Abstract

This paper presents an experimental investigation of reinforced concrete (RC) beams with double layered longitudinal bars, bond-strengthening hooks and 1300 MPa-class spirals. The hooks made of mild steel increased the bond strength along longitudinal bars. This experiment is aimed at quantification of not only the effectiveness of the spirals and bond-strengthening hooks on the RC members consisting of double layered longitudinal bars but also the influence of cut-off bars in an inner (second) layer. Twelve specimens were prepared and subjected to cyclic shear loads. All the specimens failed because of bond splitting along the longitudinal bars. The hooks, made of mild steel, increased the bond strength along the longitudinal bars. Applying the bond-strengthening hooks was found to improve the shear capacities of the specimens with the cut-off bars. The authors developed a truss mechanism model for members with longitudinal bars in an inner (second) layer. The model showed that the shear strength depends on the span/depth ratio \( L/D \) and inner (second) layer bar length \( L_d \).

Keywords: reinforced concrete member; 1300 MPa-class spiral; shear strength; bond strength; double-layered longitudinal bars

1. Introduction

The 1300 MPa-class spirals improve the shear capacities of RC members, and have successfully improved the seismic performance and even the workability in the construction of RC buildings. Use of the spiral reduces the cross-sectional area ratio of shear reinforcement \( p_w \) without reducing the term of \( \sigma_{wy} \), where \( \sigma_{wy} \) = yield stress of the spiral. However, the reduction of \( p_w \) is accompanied by a reduction of the bond strength along the longitudinal reinforcing bars. The authors' previous work discussed a bond-strengthening method using supplemental U-shaped hooks (Sato 2004). The effectiveness of the hooks on the bond strength \( \tau_b \) was quantified based on shear tests of 28 RC beams and columns with single-layered longitudinal bars (Sato 2006).

Beams and columns in actual buildings, however, usually contain multi-layered longitudinal bars. The proposed shear strength equation in the authors' previous research considers the bond strength of outer (first) layer bars while neglecting the contribution of the bond of inner (second) layer bars. In addition, it is usually not possible to cut off the inner (second) layer bars in relatively short beams or columns (i.e. \( a/D < 2 \)) designed in accordance with current RC design code (AIJ 1999a). This design limitation might cause an overuse of expensive high-strength materials, and result in defective construction due to the high content of reinforcements.

For this reason, 12 RC beam specimens with double-layered longitudinal bars were subjected to shear tests to evaluate the following five factors:

1. Bond strength of inner (second) layer bars;
2. Bond-strengthening effectiveness of the hooks on the inner (second) layer bars;
3. Spiral content \( (\rho_s = 0.60\% \text{ or } 0.89\%) \);
4. Inner (second) layer bar content (four bars or six bars); and
5. Cut-off of inner (second) layer bars \( (L_d = L/3, L/2, \text{ or non-cut-off, where } L_d = \text{inner-bar length and } L = \text{beam span}) \).

2. Modified BCJ-C1166 Guideline

The authors had proposed to use BCJ-C1166 Guideline (BCJ 1999) for the design of RC members with the 1300 MPa-class spirals because it rationally evaluates the contribution of high-strength spirals. (Sato...
This guideline is adopted again to estimate the shear strength of members with double-layered longitudinal bars. Equations (1) – (11) maintain the effective ratio of bond-strengthening hooks, the bond strength of inner (second) layer bars $\tau_{s2}$, and effective factor of inner (second) layer bars $c_v$.

$$V_c = \text{shear strength}$$
$$= \text{min}(V_1, V_2)$$

(1)

$$V_s = \text{shear strength determined by either spiral strength or concrete strength}$$
$$= b j_i k_0 + k_1 (1-k_2) b D v f'_c$$

(2)

$$V_b = \text{shear strength determined by bond strength along longitudinal bars}$$
$$= j_i (\Sigma \Psi_1 + c, \tau_{s2} \Psi_2) + k_1 (1-k_2) b D v f'_c$$

