratio \(\frac{63}{102} = \frac{\sqrt{5} - 1}{2}\), that is the ratio of the distance between the mid-line of two side-doors in the first layer to the overall height of the elevation is in accordance with the golden ratio, forming the golden main rectangle 1: A2-A6-F6-F2.

Sub-Rectangle 1: In Main Rectangle 1, Line segment C2-F2 in the height direction is 39 palmos long; Line segment C2-C6 in the width direction is 63 palmos long. The ratio of the height to the width is in accordance with the golden ratio \(\frac{39}{63} = \frac{\sqrt{5} - 1}{2}\), that is the ratio of the height gap between the third and fifth floor to the distance between the mid-line of two side-doors is in accordance with the golden ratio, forming the golden sub-rectangle 1: C2-C6-F6-F2.

Square 1: Line segment A2-A6 is 63 palmos long; Line segment A2-C2 is 63 palmos long; both are in accordance with the ratio of square, 1 to 1, that is the distance between the mid-line of two side-doors in the first layer and the height of the first and second layer form the square 1: A2-A6-C6-C2.

2) Main Rectangle 2 = Sub-Rectangle 2 + Square 2

Main Rectangle 2: in the width direction, Line segment A1-A3 is 33.5 palmos long; in the height direction Line segment A1-K1 is 54 palmos long. The ratio of width to height is in accordance with the golden ratio \(\frac{33.5}{54} = \frac{\sqrt{5} - 1}{2}\), that is the ratio of one third of the width of the elevation to the height of the first layer in accordance with the golden ratio, forming the golden main rectangle 2: A1-A3-K3-K1.

Sub-Rectangle 2: In Main Rectangle 2, Line segment B1-K1 in the height direction is 20.5 palmos long; Line segment B1-B3 in the width direction is 33.5 palmos long. The ratio of the height to the width is in accordance with the golden ratio \(\frac{20.5}{33.5} = \frac{\sqrt{5} - 1}{2}\), that is the ratio of the height of the second floor (without the overlapping part) to one third of the width of the elevation is in accordance with the golden ratio, forming the golden sub-rectangle 2: B1-B3-K3-K1.

Square 2: Line segment A1-A3 in the width direction is 33.5 palmos long; Line segment A1-B1 in the height direction is 33.5 palmos long; both are in accordance with the ratio of square, 1 to 1, that is one third of the width of the elevation and the height of the first layer form the square 2: A1-A3-B3-B1.

Besides, A3-A5-K5-K3, A5-A7-K7-K5 are all similar in the width, so could be both classified as Type Main Rectangle 2.

3) Main Rectangle 3 = Sub-Rectangle 3 + Square 3; Sub-Rectangle 3 = Sub-Rectangle 3.1 + Square 3.1

Main Rectangle 3: in the width direction Line segment A3-A8 is 40 palmos long; in the height direction Line segment A3-C3 is 63 palmos long. The ratio of width to height is in accordance with the golden ratio \(\frac{40}{63} = \frac{\sqrt{5} - 1}{2}\), forming the golden main rectangle 3: A3-A8-C8-C3.

Sub-Rectangle 3: In Main Rectangle 3, Line segment J3-C3 in the height direction is 24 palmos long; Line segment J3-J8 in the width direction is 40 palmos long. The ratio of the height to the width is in accordance with the golden ratio \(\frac{24}{40} = \frac{\sqrt{5} - 1}{2}\), forming the golden sub-rectangle: J3-J8-C8-C3.

Square 3: Line segment A3-A8 in the width direction is 40 palmos long; Line segment A3-J3 in the height direction is 39 palmos long; both are in near accordance with the ratio of square, 1 to 1, forming the square: A3-A8-J8-J3.
Sub-Rectangle 3.1: In Sub-Rectangle 3, Line segment J3-C3 in the height direction is 24 palmos long; Line segment J3-J4 in the width direction is 16.5 palmos long. The ratio of the width to the height is in accordance with the golden ratio \( \frac{15}{24} = \frac{\sqrt{5} - 1}{2} \), forming the golden sub-rectangle: J3-J4-C4-C3.

Square 3.1: Line segment J4-J8 in the width direction is 23.5 palmos long; Line segment J4-C4 in the height direction is 24 palmos long; both are in near accordance with the ratio of square, 1 to 1, forming the square: J4-J8-C8-C4.

4) Main Rectangle 4=Sub-Rectangle 4+Square 4
Main Rectangle 4: in the width direction, Line segment K4-K8 is 23.5 palmos long; in the height direction, Line segment K4-E4 is 37 palmos long. The ratio of width to height is in accordance with the golden ratio \( \frac{23.5}{37} = \frac{\sqrt{5} - 1}{2} \), forming the golden main rectangle: K4-K8-E8-E4.

