Comparison of Deterministic Calculation and Fuzzy Arithmetic for Two Prediction Model Equations of Corrosion Initiation

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Abstract

The existing deterministic solution for prediction model of corrosion initiation cannot reflect input variable uncertainties very well. Thus there is a growing tendency for a stochastic model based on the probabilistic method to be developed and applied. This paper presents an approach to the fuzzy arithmetic based modeling of the chloride-induced corrosion of reinforcement in concrete structures that takes into account the uncertainties in the physical models of chloride penetration into concrete and corrosion of steel reinforcement, as well as the uncertainties in the governing parameters, including concrete diffusivity, concrete cover depth, surface chloride concentration and critical chloride level for corrosion initiation. There are a lot of models for predicting the onset time of reinforcement corrosion of structures exposed to corrosion environment. In this work, the RILEM model formula and Crank's solution of Fick's second law of diffusion are used. The parameters of the models are regarded as fuzzy numbers with proper membership function adapted to statistical data of the governing parameters, while the fuzziness of the corrosion time is determined by the fuzzy arithmetic of the interval arithmetic and extension principle. An analysis is implemented by comparing the deterministic calculation with fuzzy arithmetic for the above two prediction models.

Keywords: corrosion; service life; prediction; fuzzy number; stochastic model

Introduction

In the case of attack on the reinforcement embedded in concrete, contrary to the case of concrete degradation, the corrosion of steel reinforcement in concrete structures leads positively to concrete fracture, loss of bond between steel and concrete, and reduction in strength and ductility. As a result, the safety, serviceability and durability of structures are reduced, while their life cycle costs are increased. Normally, concrete protects steel reinforcement from corrosion by forming a passive film around the steel due to the high alkalinity of the concrete pore solution. When chloride ions from deicing salts or seawater penetrate into the concrete and reach the steel surface, they disrupt the passive film and initiate corrosion. The corrosion of steel reinforcement will start immediately after the chloride content of concrete near the embedded steel reaches a critical level, which defines the resistance of steel to corrosion. Consequently, the onset of corrosion is governed by the surface chloride concentration, concrete diffusivity, concrete cover depth of the steel, corrosion critical level, as well as moisture level in terms of the pore solution, and the availability of oxygen.^{1), 2)}

In this regard, prediction of the onset of corrosion is very important in order to reduce life cycle costs and enlarge the service life of RC structures as stated. Thus, a reliable prediction model of chloride penetration into reinforced concrete structures is critical for predicting the time to onset of corrosion of steel reinforcement.⁵⁾

Mathematical models of chloride ingress currently being developed are primarily based on chloride diffusion, which can be used as starting points in the development of service life prediction tools and performance-based specifications. Even if chloride ingress into concrete is complex, models are constructed around Fick's second law of diffusion and the error function solution by Crank. Fick's second law of diffusion concerns the rate of change of concentration with respect to time as follows:

$$\frac{\delta C}{\delta t} = D \frac{\partial^2 C}{\partial x^2} \quad (1)$$

with boundary condition of $C_x = 0$ at $t = 0$ and $0 < x < \infty$, $C_x = C_s$ at $t = 0$ and $0 < x < \infty$.

Crank's solution of Fick's second law of diffusion can be stated as follows, using an apparent diffusion coefficient:

$$\frac{C_x}{C_s} = 1 - \operatorname{erf} \left[\frac{x}{2\sqrt{D_{ca}t}} \right] \quad (2)$$

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where C_x = chloride concentration at depth x at time t
 C_s = surface chloride concentration
 D_{ca} = apparent diffusion coefficient
 t = time of exposure
 erf = error function

It can be noted from Eq. (1.2) that the determination of time to corrosion initiation requires the values of D_{ca} , C_s , C_x and x . These variables of the deterministic prediction model assume the uncertainty associated with governing parameters such as exposure condition, type and quality of concrete, and quality of construction. In recent years, there has been much study about processing the uncertainty of the variables by using Monte Carlo (MC) simulation, by which the parameters of the models are modeled as random variables and the distribution of the corrosion time and probability of corrosion are determined. MC simulation proves to be very applicable but it is very difficult to be calculated manually because it generates the relevant random values of a number of variables.¹⁾

