Interaction Curve for High Performance Ferrocement in Biaxial State of Tension

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Abstract
Ferrocement's attribute as a thin reinforced concrete and laminated cement based composite enables numerous applications where strong and tough protective shell is needed. It is used in the construction of cylindrical and spherical shells as liquid retaining structures. The stress condition at a point in such shell structures is biaxial state of tension-tension. A study on the behaviour of ferrocement in biaxial state is important for establishing a rational design. The authors carried out a full-fledged investigation on ferrocement under different biaxial stress conditions. The present investigation deals with the study of ferrocement in biaxial tension-tension based on hollow cylindrical specimens. An interaction curve is proposed to estimate the strength of ferrocement in biaxial tension-tension. The relationship between octahedral normal and shear stress is also presented.

Keywords: high performance ferrocement; biaxial; tension-tension; interaction curve; analytical model

Introduction
Ferrocement is a versatile composite construction material, consisting of small diameter wire meshes dispersed spatially in the mortar matrix (ACI committee 549, 1993, IFS Committee 10, 2001). It has an excellent property of mouldability into any structural form and shape. Being thin walled in nature, it is an ideally suitable material for construction of shell, folded plate structures etc. It has a great potential to be used for liquid retaining structures. An element isolated from such a type of structure, is a two dimensional element subjected to two in-plane normal stresses and one in-plane shear stress. As the applied loads are primarily carried through the development of membrane action, the behavior of ferrocement under in plane stresses needs to be established. The use of high performance mortar as the matrix for ferrocement improves strength and durability characteristics such as permeability and cracking of the composite (P. R.Kumar et al, 2002a, P.R.Kumar et al, 2002b, P. R.Kumar et al, 2004). The closer distribution and uniform dispersion of reinforcement transforms the brittle mortar into a distinctly different material from RCC (Naaman, 2000). The commonly adopted specimens for conducting tests in biaxial stress state are: cubes, solid cylinders, hollow cylinders and plates. In the present investigation, hollow cylindrical specimens were used to investigate the strength of high performance ferrocement in biaxial tension-tension (P. R. Kumar, 2003, P. R. Kumar 2004).

Experimental Program
Hollow cylinders can be tested easily in all biaxial stress combinations. The test specimens were 200mm in height, 75mm in outer radius and 18mm in thickness. Slag cement, sand passing through JIS sieve No. 1.2 (1.18mm), optimum dosages of silica fume and a chemical admixture were used in the preparation of the high performance mortars. The mix proportion was cement to sand 1:1 with a water-cementitious ratio of 0.34. The silica fume was used as a partial replacement of 10% by weight of cement (P. R Kumar et al 2001). The use of silica fume increased the surface area of the mortar matrix enabling reduction in the workability. Water-reducing chemical admixture (superplasticizer) compensated the water demand (P. R. Kumar et al 2001). A number of 50mm diameter mortar cubes were cast with the same mortar as used for ferrocement hollow cylindrical specimens. The 28-day strength of the mortar was 60MPa. The properties of the wire mesh used are given in Table 1.

Table 1. Properties of Mesh Wire Used

<table>
<thead>
<tr>
<th></th>
<th>Axial Direction of the Specimen</th>
<th>Hoop Direction of the Specimen</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter of wire (mm)</td>
<td>0.47</td>
<td>0.47</td>
</tr>
<tr>
<td>Yield strength (MPa)</td>
<td>512.5</td>
<td>510</td>
</tr>
<tr>
<td>Ultimate strength (MPa)</td>
<td>525</td>
<td>520</td>
</tr>
</tbody>
</table>

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The variable in the investigation was specific surface factor \( (S_F) \) and was achieved by varying the number of layers of wire mesh. It was identified as an effective parameter influencing the mechanical properties of ferrocement in tension (C.B.K.Rao 1994) and compression (P.R.Kumar and G.R.Kumar 2002). The specific surface factor is given by the expression

\[
S_F = s \cdot \frac{\sigma_y}{\sigma_p}
\]

(1)

where, \( s \) is the specific surface ratio. Specific surface ratio is the total surface area of contact of the reinforcement wires present in the direction of application of force in a given width and thickness of the specimen, to the volume of the mortar per unit length of the specimen, in the direction of loading with the same width and thickness. In the present case the total number of wires were 584, 730 and 876 respectively for four, five and six layers in the axial direction, while it was 583, 729 and 874 in the hoop direction. The specific surface ratio was hence calculated based on the number of wires in the perimeter and volume of the mortar and the values were found to be 1.066, 1.320 and 1.480 respectively for four, five and six layered specimens. If \( \sigma_y \) is the yield strength of the wires in the direction of force and \( \sigma_p \) is the plain mortar strength in compression, the specific surface factor can be calculated based on Eq-1.

