Bond Action Effect of Longitudinal Cut-Off Bars on Shear Transfer Mechanism of RC Beams

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Abstract
A method is proposed to estimate the required length of double-layered flexural reinforcing bars for reinforced concrete (RC) beams. This research deals with deep beams in which the shear reaction is dominant and the plastic hinge is not formed. The ACI (American Concrete Institute) and AIJ (Architectural Institute of Japan) guidelines propose several methods to determine the cut-off length of double-layered bars for typical foundation beams. However, the theoretical and experimental bases are not yet presented.

A series of experimental works was conducted to measure the bond strength of double-layered bars. A method to estimate the double-layered length and the shear strength of beams with small shear span-to-depth ratio is then developed based on the concept of the partial truss mechanism.

Keywords: Shear transfer mechanism; bond stress; cut-off bar; partial truss mechanism

1. Introduction
Reinforced concrete beams, which are subject to anti-symmetrical moment and shear force, exhibit tension shift due to the diagonal tension cracks occurring at the ends of the members. Due to this phenomenon, bond stress begins to develop at the outside of the tension shift region. Thus, the cut-off location of the main re-bars should be determined to secure the necessary development length at the outside of the tension shift region. There are several methods proposed by ACI1, AIJ2, and KCI3 Codes for the determination of the cut-off location on the main re-bars. However, these methods cannot fully reflect the mechanical behavior.

For example, which illustrates the importance of the determination of the cut-off location, is the foundation beam of the frame structure. The foundation beam must resist the elastic bending moment at the hinge of the pedestal of foundation piles. However, because the depth of the beam is very deep and the method to determine the cut-off location in this situation is not established yet, the cut-off on the main re-bars is not exercised and the re-bars are laid out throughout the entire length of the beam.

This study assumes a foundation beam, which does not form the plastic hinge, to propose a method to determine the location of the cut-off on the main re-bars. The applicability of the proposed method in this study for the members forming the plastic hinge is left to future research.

2. Outline of Experiments
2.1 Test program
Three test specimens were constructed. Fig.1. and Table 1. show the details of the test specimens. All the specimens have a concrete cover of 16 mm on the cross section, a shear span ratio of 1.5, and a spacing of 50 mm between the shear reinforcements. Each specimen has a central test region with 200x300 mm sectional dimension and loading stubs at both beam ends. To measure the strains of the longitudinal reinforcements and stirrups, wire strain gauges were attached to their surfaces. From these strain measurements, the tension force of the stirrups and bond force of the longitudinal re-bars were computed.

The N-4-2L specimen is the regular specimen without any cut-off re-bars. Two specimens, N-2-2L and N-3-2L, with the cut-off main re-bars in the second layer at the center and the 3/4 length of the span of the test region, respectively, were designed to observe the variation of the bond stress of the cut-off main re-bars.

Deformed bars with a diameter of 16 mm and a yield strength of 795 MPa were arranged at both top and bottom sections of all beams. These longitudinal bars had enough yield strength to avoid flexural yielding. As the transverse reinforcement, welded closed stirrups of 6 mm in diameter-deformed bars with the yield strength of 305 MPa were arranged. Normal weight concrete was used and its compressive strength at
beam loading tests was 30.6 MPa.

Fig. 2 illustrates the loading equipment and the test setup. This loading method was developed by Jirsa et al.\textsuperscript{30}, where the rotation of the top stub of a specimen was restricted by two vertical jacks and results in the anti-symmetric moment distribution of the central beam test region. In this system vertical movement of the top stub was free, that is, no additional vertical load was applied to the specimen. Lateral shear force was applied by two parallel horizontal jacks. During the test the rotation angle of the member (drift) was measured by two electronic displacement transducers and recorded by the computer controlled data acquisition system.

2.2 Test results

Table 1. shows the maximum shear force and failure mode observed from this experiment. It exhibited that bond failure occurred for all test specimens. Fig. 3. shows the load versus displacement relationships obtained from the experiment. All the test specimens showed the maximum load around the displacement of 9 mm (drift angle $R=1/100$), and gradual decrease in the load after the peak load. There was sudden decrease in the load around the drift angle of $R=1/50$, whereas the downward trend of load was maintained up to the maximum displacement of 30 mm. Additionally, a decrease in the initial stiffness was not observed regardless of whether or not the main re-bar was cut-off. A reduction of 10% for the peak load was observed as the cut-off length increased by $L/4$, where $L$ is the whole span of the beam.