In this paper, the uncertainties of the deterministic model associated with various conditions are treated by fuzzy arithmetic which is a successful tool for solving engineering problems with uncertain parameters. The shape of the variable derived from measured data is modeled as the standard form of triangular fuzzy numbers (TFN) which are just a rough approximation of the existing uncertainty.³⁾ The prediction capability of the fuzzy variable treated-prediction model is illustrated in the case study of a reinforced concrete building structure in a coastal environment.

Uncertainty in prediction model and its process

To achieve reliable results for the numerical solution of the deterministic prediction problem, exact values for the parameters for the problem equations should be available. In practice, however, exact values cannot be provided. The model parameters exhibit variability, e.g. due to both irregularities in manufacturing when considering the physical properties of a material and uncertainties in measuring when considering the environmental condition.

According to Z. Lounis *et al.*, these uncertainties can be grouped into aleatoric uncertainty and epistemic uncertainty. The aleatoric uncertainty arises from the physical or inherent uncertainty identified with the random nature of the basic parameters that govern the chloride penetration and corrosion mechanisms. This uncertainty is associated with variability of the concrete cover depth, uncertainty of the chloride concentration at the surface, and uncertainty of the chloride diffusion coefficient.

The epistemic uncertainty arises from the uncertainty in the models concerning chloride transport and corrosion initiation. The model uncertainty results from the use of a simplified physical model of the actual phenomenon, such as assumption of chloride transport mechanism governed by diffusion, use of simplified models of the diffusion coefficient and driving chloride concentration and use of simplified chloride critical level to define the corrosion resistance of steel reinforcement. The epistemic uncertainty also arises from statistical uncertainty due to estimating

statistical representative value of an average from a limited sample size. Thus, it is clear that a deterministic prediction model can be quite inappropriate in predicting the actual structural response to a particular environmental condition.

To solve this limitation, the application of fuzzy set theory proves to be a practical approach. More specifically, the uncertainties in the model parameters can be taken into account by representing the effects of scatter by fuzzy numbers with their shape derived from statistical data. The elementary mathematical operations like addition, multiplication, etc. must then be carried out using generalized versions of the operations that ensure the handling of fuzzy numbers. By this technique, one can demonstrate how initially assumed uncertainties are processed through the calculation procedure leading finally to fuzzy results that reflect the reliability of the problem solution. Additionally, the fuzzy results allow the computation of a crisp value as the most likely result for the problem which in general differs from the result achieved by an initially non-fuzzy approach using only crisp parameters.

Implementation of fuzzy numbers

Definition of fuzzy number and fuzzy arithmetic

To qualify as a fuzzy number, a fuzzy set A on real numbers must be normal and convex. The fuzzy set must be normal, since the concept of a set of "real numbers close to a given real number R " is fully satisfied by R itself; hence the membership grade of R in any fuzzy set that attempts to capture a fuzzy number must be 1. The bounded support of a fuzzy number and all its α -cuts for $\alpha \neq 0$ must be closed intervals to allow definition of arithmetic operations on fuzzy numbers in terms of standard arithmetic operations on closed intervals. Since α -cuts of any fuzzy number are required to be closed intervals for all $\alpha \in [0, 1]$, every fuzzy number is a convex fuzzy set.

A fuzzy number is represented as an ordered set of confidence intervals, each of them providing the related numerical value at a given presumption level $\alpha \in [0, 1]$. These confidence intervals should comply with the relation $\alpha_1 > \alpha_2 \Rightarrow {}^{\alpha_1}A \subset {}^{\alpha_2}A$.

where $\alpha_1 > \alpha_2 \in [0, 1]$ and ${}^{\alpha_1}A$, ${}^{\alpha_2}A$ are the confidence intervals at presumption levels α_1 and α_2 respectively.