(3)

where

$$v = 0.7 - f'_c / 200$$

(4)

$$k_0 = \min[p_a \sigma_{sy}, v f'_c / 2]$$

(5)

$$k_1 = \left(\sqrt{(L/4)^3} + 1 - L / D\right)/2$$

(6)

$$k_2 = 2 k_0 / (v f'_c)$$

(7)

$$k_3 = 2 (\tau_{s1} \Sigma \Psi_1 + c, \tau_{s2} \Psi_2) / (v f'_c)$$

(8)

$$p_{bw} = \min[N_{bw} \pi d_{bw}^2 / (4 b s_b), 0.012]$$

(9)

$$\Psi_1 = N_1 \pi d_b$$

(10)

$$\Psi_2 = N_2 \pi d_b$$

(11)

$$b = \text{width of member (mm)}$$

$$D = \text{depth of member (mm)}$$

$$d_b = \text{nominal diameter of longitudinal bar (mm)}$$

$$d_{bw} = \text{diameter of spiral (mm)}$$

$$f'_c = \text{compressive concrete strength (MPa, f'c>0)}$$

$$j_i = \text{distance between centers of tensile and compressive longitudinal bars in outer (first) layer (mm)}$$

$$L = \text{clear span length (mm)}$$

$$N_1 = \text{number of longitudinal bars in outer (first) layer}$$

$$N_2 = \text{number of longitudinal bars in inner (second) layer}$$

$$N_w = \text{number of legs of spiral in cross section}$$

$$s_w = \text{spiral spacing (mm)}$$

$$\sigma_{sy} = \text{yield stress of spiral (MPa)}$$

$$\tau_{s1} = \text{bond strength along longitudinal bar in outer (first) layer (MPa); and}$$

$$\tau_{s2} = \text{bond strength along longitudinal bar in inner (second) layer (MPa).}$$

The bond strengths $\tau_{s1}$ and $\tau_{s2}$ are given by Eqs. (12) – (19).

$$\tau_{s1} = \left[0.096 b_{s1} + 0.134 + 7.79 h_1 b (p_{bw} + p_{sw}) / (N_1 d_b) \right] \sqrt{f'_c}$$

(12)

$$\tau_{s2} = 0.6 \left[0.096 b_{s2} + 0.134 + 7.79 h_2 b (p_{bw} + p_{sw}) / (N_2 d_b) \right] \sqrt{f'_c}$$

(13)

where

$$b_{s1} = b / (N_1 d_b) - 1$$

(14)

$$b_{s2} = b / (N_2 d_b) - 1$$

(15)

$$h_1 = 1 + 0.85 (N_u - 2 + N_b) / N_1$$

(16)

$$h_2 = 1 + 0.85 (N_u - 2 + N_b) / N_2$$

(17)

$$p_{bw} = N_{bw} \pi d_{bw}^2 / (4 b s_b)$$

(18)

$N_{bw}$ = effective number of hook anchors

$$\min[N_{bw}, b / (15 d_{bw})]$$

(19)

$$d_{bw} = \text{diameter of bond-strengthening hook (mm)}$$

$N_b$ = total number of anchors of bond-strengthening hooks; and

$s_w$ = hook spacing (mm).

The effective factor of the inner (second) layer bars $c_v$ is given by Eqs. (20) – (24):

(1) For a member without inner (second) layer bars;

$$c_v = 0$$

(20)

(2) For a member with continuous inner (second) layer bars;

$$c_v = 1$$

(21)

(3) For a member with cut-off inner (second) layer bars and $L > 2.8D$;

$$c_v = 0 \quad \text{for } L_a \leq L/2 - 1.4D \quad (22)$$

$$c_v = 1 + (L_a - L/2) / (1.4D) \quad \text{for } L_a > L/2 - 1.4D \quad (23)$$

(4) For a member with cut-off inner (second) layer bars and $L \leq 2.8D$;

$$c_v = 2 L_a / L \quad (24)$$

where

$L_a = \text{length of cut-off inner (second) layer bar.}$

Flexural strength $V_y$ is given by Eqs. (25) – (28):

$$V_y = 2M_u / L \quad (25)$$

$$M_u = 0.9 \Sigma a_i \sigma_{sy} d$$

(26) for $F_s = 0$

$$M_u = 0.8 \Sigma a_i \sigma_{sy} D + 0.5 F_s D \{1 - F_s / (b D f'_c)\}$$

(27) for $F_s > 0$

$$\Sigma a_i = (N_1 + N_2) \pi d_{bw}^2 / 4 \quad (28)$$

$d$ = effective depth (mm);

$F_s$ = compressive axial force ($F_s > 0$ (N)); and

$\sigma_{sy}$ = yield stress of flexural reinforcing bar (MPa).
The basis of these equations is described in the following sections.