![Fig.1. Details of Specimens](image)

![Fig.2. Test Setup](image)

![Fig.4. Crack Patterns](image)

![Table 1. Properties of Specimens and Test Results](image)
and longitudinal reinforcements for each test specimen. The thick line in the figure represents the strain at the maximum load, and the x-axis indicates the location of the measured strain.

The shear reinforcements, as shown in the figure, yielded for all test regions regardless of whether or not the main re-bar was cut-off. In addition, the characteristic on the distribution of the strain of the longitudinal re-bars was that the phenomenon of tension shift, in which the tensile strain at a distance of \( d \) from the end of the beam increased as far as the strain at the beam end, was exhibited in all the specimens. Where \( d \) is the effective depth of the beam.

The tension shift phenomenon was maintained for the first layer main re-bars even after the maximum shear force, whereas it was not observed for those in the second layer.

3. Bond Behavior of Main Re-bars

Fig.6. shows the relationships between the measured

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**Fig.3. Load-Displacement Relationships**

**Fig.4. Crack Patterns at Maximum Shear Force**

**Fig.5. Distributions of Strain**

(a) N-4-2L  
(b) N-3-2L  
(c) N-2-2L
bond stress of the main re-bars and the displacement. Here, the actual measured value of the bond stress was obtained from the average bond stress using the strain of the gauges located in the B-D region excluding the A region, which exhibited tension shift, as illustrated in Fig.5.

Fig.6.(a) shows the bond stress of the first layer main re-bar, and it can be seen that it is different from that of the second layer main re-bar due to the degree of cut-off. The specimen N-4-2L, of which the re-bar was not cut off, exhibited the same bond stress at the range from the displacement of 9 mm (drift angle of R=1/100) to the maximum displacement of 30 mm (R=1/30). However, in the case of the second layer in which the main re-bar was cut-off, the bond stress of the first layer cut-off main re-bars decreased significantly after the displacement of 9 mm (R=1/100) for the test specimens N-3-2L and N-2-2L. On the other hand, although the bond behavior of the second layer main re-bars showed a little deviation depending on the experimental variable as shown in Fig.6.(b), it indicated a similar behavior overall. The bond stress reached the peak around the displacement of 9 mm, and then decreased as the displacement progressed.

4. Shear Transfer Mechanism of Beam Members with Cut-off Bars

Considering the crack pattern (Fig.4.) and the distribution of strain on the main re-bars (Fig.5.), an analytical model to predict the shear behavior considering the bond failure is proposed as shown in Fig.7. The following assumptions are adopted.

1) The concrete is assumed to reach the effective compressive strength at the drift angle of 1/100 ≤ R≤1/30.
2) The bond stress of the main re-bars is determined based on the experimental results as shown in Fig.8. The following indicates each shift point and the route of bond stress by the different layout of the re-bar.

\[
\begin{align*}
\text{Point A: } & \tau_{b1A} = 0.8\tau_{fm} \\
\text{Point B: } & \tau_{b1B} = 0.8\tau_{co} \\
\text{Point C: } & \tau_{b1C} = 0.25\tau_{co} + \tau_{st} \\
\text{Point D: } & \tau_{b1D} = 0.6\tau_{fm} \\
\text{Point E: } & \tau_{b1E} = \tau_{st}
\end{align*}
\]

where \(\tau_{fm}\) is the bond stress calculated from the Fujii-Morita equation \(^5\) and \(\tau_{co}\) and \(\tau_{st}\) are the bond stresses provided by the concrete and the shear reinforcement, respectively. Using the equation, the values of \(\tau_{fm}\), \(\tau_{co}\), and \(\tau_{st}\) for each specimen are 2.33 MPa, 1.90 MPa, and 0.43 MPa, respectively.

- **Bond stress \((\tau_{b1})\) of the first layer cut-off re-bar**
  
  Because the bond stress of the first layer cut-off re-bar deteriorates after the drift angle of 1/100 of the second layer cut-off main re-bars, the stress is redistributed. In the case that the re-bar is not cut off, 80% of the bond stress is maintained past the drift angle of 1/100 according to the equation proposed by Fujii and Morita \(^5\). However, if the second layer main re-bar is cut off, the bond stress of the first layer cut-off main re-bar is decreased and follows the following route.

\[
\begin{align*}
\text{i) } & R \leq 1/100 : \text{route OA} \\
\text{ii) } & 1/100 \leq R \leq 1/30 : \text{route AB} \\
\text{(in the case of no second layer cut-off)} & : \text{route AC} \\
\text{(in the case of a second layer cut-off)} & : \text{route OD} \\
\text{ii) } & 1/100 \leq R \leq 1/30 : \text{route DE}
\end{align*}
\]