The four basic arithmetic operations on fuzzy numbers (addition, subtraction, multiplication, and division) can be described as sequences of operations among confidence intervals. In particular, let A and B be fuzzy numbers and let $\otimes$ be a generic arithmetic operator.

The fuzzy number $A \otimes B$ is obtained by computing the operation ${}^{\alpha}A \otimes {}^{\alpha}B$ for each $\alpha \in [0, 1]$, where ${}^{\alpha}A$ and ${}^{\alpha}B$ are the confidence interval of A and B at presumption level α . It was proved that this approach complies with the extension principle of Zadeh as

$$C(z) = \sup_{z=x \otimes y} \min[A(x), B(y)] \quad (3)$$

Fig.1. Normal Distribution of any Parameter and its Adaptation to the Membership Function of Triangular Fuzzy Number

Fig.2. Lognormal Distribution of any Parameter and its Adaptation to Membership Function of Triangular Fuzzy Number

Triangular Fuzzy Number (TFN)

To include uncertainties into the solution procedures of the deterministic prediction model, the fuzzy numbers that are used to represent the uncertain model parameters were implemented in a standard form of TFN due to the simplicity in both calculation and the fact that there were just three components.

Considering a definite uncertain parameter A, measured data for the parameter are assumed to be available from which a normalized distribution function $N_A(x)$ can be derived that expresses the frequency of occurrence of a certain measured value x for the parameter A within the interval Δx . In most cases, these data approximately show Gaussian

distribution approximately, i.e. normal distribution. The uncertainty in the parameter A can then be approximately modeled by a fuzzy number $\tilde{A}$ with the membership function $A(x)$ of equation (4), which has the support of $2 \times 2\sigma_a$ set up for around 95% confidence interval of a normalized distribution function $N_A(x)$.

$$A(x) = \begin{cases} \frac{x-a}{m-a}, & \text{if } a < x \leq m \\ \frac{b-x}{b-m}, & \text{if } m < x \leq b \\ 0, & \text{otherwise} \end{cases} \quad (4)$$

where m is the mean value of the normal distribution in Fig.1. and a and b is lower bound and upper bound that obtained from lower and upper bound of 5% of the normal distribution in Fig.1.

Considering an uncertain parameter B showing lognormal distribution, similarly to an uncertain parameter A of normal distribution, a triangular form of membership function is identified as shown in Fig.2.

Prediction model by means of fuzzy arithmetic (Example)

Calculation of T_{corr} based on the RILEM model

Fig.3. illustrates the calculation procedure of fuzzy arithmetic for prediction model of the RILEM model and Crank's solution. In implementing fuzzy arithmetic for the RILEM model and Crank's solution, the method treating input variables with fuzzy numbers is the same. Just such a difference stems from selecting the deterministic formula.

As stated earlier, the prediction model by Fick's 2nd law like Eq. (1) has been widely used due to its simplicity. To calculate the deterministic prediction model by using fuzzy arithmetic in this study, the approximation model of Crank's solution of Fick's second law presented by RILEM like Eq.(5) is used.⁴⁾

$$C_{cr} = (C_0 - C_{init}) \cdot \left\{ 1 - \frac{x}{2\sqrt{3D \cdot T_{corr}}} \right\}^2 + C_{init} \quad (5)$$

Equation (5) is rewritten as follows:

$$T_{corr} = \frac{x^2}{12D} \cdot \left(\frac{1}{1 - \sqrt{\frac{C_{cr}}{C_0}}} \right)^2 \quad (6)$$

Thus fuzzy arithmetic is applied to calculate the time T_{corr} to initiation of reinforcement corrosion separated into two parts as represented in Fig.4. Normally, in stochastic model by Monte Carlo Simulation (MC simulation), all parameters of Eq. (6) are taken into consideration by modeling them as random variables which have probabilistic density functions (PDF) that are obtained from field measurements or from the survey analysis. However in this study, they are treated as fuzzy variables with proper core and support by

Fig.3. Calculation Procedure of Fuzzy Arithmetic for Prediction Model of the RILEM Model and Crank' Solution

conforming to the procedure illustrated in Figs. 1 and 2.