3. Shear Tests

Fig.1. shows the configurations of the specimens, which represent beams at the lower stories of high-rise RC buildings. The cross section is 400 mm square, and the shear span to depth ratio a/D = 1.5. The concrete strength $f'_c$ is approximately 45 MPa. However, Specimens Z15, Z16, Z19, and Z20 were made of lower-strength concrete ($f'_c = 37.3$ MPa) because of inadequate mixing control. Twenty longitudinal bars were provided for Specimens Z9 to Z14 ($\rho_1 = 2.06\%$), and twenty-four bars for Specimens Z15 to Z20 ($\rho_1 = 2.47\%$). Specimens Z13, Z14, Z17, Z18, Z19, and Z20 contained cut-off bars. The cut-off position did not meet the design code (AIJ 1999a) because this experiment is intended to observe the contribution of short cut-off bars to the development of the bar stress at the critical cross section. The bond-strengthening hooks were inserted to a depth of 185 mm into the specimens. The depth of 185 mm was determined as $D/3$ (Sato 2006) plus the distance between the first and second bar layers. Table 1. summarizes the specifications and test results, and Table 2. lists properties of the deformed bars used.

The specimens were loaded by an apparatus used in a previous test (Sato 2006). The reversed cyclic loads were applied twice at each rotation angle of 0.25%, 0.5%, 1.0%, 2.0%, 3.3%, and 5.0%. Strain gauges were applied to measure the reinforcement strain.

4. Test Results

All specimens failed because of bond splitting along the longitudinal bars. Fig.2.(a) shows the crack patterns of Specimens Z19 and Z20 at rotation angle $R = 2\%$. The thick lines in Fig.2.(a) indicate cracks that occurred at $R = 2\%$. Specimen Z20 with the hooks shows relatively discrete crack patterns in comparison to Specimen Z19 without the hooks, in which the cracks form very continuous patterns. The applied hooks prevented bond splitting along the longitudinal bars at smaller rotation angles ($R < 2\%$) although the splitting cracks propagated after the angle exceeded 2%.

Fig.3. shows envelopes of relationships between the shear force and rotation angle of the specimens. The thinner lines represent specimens without the hooks while thicker lines those with the hooks. Of the 12 specimens, 9 achieved the maximum shear force at a rotation angle of 2%, with the exception being Specimens Z9, Z10, and Z14, which achieved it at 3.3%.