A total of 72 specimens were cast and tested, 36 for constant force in the horizontal direction and 36 for constant force in the vertical direction.

Preparation of specimen

The standard 150mm diameter and 300mm cast iron cylindrical mould was fitted with a 100mm high teak wood cylinder at the bottom. One more cylinder made of galvanized iron of a diameter equal to the inner diameter of the hollow cylindrical ferocement specimen and 100mm more in height than the specimen, was placed in the central recess of the bottom cylinder to obtain the required uniform thick annular space over a height of 200mm. A typical mould used for casting the specimens is shown in Fig.1.

The required size of mesh was 195mm wide and of length slightly more than that required to obtain the number of layers, was cut from the standard mesh roll. The 195 mm long transverse wires formed the vertical axis of the specimen. The mesh was wound tightly to confirm with the cylindrical shape of the specimen. The fabricated mesh was placed in the annular space of the mould (Fig.2). Spacer rods were placed between the mesh layers and cover pieces were provided on the inner and outer surfaces. Mortar was filled in between the mesh layers, simultaneously vibrating the moulds. After about four hours of casting, the specimens were cured with damp burlap to prevent moisture loss. The specimens were stripped off the moulds 28 days after casting and then air cured before testing. Companion plain hollow cylindrical specimens were cast to obtain direct compression strength.

Test Set-up

The test setup for biaxial tension-tension consisted of an independent arrangement for applying tension in the hoop direction and vertical direction (Fig.3).
Specimens were subjected to circumferential tension by applying radial pressure by inserting a rubber tube in the annular space between the cast iron pipes and inflating the same by pumping oil through the inlet valve. The top and bottom steel plates restrain the expansion of the rubber tube, and thus the radial force was transferred to the inner surface of the ferrocement cylinder as a reaction. The tensile force in the vertical direction was applied using friction grips in the Universal Testing Machine of 100 Tonnes Capacity. Each system can be operated independently. Circular wedge friction grips were attached at the ends (Fig. 4a) and direct force was applied using a universal testing machine (Fig.4b).

Method of Testing
The general method of testing was to apply a constant force in one of the principal directions and increase the force in the orthogonal direction. Four discrete levels were taken in steps of 20% ranging from 20% to 80% of the theoretical axial and hoop tensile strength.

A predetermined force in one of the principal directions i.e. circumferential or axial was applied at a particular discrete level and maintained constantly. The force in the other principal direction was then increased gradually until failure occurred. In the present investigation ferrocement specimens were tested by
both alternatives; method (a) with constant force in the horizontal direction; and method (b) with constant force in the vertical direction. It is known that the uniaxial tensile strength in the direction of force is equal to the load carrying capacity of the reinforcing wires in that direction (Naaman, 2000).

\[
\sigma_t A_f = \sigma_{us} A_s \tag{2}
\]

where, \(\sigma_t\) is the ultimate tensile strength of ferrocement, \(A_f\) is the cross sectional area of ferrocement, \(\sigma_{us}\) is the ultimate tensile strength of the reinforcing wires in the direction of the force and \(A_s\) is the area of reinforcing wires in the direction of force.

The uniaxial strength of ferrocement in the principal directions i.e.; circumferential direction (\(\sigma_{th}\)) and axial direction (\(\sigma_{tv}\)) have been calculated from the above equation. It is known that the circumferential stress on the cross section of a thick cylinder subjected to internal pressure is non-uniform. However, the thin walled ferrocement specimens under investigation were treated as experiencing uniform stress such that the total force acting on the section was the same as that of non-uniform stress. Tensile stress in the circumferential direction satisfying the above assumption is given by the expression

\[
(\sigma_{th}) = \frac{P_i}{R_0 - R_i} \tag{3}
\]

where, \(P_i\) is the radial pressure and \(R_i\) is the internal radius and \(R_o\) is the outer radius of the cylinder. The uniaxial strength in the hoop/vertical direction is calculated using Eq. (2) and the radial pressure \(P_i\) to be applied is calculated using Eq. (3).