- **Bond stress \((\tau_{b2})\) of the second layer cut-off re-bar**
  
  At the drift angle of R=1/100, the bond stress of the second cut-off main re-bars reaches the peak load. As the displacement progresses, the bond stress decreases. When the displacement angle is R=1/100, the bond stress is assumed to be 60% of the bond strength calculated from the Fuji-Morita equation based on the AIJ Guideline \(^6\). Moreover, at the maximum displacement and drift angle of R=1/30, the contribution of concrete \((\tau_{co})\) of the Fuji-Morita equation does not exist and only the contribution of shear reinforcement is considered.

\[
\begin{align*}
\text{i) } & R \leq 1/100 : \text{route OD} \\
\text{ii) } & 1/100 \leq R \leq 1/30 : \text{route DE}
\end{align*}
\]
At the drift angle of $1/100 \leq R \leq 1/30$, the shear reinforcement yields for all test regions.

### 4.1 Shear transfer mechanism

Based on the bond stress of the aforementioned model and three basic assumptions, it can be considered that the shear force is transferred through three combined actions of shear resistance, that is, partial truss actions (formed by the bond action of the first and second layers of the cut-off main re-bars) and arch strut as shown in Fig.9. After the drift angle of $R=1/100$, the following relational equations can be obtained based on the basic assumptions 1) and 2).

\[
\rho_w' \sigma_{w1} + \rho_w' \sigma_{w2} = \rho_w' \sigma_{wy} \tag{2}
\]

\[
\sigma_{t1} + \sigma_{t2} + \sigma_a = \nu \sigma_B \tag{3}
\]

where $\rho_w' \sigma_{w1}$ and $\sigma_{t1}$ indicate the stresses of the shear reinforcement and the diagonal strut of the concrete used in the partial truss action formed by the first layer cut-off main re-bars, respectively; $\rho_w' \sigma_{w2}$ and $\sigma_{t2}$ signify those formed by the second layer cut-off ones, respectively; $\sigma_a$ is the concrete compressive stress used in the arch action; $\nu$ is the softening coefficient of the concrete; and $\sigma_B$ is the effective compressive stress of the concrete$^7,8$.

The analytical model is able to compute the shear force transferred by the truss action formed by the first and second layers cut-off main re-bars and the arch action in accordance with the basic assumption 3). The status of stresses by the truss and arch actions formed by the first and second layers of the cut-off main re-bars are examined in detail in the next chapter.

#### 4.2 Truss action formed by the first layer cut-off main re-bars

The stress of the truss action formed by the first layer cut-off main re-bar is shown in Fig.9.(a). In the figure, the shear force is transferred by the truss action of the concrete compressive strut with the angle $\phi_1$ formed by the resultant force of the bond stress ($\tau_{b1}$) of the first layer cut-off main re-bars and the tensile force ($\rho_w' \sigma_{w1}$) of the shear reinforcements.

Based on the equilibrium condition illustrated in Fig.9.(a), the angle of the truss action ($\phi_1$) and the compressive stress of concrete strut ($\sigma_{t1}$) can be expressed as

\[
\cot \phi_1 = \frac{\Sigma \psi_1 \tau_{b1}}{b \rho_w' \sigma_{w1}} \tag{4}
\]

\[
\sigma_{t1} = \frac{\tau_{b1} \Sigma \psi_1}{b \sin \phi_1 \cdot \cos \phi_1} \tag{5}
\]

where $\Sigma \psi_1$ is the sum of the perimeters of the first layer cut-off main re-bars.

Moreover, the shear force, $V_{ib1}$, of the truss action formed by the bond action of the first layer cut-off main re-bar on the B-B' cross section can be derived as follows:

\[
V_{ib1} = \tau_{b1} \Sigma \psi_1 \cdot j_i \tag{6}
\]

where $j_i$ is the distance between the centers of the first layer cut-off main re-bars located in the top and bottom of the section.

#### 4.3 Partial truss action formed by the second cut-off main re-bars

The stress of the partial truss action formed by the second layer cut-off main re-bars is shown in Fig.9.(b). In the figure, the shear force is transferred by the truss action composed of the concrete compressive strut of the angle $\phi_2$ formed by the composite action of the bond stress ($\tau_{b2}$) on the second layer cut-off
main re-bars and the tensile force ($\rho_w \sigma_w$) of the shear reinforcements. However, in the case of the second layer cut-off main re-bars (N-2-2L and N-3-2L), the bond stress as the composite element of truss action is not formed because there is no main re-bar in the compression zone between A-B sections.