### Table 1. Specifications of Specimens and Test Results

<table>
<thead>
<tr>
<th>$f'_c$</th>
<th>$N_1$</th>
<th>$d_{so}$</th>
<th>$s_w$</th>
<th>$d_{j}$</th>
<th>$L_1$</th>
<th>$\alpha_{w}$</th>
<th>$\alpha_{wexp}$</th>
<th>$\tau_1$</th>
<th>$\tau_{1\exp}$</th>
<th>$\tau_{2\exp}$</th>
<th>$\sigma_{w}$</th>
<th>$\sigma_{w}\exp$</th>
<th>Ave.</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPa</td>
<td>mm</td>
<td>mm</td>
<td>%</td>
<td>mm</td>
<td>kN</td>
<td>kN</td>
<td>kN</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td>MPa</td>
<td></td>
</tr>
<tr>
<td>Z9</td>
<td>46.4</td>
<td>4</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>0</td>
<td>0</td>
<td>0.88</td>
<td>3.6</td>
<td>3.7</td>
<td>0.52</td>
<td>0.52</td>
<td>0.53</td>
</tr>
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<td>46.4</td>
<td>4</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>2</td>
<td>0.32</td>
<td>4.97</td>
<td>3.2</td>
<td>4.2</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
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<tr>
<td>Z11</td>
<td>46.4</td>
<td>4</td>
<td>7.1</td>
<td>45</td>
<td>0.89</td>
<td>0</td>
<td>0.57</td>
<td>5.94</td>
<td>3.3</td>
<td>3.3</td>
<td>0.39</td>
<td>0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>Z12</td>
<td>40.4</td>
<td>4</td>
<td>7.1</td>
<td>45</td>
<td>0.89</td>
<td>2</td>
<td>0.36</td>
<td>6.07</td>
<td>3.5</td>
<td>3.5</td>
<td>0.40</td>
<td>0.40</td>
<td>0.40</td>
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<tr>
<td>Z13</td>
<td>46.1</td>
<td>4</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>0</td>
<td>0</td>
<td>600</td>
<td>9.10</td>
<td>9.10</td>
<td>3.9</td>
<td>3.9</td>
<td>3.9</td>
</tr>
<tr>
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<td>46.1</td>
<td>4</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>2</td>
<td>0.32</td>
<td>600</td>
<td>9.10</td>
<td>9.10</td>
<td>3.9</td>
<td>3.9</td>
<td>3.9</td>
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<tr>
<td>Z15</td>
<td>57.3</td>
<td>6</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>0</td>
<td>0</td>
<td>800</td>
<td>12.8</td>
<td>12.8</td>
<td>4.8</td>
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<td>Z16</td>
<td>57.3</td>
<td>6</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>2</td>
<td>0.32</td>
<td>753</td>
<td>12.3</td>
<td>12.3</td>
<td>4.8</td>
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<td>4.8</td>
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<tr>
<td>Z17</td>
<td>46.1</td>
<td>6</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>0</td>
<td>0</td>
<td>600</td>
<td>9.10</td>
<td>9.10</td>
<td>3.9</td>
<td>3.9</td>
<td>3.9</td>
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<tr>
<td>Z18</td>
<td>46.1</td>
<td>6</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>2</td>
<td>0.32</td>
<td>600</td>
<td>9.10</td>
<td>9.10</td>
<td>3.9</td>
<td>3.9</td>
<td>3.9</td>
</tr>
<tr>
<td>Z19</td>
<td>46.1</td>
<td>6</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>2</td>
<td>0.32</td>
<td>400</td>
<td>7.5</td>
<td>7.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
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<tr>
<td>Z20</td>
<td>46.1</td>
<td>6</td>
<td>6.2</td>
<td>50</td>
<td>0.60</td>
<td>2</td>
<td>0.32</td>
<td>400</td>
<td>7.5</td>
<td>7.5</td>
<td>0.4</td>
<td>0.4</td>
<td>0.4</td>
</tr>
</tbody>
</table>

\*1 $C_n$ = continuous bar. \*2 $\alpha_{wexp}$ = maximum stress of spiral. \*3 $\alpha_{w\exp}$ = maximum stress of bond-strengthening hook.

Common parameters: $b = 400$, $d_j = 200$, $d_w = 1.5$, $d_j = 19.1$, $N_1 = 6$, $N_2 = 4$, $d_{so} = 6$, $d_{j1} = 300$, $d_{j2} = 200$.  

Fig.1. Configurations of Specimens (Unit: mm)
The number of inner (second) layer bars did not have a resulting significant influence on the behavior of the specimens. The strength and ductility of Specimens Z15, Z16, Z19, and Z20, which contained six inner layer bars, were not superior to those of Specimens Z9, Z10, Z13, and Z14, with four inner layer bars.

In contrast, the cutting-off of the inner-layer bars considerably reduced the shear strength as observed between Specimens Z15 (continuous, \( V_{\text{exp}} = 1165 \text{ kN} \)) and Z19 (\( L_d = 400 \text{ mm}, V_{\text{exp}} = 1047 \text{ kN} \)).

5. Bond Stresses

The measured steel bar strains were converted into stresses using a modified Ramberg-Osgood model.