**Behaviour under Load**

*Method (a) with constant force in the horizontal direction*

The first crack was generally in the horizontal direction due to force in the vertical direction. Thereafter, the cracks formed at different sections and the number of cracks increased as the force increased. New cracks continued to form till the force in the vertical direction reached to about 90% of the peak value, while the earlier cracks widened and propagated in a direction inclined to the horizontal. The force in the vertical direction was nearly constant for a brief period on reaching a peak value. Attempts to increase the force further resulted in widening of the cracks. At this stage the reinforcing wires in the vertical direction snapped and the test was stopped after this peak value to avoid bursting of the tube. It was noticed that the force in the vertical direction at the first crack stage in biaxial condition was the same as uniaxial ultimate tensile strength in that direction. The number of cracks increased with an increase in the steel content. Cracks similar to those at the outer surface were observed at the inner surface also. It was noted that the behaviour of ferrocement specimens was similar in biaxial tension-tension to that in the alternate methods of testing.

*Method (b) with constant force in the vertical direction*

A predetermined force in the vertical direction was applied and maintained constantly while the force in the horizontal direction was increased later. The first cracks were observed in the vertical direction. The number of cracks increased with an increase in the fluid pressure. New cracks were formed until the fluid pressure reached 90% of the peak value, while the cracks formed earlier widened and propagated in the same direction (Fig.6).

**Interaction Diagram**

The uniaxial ultimate strength in the vertical direction (\(\sigma_{tv}\)) and the horizontal direction (\(\sigma_{th}\)) of a typical four-layered ferrocement specimen is given in columns four and five of Table 2. For the values noted at the peak stage during test under the biaxial state, the tensile strength in the horizontal direction (\(\sigma_{tv}\)) is observed at the inner surface also.
calculated using Eq. (3) and the strength in the vertical direction ($\sigma'_v$) is calculated conventionally. The values are reported in columns six and seven of the table respectively. The prefixes A, B, C and D indicate the discrete levels of 20%, 40%, 60% and 80% of the ultimate tension in the hoop direction, while E, F, G and H are the discrete levels for the same percentage of maximum tension in the axial direction. 41, 42 and 43 indicate the number of the four-layered specimens for that discrete level respectively. Similar results were obtained for five and six layered ferrocement specimens also.

An examination of the values obtained shows that the tensile strength in the direction orthogonal to the direction of application of predetermined constant force is always greater than the uniaxial tensile strength in that direction owing to the Poisson's effect. It increased initially and decreased later. This was found to be true in the case of five and six layered specimens also. The trend was the same irrespective of the specific surface factor and the method of testing viz., method (a) and (b). The trend is depicted in Fig.7.

To normalize the effect of specific surface factor, tensile strength in the biaxial states of each direction are divided by the uniaxial ultimate strength in their respective directions. The superposed normalized interaction diagrams $\sigma'_v / \sigma_v$ vs. $\sigma'_h / \sigma_h$ of four, five and six layers are shown in Fig.8.

<table>
<thead>
<tr>
<th>S.No</th>
<th>Specimen Desg.</th>
<th>Plain Mortar Strength(MPa)</th>
<th>Analytical Uniaxial Tensile Strength (MPa)</th>
<th>Experimental Biaxial Tensile Strength (MPa)</th>
<th>Non-Dimensionalised Strength values</th>
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<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3) (4) (5)</td>
<td>(6) (7) (8) (9)</td>
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<tr>
<td>1</td>
<td>TTA41</td>
<td>58.5</td>
<td>5.59</td>
<td>5.62</td>
<td>6.73  1.16  1.2  0.21</td>
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<td>2</td>
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<td>58.5</td>
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<td>5.62</td>
<td>7.1   1.14  1.27 0.2</td>
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<td>3</td>
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<td>5.59</td>
<td>5.62</td>
<td>6.24  1.17  1.12 0.21</td>
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<td>5.59</td>
<td>5.62</td>
<td>6.86  2.16  1.23 0.38</td>
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<td>5.62</td>
<td>7.1   2.16  1.27 0.38</td>
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<td>5.62</td>
<td>7.43  3.53  1.33 0.63</td>
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<td>5.62</td>
<td>7.6   3.34  1.36 0.59</td>
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<td>5.62</td>
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<td>56.5</td>
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<td>5.62</td>
<td>1.1   7.87  0.2  1.4</td>
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<td>14</td>
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<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>1.1   6.39  0.2  1.14</td>
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<tr>
<td>15</td>
<td>TTE43</td>
<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>2.17  6.96  0.39 1.24</td>
</tr>
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<td>16</td>
<td>TTF41</td>
<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>2.17  7.85  0.39 1.4</td>
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<td>17</td>
<td>TTF42</td>
<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>2.17  7.12  0.39 1.27</td>
</tr>
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<td>18</td>
<td>TTF43</td>
<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>3.32  7.35  0.59 1.31</td>
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<td>19</td>
<td>TTG41</td>
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<td>5.59</td>
<td>5.62</td>
<td>3.32  8.31  0.59 1.48</td>
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<td>20</td>
<td>TTG42</td>
<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>3.32  8.12  0.59 1.44</td>
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<tr>
<td>21</td>
<td>TTG43</td>
<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>4.34  6.9  0.78 1.23</td>
</tr>
<tr>
<td>22</td>
<td>TTH41</td>
<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>4.44  7.19  0.79 1.28</td>
</tr>
<tr>
<td>23</td>
<td>TTH42</td>
<td>56.5</td>
<td>5.59</td>
<td>5.62</td>
<td>4.47  7.57  0.8  1.35</td>
</tr>
</tbody>
</table>
It can be seen from Fig. 8 that after normalization there is no influence of specific surface factor or steel content on the interaction diagram. The trend of the interaction diagram is the same in either method of testing, meaning that the behaviour of ferrocement specimens in biaxial tension-tension is symmetrical about a $45^\circ$ line. Method (b) gave slightly higher values of $\sigma_0^*/\sigma_0$ than its symmetrical value of $\sigma_0^*/\sigma_0$ in method (a). This difference may be due to the effect of the wedge grips, which might have induced additional restraint on the edges of the specimen as evidenced by bulging in the central portion of the specimen. The assumptions of symmetry of behaviour about a $45^\circ$ line, provides a conservative and safe condition.