The mean value and standard deviations of all parameters which are used to apply fuzzy arithmetic to solving corrosion prediction problems with uncertain parameters are listed in Table 1.

Overall procedure of calculating the time T_{corr} to initiation of reinforcement corrosion by Eq. (6) based on fuzzy arithmetic is represented in Fig.4. Crisp value of time to corrosion initiation is acquired by multiplying the defuzzified value of membership function of $\tilde{x}^2/12\tilde{D}$ by the crisp value of $\left[1 / \left(1 - \sqrt{\tilde{C}_{cr}/\tilde{C}_o} \right) \right]^2$ calculated by inserting the defuzzified value of $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$, where each membership function is defuzzified by fuzzy centroid method, i.e. Center of Area (CoA), by way of the following Eq. (7).

$$\bar{y} = \frac{\int A(y) \cdot y \, dy}{\int A(y) \, dy} \quad (7)$$

In this study, arithmetic operation of addition, subtraction, division, and multiplication on fuzzy numbers is carried out by using fuzzy interval arithmetic. By mapping fuzzy numbers via functions, the extension principle is applied to those transformations.

Fig.5. represents the shape of membership function of fuzzy number $\tilde{C}_{cr}$ and $\tilde{C}_o$ as well as the fuzzy arithmetic procedure of $\sqrt{\tilde{C}_{cr}/\tilde{C}_o}$ based on fuzzy interval arithmetic and extension principle.

Table 1. Statistic Properties of all Parameters in Corrosion Prediction Model

Variable name	Statistical properties		
Cover thickness:	Mean value	4.51	Normal
	Standard variation	1.59	
Surface concentration	Mean value	3.09	Lognormal
	Standard variation	0.44	
Critical concentration	Mean value	1.25	Normal
	Standard variation	0.23	
Diffusion coefficient	Mean value	1.26	Lognormal
	Standard variation	0.37

Fig.4. Calculation Procedure of Time to Corrosion Initiation

Fig.5. Membership Function of Fuzzy Number Critical Chloride Concentration (Ccr) Divided by Fuzzy Number Surface Chloride Concentration (Co) and its Square Root

Fig.6. represents the membership function of fuzzy number $\widetilde{x}$, $\widetilde{D}$ and also those transformations via each function. They show that mathematical operation of division increases the variability that each parameter possesses.

As shown in Fig.5., the defuzzification of $\sqrt{\widetilde{C}_{cr}/\widetilde{C}_o}$ with the Center of Area (CoA) leads to the crisp value

$$\sqrt{\widetilde{C}_{cr}/\widetilde{C}_o} = 0.637$$

which can be considered as the representative value for $\sqrt{\widetilde{C}_{cr}/\widetilde{C}_o}$.

By inserting the above expected value of $\sqrt{\widetilde{C}_{cr}/\widetilde{C}_o}$ into $\left[1/\left(1-\sqrt{\widetilde{C}_{cr}/\widetilde{C}_o}\right)\right]^2$, the crisp value is as follow:

$$\left[1/\left(1-\sqrt{\widetilde{C}_{cr}/\widetilde{C}_o}\right)\right]^2 = 7.589$$

Fig.7. represents the membership function of $7.589 \times (\widetilde{x}^2/12\widetilde{D})$ and its cumulative distribution function,

Fig.6. Membership Function of Cover Thickness (x) and Diffusion Coefficient (C), and Transformation to $0.632 (\text{Cover Thickness})^2$ Based on the Extension Principle

and defuzzified $7.589 \times \left(\frac{\tilde{x}^2}{12\tilde{D}} \right)$ by CoA is as follow:

$$7.589 \times \frac{\tilde{x}^2}{12\tilde{D}} \longrightarrow \frac{0.632 \cdot \tilde{x}^2}{\tilde{D}} = 9.912$$