Table 2. Properties of Deformed Steel Bars

<table>
<thead>
<tr>
<th>Steel bar</th>
<th>Cross section, mm²</th>
<th>Yield stress, MPa</th>
<th>Tensile strength, MPa</th>
<th>Elastic modulus, GPa</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal bar φ19</td>
<td>286.5</td>
<td>1019</td>
<td>1102</td>
<td>201</td>
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<tr>
<td>Spiral φ 6-4</td>
<td>30.0</td>
<td>1480</td>
<td>1510</td>
<td>200</td>
</tr>
<tr>
<td>Spiral φ 7.1</td>
<td>40.0</td>
<td>1407</td>
<td>1437</td>
<td>194</td>
</tr>
<tr>
<td>Bond-strengthening</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>hook φ 6</td>
<td>31.6</td>
<td>425</td>
<td>591</td>
<td>215</td>
</tr>
</tbody>
</table>

(a) Crack patterns (Thick lines indicate cracks at \( R = 2\% \))

(b) Distributions of truss struts

Fig.2. Typical Crack Patterns and Distributions of Truss Struts at Rotation Angle \( R = 2\% \) (Specimens Z19 and Z20)

Fig.3. Relationships between Shear Force and Rotation Angle (Envelopes)
The bond stresses were computed from the stress gradient, and they were averaged in the interest of design practices (Sato 2006). Fig.4.(a) shows the relationship between the average bond stress along the outer (first) layer longitudinal bars and the rotation angle, and Fig.4.(b) shows it for the inner (second) layer bars. Figs.4.(a) and (b) plot only bond stresses at the peak rotation angles of each cycle.

For the maximum bond stresses of outer (first) layer bars $\tau_{b1\text{exp}}$, larger stresses were observed in specimens with the hooks (Z12, Z14, Z16, and Z18), except Specimens Z10 and Z20. However, the hooks did not always increase the bond stresses of the inner (second) layer bars $\tau_{b2\text{exp}}$. The average ratio of $\tau_{b2\text{exp}}$ to $\tau_{b1\text{exp}}$ is 0.53 (Table 1.). This value roughly agrees with the coefficient 0.6 in Eq. (13), which was determined in AIJ's Guidelines (AIJ 1999b).

### 5.1 Stresses of Spiral and Bond-Strengthening Hooks

Fig.4.(c) shows the relationship between spiral stress and rotation angle. The stresses shown in Fig.4.(c) are the maximum values in the clear span. The spiral stresses range from 500 MPa to 800 MPa. However, a significant difference was observed between Specimens Z19 and Z20, in which the inner (second) layer bars were cut-off at $L_d = 400$ mm. The maximum spiral stress of Specimen Z19 without the hooks was 362 MPa. This was the lowest value among the 12 specimens and indicates that the specimen failed prematurely because the anchorage capacity of the cut-off bars was insufficient. In addition, the strain gauges of spirals were located between two parallel cracks, which were supposed to reduce the spiral strain due to the local bond action. Conversely, the spiral stress of Specimen Z20 was increased to 958 MPa, which was the highest among the 12 specimens because the hooks improved the inner bars' anchorage, resulting in steeper and much discrete cracks.

Fig.4.(d) shows relationships between the bond-strengthening hook stress and rotation angle. Yielding of the hooks was observed in Specimens Z16 and Z18, but only in local parts of the specimens. This observation indicates that the hooks need not be made of high strength steel for beams whose spiral content is greater than $p_w = 0.60\%$.

### 5.2 Developed Stress Force in Inner (Second) Layer Bars and Bar Lengths

Fig.5. shows the tensile stresses of inner (second) layer bars $\sigma_2$ at the end of the beam span, in terms of the bar length $L_d$. The stresses were adopted from
Specimens Z15 to Z20 at a rotation angle of 2%. The tensile stress in Fig.5. is given as the ratio \( r_2 \) of the inner bar stress \( \sigma_2 \) to outer bar stress \( \sigma_1 \), which is modified considering the strain gradient along beam depth as expressed by Eq. (29).

\[
r_2 = \frac{\sigma_2}{\sigma_1 \times (7/8d - 50)/(7/8d)}
\]

(29)

The denominator \((7/8d - 50)/(7/8d)\) is given based on an assumption that the cross sections of specimens remain on the same plane, in the interest of simplicity, although they might not be entirely on the same plane at a rotation angle of 2%. The inner (second) layer bar stress ratio \( r_2 \) of Specimens Z19 and Z20 with the shortest bar length \( L_d = 400 \text{ mm} \) decreased by 21% and 26%, respectively, in comparison to those of Specimens Z15 and Z16 with continuous bars.