Sub-Rectangle 4: In Main Rectangle 4, Line segment D4-E4 in the height direction is 14 palmos long; Line segment D4-D8 in the width direction is 23.5 palmos long. The ratio of the height to the width is in accordance with the golden ratio \( \frac{14}{23.5} = \frac{\sqrt{5} - 1}{2} \), forming the golden sub-rectangle: D4-D8-E8-E4.

Square 4: Line segment K4-K8 in the width direction is 23.5 palmos long; Line segment K4-D4 in the height direction is 23.5 palmos long; both are in near accordance with the ratio of square, 1 to 1, forming the square: K4-K8-D8-D4.

5) Main Rectangle 5=Sub-Rectangle 5+Square 5
Main Rectangle 5: in the width direction, Line segment J2-J3 is 15 palmos long; in the height direction, Line segment J2-C2 is 24 palmos long. The ratio of width to height is in accordance with the golden ratio \( \frac{15}{24} = \frac{\sqrt{5} - 1}{2} \), forming the golden main rectangle: J2-J3-C3-C2.

Sub-Rectangle 5: In Main Rectangle 5, Line segment K2-K3 in the width direction is 15 palmos long; Line segment K2-C2 in the height direction is 9 palmos long. The ratio of the height to the width is in accordance with the golden ratio \( \frac{9}{15} = \frac{\sqrt{5} - 1}{2} \), forming the golden sub-rectangle: K2-K3-C3-C2.

Square 5: Line segment J2-J3 in the width direction is 15 palmos long; Line segment J2-K2 in the height direction is 15 palmos long; both is in near accordance with the ratio of square, 1 to 1, forming the square: J2-J3-K3-K2.

We can find the counterparts of the partitioned golden rectangles mentioned above through the symmetry of the middle axis. Accordingly, we can see that the golden rectangle is a significant underlying basis for the design drawing of each of the control points and lines, and for the adjustment of measurement rounding.

6. Conclusions
Built at the beginning of the opening of the Macao port in the Age of Navigation, St. Paul’s Church was different from the Renaissance-style Guia Church which was constructed during the same period. To this day, St. Paul’s Church is not only the symbol of the city of Macao, but also a masterpiece of the early-stage Baroque-style architectures in both East Asia and the world. Its designer, Father Carlo Spinola applied palmos as the basic measurement in the designing of the facade of the Front Wall. The elevation could be stratified into five layers, while the remaining design drawing of the facade in historical documents could only show us the line of deduction of each control point in the geometric drawing. Through further digital analysis of the drawing of the facade in historical documents, we have obtained the underlying Mathematics-Aesthetics relation of "Digitally-derived Geometric Design" that guided the design drawing of the controlling points and lines of the five layers of the elevation and number-rounding adjustment of the constructing dimensions: the application of the Law of Equal-partition in the horizontal direction, the Law of Successive-subtraction in Stratification in the vertical direction, and the complicated combination of golden-rectangle control designs. The latter two applications are key geometrical features of Baroque Architecture.

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The vertical interlocked golden rectangles in the vertical direction are a significant geometric characteristic that is different from the upward successive-subtraction in stratification of the Baroque style.

Counterparts of the golden rectangles in the graph could be obtained through the symmetry of the middle axis.

The vertically interlocked golden rectangles in the vertical direction (bilaterally symmetric golden rectangles in the horizontal direction, and vertically interlocked golden rectangles in the vertical direction). Thus classified into the type of Main Rectangle 2. Counterparts of the same type; only one of the same type is listed in the graph, such as the golden rectangle A1-A3-K3-K1, A3-A5-K5-K3, A5-A7-K7-K5. These rectangles have a similar or the same width and are thus classified into the type of Main Rectangle 2. Counterparts of the golden rectangles in the graph could be obtained through the symmetry of the middle axis.

Notes

3. File 2901/90 from the Macao Cartography and Cadastre Bureau (Governo da Região Administrativa Especial de Macao, Direcção dos Serviços de Cartografia e Cadastro, Archive 2901/90).
11. Guia Church (Igreja de nossa Senhora da Guia; its construction completed before 1622) was basically built during the same period as St. Paul's Church, and it is the only remaining Renaissance-style church in Macao. The front facade of Guia Church is controlled by 4 pairs of golden rectangles (bilaterally symmetric golden rectangles in the horizontal direction, and vertically interlocked golden rectangles in the vertical direction). These golden rectangles in the vertical direction are a significant geometric characteristic that is different from the upward successive-subtraction in stratification of the Baroque style.

References