Finally, the defuzzification of the time T_{corr} to initiation of reinforcement corrosion determined is as follows:

$$T_{corr} = \left(\frac{\tilde{x}^2}{12\tilde{D}} \right) \times \left(\frac{1}{1 - \sqrt{\frac{\tilde{C}_{cr}}{\tilde{C}_o}}} \right)^2 = 9.912$$

Therefore, from the result of fuzzy arithmetic for the RILEM formula, it can be understood that the time to corrosion initiation ranges from 2.9 years to 16 years and the representative value of that distribution is about 9.9 years.

Fig.7. Membership Function of $\tilde{C}_{cr}/\tilde{C}_o$

This means that reinforcement corrosion will be initiated after 2.9 to 16 years under the given boundary condition and cover thickness in the case that they are calculated by using RILEM model.

The cumulative curve of the time to corrosion initiation is generated in Fig.9. This cumulative curve is generated by representing the ratio of cumulative area of function to total area in given limitation of 2.9 to 16.

The crisp value of 9.9 years is interpreted to have the membership degree of about 0.9 in function shown in Fig.8. In other words, in view of the cumulative curve of Fig.9., it means that the possibility that reinforcement corrosion is initiated after 9.9 years can be estimated about to be 50%. From this, it is expected that corrosion will undoubtedly occur after a period of 16 years.

Explanation of calculation for T_{corr} based on Crank's solution of Fick's second law

To calculate the prediction model by using fuzzy arithmetic, here the second model is Crank's solution of Fick's second law as follows;

$$\begin{aligned} C(x,t) &= C_o \left\{ 1 - \operatorname{erf} \left[\frac{x}{2\sqrt{D \cdot t}} \right] \right\} \\ &= C_o \left\{ \operatorname{erfc} \left[\frac{x}{2\sqrt{D \cdot t}} \right] \right\} \end{aligned} \quad (8)$$

where $C(x,t)$: chloride ion concentration at a time t and a depth x in kg/m^3 , C_0 : boundary chloride concentration in kg/m^3 , x : concrete cover depth in cm and erf : statistical error function is as follows;

$$\text{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) dt$$

$$\text{erfc} = 1 - \text{erf}$$

The corrosion of steel reinforcement is activated as the chloride content of concrete near the embedded steel reaches the chloride threshold, i.e., critical corrosion level.

Thus to acquire the time to corrosion initiation Eq. (8) is rewritten as follows:

$$T_{\text{corr}} = \frac{x^2}{4D \cdot \gamma^2} \quad \text{with} \quad \gamma = \text{erfc}^{-1}\left(\frac{C_{\text{cr}}}{C_0}\right) \quad (9)$$

The prediction model used in this chapter is slightly different from the RILEM formula in that the RILEM formula is an empirical solution while Crank's solution of Eq.(8) is based on diffusion.

The calculation procedure implementing fuzzy arithmetic for Crank's solution of Fick's second law of diffusion shown in Eq.(9) is illustrated in Fig.3.

The input variable is first prepared in order to implement fuzzy arithmetic for Crank's solution, which obtained from the result of best curve fitting to the collected data from the literature survey and field measurements. Second, the input variable is treated with triangular fuzzy number with proper core and support as a method represented in Figs. 1 and 2.

The ready input data of the fuzzy number is substituted for each variable in Eq.(9) and fuzzy arithmetic is implemented using the fuzzy interval and extension principle.

Fuzzy arithmetic is implemented as parted with two. First, find the mapping of the inverse error function to the defuzzified value of $\tilde{C}_{\text{cr}}/\tilde{C}_0$ and second, implement the fuzzy arithmetic to $\tilde{x}^2/1.348\tilde{D}$ in Fig.10., in order to obtain the final output, where fuzzy arithmetic is implemented through the fuzzy interval and extension principle and each membership function is defuzzified by the fuzzy centroid method, i.e. Center of Area (CoA), by way of the following Eq. (8).