\[
\phi_1 = \tan^{-1}[b \ c_{w1} (p_a \ \tau_{wexp} + p_b \ \tau_{exp})/(\tau_{b1} \ \Sigma \ t_1)]
\]

(30)

\[
\phi_2 = \tan^{-1}[b \ c_{w2} (p_a \ \tau_{wexp} + p_b \ \tau_{exp})/(\tau_{b2} \ \Sigma \ t_2)]
\]

(31)

\[
\sigma_{t1} = \text{stress of truss strut acting on outer (first) layer longitudinal bars} = \frac{\tau_{wexp} \ \Sigma \ t_1}{b \ \sin \phi_1 \ \cos \phi_1}
\]

(32)

\[
\sigma_{t2} = \text{stress of truss strut acting on inner (second) layer longitudinal bars} = \frac{\tau_{wexp} \ \Sigma \ t_2}{b \ \sin \phi_2 \ \cos \phi_2}
\]

(33)

\[
c_{w1} = 0.625
\]

(34)

\[
c_{w2} = 0.375
\]

(35)

The spiral stress \( \sigma_{wexp} \) was assumed to act on the first and second layer bars by a ratio of 1:0.6 in accordance with the AIJ 1999 Guidelines (AIJ 1999b), which evaluates the bond strength of inner (second) layer bars to be roughly 0.6 times that of the first (outer) layer bars. Coefficients \( c_{w1} \) and \( c_{w2} \) define these ratios. The AIJ Guidelines (AIJ 1999b) classifies the truss struts into two types:

1. Strut connecting upper and lower outer (first) layer bars (1-1 strut); and
2. Strut connecting upper and lower inner (second) layer bars (2-2 strut).

Fig.2.(b) indicates the possibility of another type of strut in beams with cut-off bars:

(3) Strut connecting outer (first) layer and inner (second) layer bars on opposite sides from each other (1-2 strut).

The truss mechanism modeling described in the next section considers these three types of struts.

5.4 Truss Mechanism Modeling

Fig.6. outlines the truss mechanism model proposed in this section. This model classifies the truss strut distribution into four cases (Cases A, B, C, and D) depending on the inner (second) layer bar length \( L_d \). The boundary condition and shear strength of each case are given by Eqs. (36) – (41):

\[
V_b = \max(V_{bA}, V_{bB}, V_{bC}, V_{bD})
\]

(36)

Case A: \( 2\Delta L/(j_i - j_d) \leq \cot \phi_2 \)

\[
V_{bA} = j_i \ \tau_{b1} \ \Sigma \ t_1 + (j_a + j_o + c_{w2} j_b) \ \tau_{b2} \ \Sigma \ t_2
\]

\[
+ k_i b D \left[ v f'_c \ - \sigma_{t1} - c_{w2} \sigma_{t2} (j_a + j_o + c_{w2} j_b)/j_i \right]
\]

(37)

Case B: \( \Delta L/j_d \leq \cot \phi_2 < 2\Delta L/(j_i - j_d) \)

\[
V_{bB} = j_i \ \tau_{b1} \ \Sigma \ t_1 + (j_a + c_{w2} j_b) \ \tau_{b2} \ \Sigma \ t_2
\]

\[
+ k_i b D \left[ v f'_c - \sigma_{t1} - c_{w2} \sigma_{t2} (j_a + c_{w2} j_b)/j_i \right]
\]

(38)

Case C: \( \Delta L/(j_i + j_d) \leq \cot \phi_2 < \Delta L/j_d \)

\[
V_{bC} = j_i \ \tau_{b1} \ \Sigma \ t_1 + c_{w2} j_b \ \tau_{b2} \ \Sigma \ t_2
\]

\[
+ k_i b D \left[ v f'_c - \sigma_{t1} - c_{w2} \sigma_{t2} j_b/j_i \right]
\]

(39)

Case D: \( \cot \phi_2 < \Delta L/(j_i + j_d) \)

\[
V_{bD} = j_i \ \tau_{b1} \ \Sigma \ t_1 + k_i b D \left( v f'_c - \sigma_{t1} \right)
\]

(40)

where

\[
\Delta L = L - 2L_d
\]

\( j_d \) = distance between centers of tensile and compressive longitudinal bars in inner (second) layer; and

\[
c_{w2} = \text{reduction factor of bond strength of inner (second) layer bar} \tau_{b2}.
\]