Analytical Model

The normalized interaction diagram is not influenced by the steel content vide Fig. 8., further, there seems to be a definite trend between the direct non-dimensionalised and hoop tensile strengths. A best fit between these points, seemed to follow the path of part of a parabola. An equation of the form $y = a + bx + cx^2$ was fitted for one half the symmetrical part of the interaction diagram and the constant values were obtained. The regression equation obtained is

$$\frac{\sigma_0}{\sigma_0^*} = 1.0 + 1.3678 \frac{\sigma_0}{\sigma_0^*} - 1.2749 \left(\frac{\sigma_0}{\sigma_0^*}\right)^2$$

The path of the above equation is plotted as shown in Fig. 9. The correlation coefficient was 0.98. The other half of the interaction diagram was also similar to the first symmetrical half.

The equation can hence be used to predict the behaviour of ferrocement in biaxial tension-tension. The 10% error bars for x and y directions were plotted and it was noted from the diagrams that all the experimental points were well within the 10% range.

Failure criteria for tension-tension

Nadai’s concept of octahedral shear stress, which is a function of octahedral normal stress as a failure criterion was investigated, as it seemed applicable to ferrocement. Applying Nadai’s theory to ferrocement the octahedral normal stress ($\sigma_o$) and octahedral shear stress ($\tau_o$) were computed for the test data using formulae

$$\sigma_o = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$$  \hspace{1cm} (5)

$$\tau_o = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$$  \hspace{1cm} (6)

where, $\sigma_1$, $\sigma_2$, $\sigma_3$ are principal stresses. $\sigma_3$ is zero as the problem is a plane stress problem. The values of $\sigma_o$ and $\tau_o$ are given in Table 3. for biaxial tension-tension. TX4, TX5 and TX6 in the table indicate uniaxial specimens. The plot of $\sigma_o$ vs. $\tau_o$ is shown in Fig. 10. There is a good correlation between $\sigma_o$ and $\tau_o$. 

![Fig.8. Normalized Interaction Diagram](image)

![Fig.9. Normalized Interaction Curve](image)

![Fig.10 Octahedral Shear Stress vs Normal Stress](image)
The relationship between octahedral normal and shear stress for biaxial stress state in tension-tension can hence be expressed as

\[ \tau_0 = 1.3927 + 0.6189\sigma_0 \]  

(7)

Conclusions

The test setup designed was suitable for testing hollow cylindrical specimens in biaxial tension-tension. Some of the important conclusions obtained from the experimental investigation are:

1) Stress in one direction was found to be beneficial to strength in the orthogonal direction as observed from the normalized interaction diagram.

2) The trend of the interaction diagram was not affected by the specific surface factor (SF), whereas the individual values of strength were dependent on SF.

3) An interaction curve and hence a mathematical model was developed for the high performance ferrocement which can be used as a criterion for design of ferrocement surfaces under biaxial tension-tension.

4) From the crack pattern it may be concluded that there is adequate ductility in ferrocement as these specimens continued to resist load even after the total propagation of cracks in the thickness of the specimen.

5) Nadai’s concept of octahedral shear stress being a function of octahedral normal stress is found to be applicable to ferrocement. A relationship between octahedral normal and shear stress for high performance ferrocement is suggested.

6) High performance ferrocement is ideal for use in stressed skin surfaces owing to its superior performance in strength, ductility and crack resistance.

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