Fig.11. illustrates the fuzzy number $\tilde{C}_{\text{cr}}$ divided by the fuzzy number $\tilde{C}_0$ and the defuzzification of $\tilde{C}_{\text{cr}}/\tilde{C}_0$ leads to the crisp value

$$\tilde{C}_{\text{cr}}/\tilde{C}_0 = 0.419$$

and the inverse of complementary error function to the defuzzification value of $\tilde{C}_{\text{cr}}/\tilde{C}_0$ returns 0.5802.

Finally the membership function of is plotted in Fig.11. and its defuzzification value is about 11.6 years.

From the result of fuzzy arithmetic for Crank's solution of Fick's law of diffusion, it can be understood that the time to corrosion initiation ranges from 3 years to 19 years and the representative value of that distribution is about 11.6 years.

Fig.8. The membership Function of $0.632 \cdot \tilde{x}^2 / \tilde{D}$ which is the Final Output

Fig.9. The Cumulative Curve of $0.632 \cdot \tilde{x}^2 / \tilde{D}$ which is the Final Output

Fig.10. Membership Function of $\tilde{x}^2 / 1.348 \tilde{D}$

This means that reinforcement corrosion will be initiated after 3 to 19 years under the given boundary condition and cover thickness in the case that it is calculated by using Crank's solution. And also the cumulative curve of the time to corrosion initiation is generated and shown in Fig.12. From the cumulative distribution function, membership degree of 11.6 to the membership function shown in Fig.12. is about 0.5. In other words, in view of the cumulative curve it means that the possibility that reinforcement corrosion is initiated after 11.6 years can be estimated to be about 50%. From this, it is understood that corrosion will undoubtedly occur after a period of 19 years.

Fig.11. Cumulative Curve of $\tilde{x}^2/1.348\tilde{D}$

Fig.12. Comparison of RILEM Model and Crank's Solution (Membership Function and Deterministic Solution)

Fig.13. Comparison of the RILEM Model and Crank's Solution (Cumulative Curve and Deterministic Solution)

Comparison of deterministic solution and fuzzy arithmetic solution

Fig.12. plots each membership function of the time to corrosion initiation (T_{corr}) calculated by fuzzy arithmetic for the RILEM model and Crank's solution and the result of its deterministic solution. While Fig.13. illustrates the cumulative curve of the fuzzified output of T_{corr} obtained from fuzzy arithmetic for the RILEM model and Crank's solution, respectively.

The defuzzification value of Crank's solution is higher than that of the RILEM model and also the defuzzification value of each model formula is well coincident with the solution value of deterministic calculation using a mean as the representative value.

The result of fuzzy arithmetic for the RILEM model shows the corrosion initiation time prior to that of Crank's solution, which can interpret the RILEM model as more conservative expression than Crank's solution.

Conclusion

This paper has presented the application of a fuzzy arithmetic approach for the modeling and prediction of reinforcement corrosion in building structures that are subjected to coastal environments. The approach takes into account the uncertainties in the physical modeling, and variability of the material and structural parameters affecting the corrosion process, in addition to the statistical and decision making uncertainties.

Of the prediction model for corrosion initiation used in this work, the RILEM model is more conservative than Crank's solution and both models show good coincidence with deterministic solution.

The fuzzy arithmetic-based prediction model can output the stochastic result and make it possible for users to make more rational decisions concerning the time to corrosion initiation and predicting service life.

Thus, the proposed fuzzy arithmetic-based prediction model will overcome the shortcomings of existing deterministic prediction models. The implementation of this tool will provide more extensive predictions and enable decision-makers to select cost-effective repair strategies that will extend the life of structures and reduce the life cycle.

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