Fig.6. shows only the struts around the locations of cut-off bars. The struts in other parts are conventional.
ones (i.e. 1-1 struts and 2-2 struts). As mentioned previously, this model is characterized by the 1-2 strut, which connects first layer and second layer bars that are on opposite sides from each other. The symbol \( j_b \) represents the 1-2 strut height. As for the other symbols, \( j_{aa} \) and \( j_{a} \) represent the 2-2 strut heights, and \( j_c \) represents the height of the ineffective zone. The 1-1 strut height is usually \( j_t \), but it is not shown in Fig.6.

As the length of inner (second) layer bar becomes shorter, the truss mechanism is transferred from Case A to Case D.

Equations (37) – (40) superpose the truss actions of the 1-1 strut, 2-2 strut, 1-2 strut, and arch strut. It is supposed that the contribution of the 1-2 strut is significant in Specimens Z19 and Z20 with the shortest inner (second) layer bars. The bond strength of inner (second) layer bars \( \tau_b \) of each of Specimens Z19 and Z20 is approximately 0.5 times that of Specimens Z15 and Z16 with the continuous bars. Based on this observation, the factor \( c_{b2} (= 0.5) \) reduces \( \tau_b \) for the 1-2 strut.

The shear strength was evaluated based on the lower bound theory. \( V_{bb}, V_{bc}, V_{bc}, \) and \( V_{bd} \) are maximized by varying the strut angle \( \phi_2 \). Fig.7. shows the typical relationship between shear strength \( V_b \) by Eqs. (36) – (41) and inner (second) layer bar length \( L_d \). The thicker line represents the shear strength of beams with the hooks, while the thinner line those without the hooks.

The inner (second) layer bar is not effective if the length is shorter than 40 mm (Case D). From that point, the shear strength starts to increase almost linearly as the bar length \( L_d \) increases (i.e. Cases B and C). The increasing gradient becomes steeper when \( L_d \) exceeds 550 mm (Case A). The strength finally reaches the maximum at \( L_d = 600 \) mm, which is equivalent to that of a beam with continuous inner (second) layer bars.

5.5 Simplified Estimation of Shear Strength of Member with Cut-Off Bars

Equations (36) – (41) need iterative calculations. Simplification is therefore required for practical design. Parametric calculations were performed with a varied span \( L \) (400 mm – 6000 mm), depth \( D \) (100 mm – 1600 mm), width \( b \) (400 mm – 800 mm), cross-sectional area ratio of longitudinal bars \( p_t \) (0.16% – 2.36%) and spiral \( p_s \) (0.075% – 0.60%), and concrete strength \( f'_c \) (45 MPa – 80 MPa). The calculations indicate that the shear strength depends on the span/depth ratio \( L/D \) and inner (second) bar length/span ratio \( L_d/L \), and the influence on other variables is insignificant. Therefore, a simplified model is proposed based on the calculation results. The model is classified into two cases:

1. If \( L > 2.8D \), then the shear strength is assumed to increase linearly from \( L_d = L/2 – 1.4D \) to \( L_d = L/2 \).
2. If \( L \leq 2.8D \), then the shear strength is assumed to increase linearly from \( L_d = 0 \) to \( L_d = L/2 \). Case D does not exist in this case.

The coefficient \( c_v \) defined by Eqs. (20) – (24) has been proposed based on this simplification (see the dotted lines in Fig.7.).

Fig.8. compares the experimental strength \( V_{exp} \) of 120 specimens with the calculated strength \( V_u \) given by Eqs. (1) – (24). These specimens were adopted from 13 published works (Matsubara 1991, Oyado 1991, Furukawa 1992, Oyado 1993, Yamada 1994, Nagai 1995, Tsuihiji 1997a, Tsuihiji 1997b, Zhou 1998, Kim 2001, Hamada 2001, Tabata 2001, and Tabata 2002). The strengths were normalized by dividing them by the flexural strength \( V_f \) given by Eqs. (25) – (28). The average of \( V_{exp} / V_u \) is 1.41, and the coefficient of variation is 0.27. An evaluation with results from the calculations demonstrate that the test results overall are safe.
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References