To satisfy the equilibrium condition, this study assumed that the difference of compressive stress carried by the concrete acted uniformly at the compression zone in the A-B region with no second layer main re-bars, instead of the bond stress of the second layer ones necessary for the formation of the truss action, as shown in Fig.7. On the other hand, the tension zone in the A-B region with the second layer cut-off main re-bars exhibited comparatively small change in the compressive stress carried by the concrete due to the tension shift effect (Fig.7.). Therefore, a partial truss action can be formed by superimposing the difference of the compressive stress carried by the concrete due to the tension shift effect of the first layer cut-off main re-bars.

The angle ($\phi_2$) of the partial truss action formed by the second layer cut-off main re-bars is given as the following equation in accordance with Fig.9(b).

$$\cot \phi_2 = \frac{(\sum \psi_2 \tau_{32})}{b \cdot \rho_w \sigma_w} = \frac{\sum \psi_2 \tau_{32}}{b(\rho_w \sigma_{wy} - \rho_w \sigma_{wy})}$$  (7)

Then, the compressive stress of the concrete strut ($\sigma_{t2}$) is expressed as follows:

$$\sigma_{t2} = \frac{\tau_{32} \cdot \sum \psi_2}{b \cdot \sin \phi_2 \cdot \cos \phi_2}$$  (8)

Moreover, the shear force, $V_{tb2}$, of the truss action formed by the bond action of the second layer cut-off main re-bars on the B-B' cross section is expressed as

$$V_{tb2} = \tau_{32} \sum \psi_2 \cdot j_2$$  (9)

where $j_2$ is the distance between the centers of the second layer cut-off main re-bars, and $\sum \psi_2$ is the sum of the perimeter of the first layer cut-off main re-bars. As shown in Fig.9(c), the bond length that contributes to the partial truss action is shortened as the length of the cut-off re-bar is decreased. Thus, $j_2$ is constrained by the following equation.

$$j_2 = \min(j_2 \frac{L_2 - d}{\cot \phi_2})$$  (10)

where $L_2$ is the length of the second layer cut-off main re-bars.

Additionally, when the partial truss action, formed by the first and second layers, is superimposed based on the theory of plasticity, the angle of the concrete compressive strut $\phi$ can be obtained as the minimum...
value of 26.6° (\(\cot \phi = 2\)), and the following equation is derived from Figs. 9(a) and (b).

\[
0 < \cot \phi = \frac{(\sigma_1 \sin \phi_1 \cos \phi_2 + \sigma_2 \sin \phi_1 \cos \phi_2)}{\sigma_1 \sin^2 \phi_2 + \sigma_2 \sin^2 \phi_2} \leq 2
\]  

(11)

4.4 Arch action

Fig. 9(d) shows the stress status due to the arch action. From the figure, the shear force due to the arch action can be expressed as follows:

\[
V_a = D \frac{\sigma_b \tan \theta}{2}
\]  

(12)

\[
\sigma_a = \nu \sigma_b - \sigma_{t1} - \sigma_{t2}
\]  

(13)

where \(\theta\) is the angle of the arch action.

Thus, the shear force transferred by the three overlapping forces of two partial truss actions and an arch action can be given by the following summation equation.

\[
V = V_{t1} + V_{t2} + V_a
\]  

(14)

The flow chart of the solution procedure to calculate the shear force using the proposed method is illustrated in Fig.10.

5. Analytical Results and Discussions

Using the bond stress versus displacement relationship based on the experimental results, a computational analysis for the post-peak behavior (\(1/100 \leq R \leq 1/30\)) of the beam members with cut-off re-bars was carried out to obtain the analytical results as described in Fig.11.

As shown in Fig.11(a), the test specimen without cut-off re-bar, N-4-2L, exhibited a decrease in the bond stress only for the second layer cut-off main re-bars after the drift angle of \(R=1/00\), whereas the shear force transfer by the truss action of the first layer cut-off main re-bars maintained the stress. Only the shear resistance by the partial truss action formed by the second layer cut-off main re-bar deteriorated. On the other hand, the shear force transferred to the arch action increased a little, because the compressive stress of the strut used in the arch action increased as much as that used in the partial truss action of the second layer cut-off main re-bars (\(\sigma_{t0} = \sigma_{t1} + \sigma_{t2} + \sigma_c\)). It can be seen that the predicted response was in excellent agreement with the experimental post-peak behavior up to the maximum displacement.

It can be seen from Figs.11(b) and (c) that the shear force transfer by the truss action decreased. This is due to the fact that the bond stresses of the specimens, of which the second layer main re-bar was cut off at the critical section, N-2-2L and N-3-2L, decreased at both ends of the first and second layers of the cut-off main re-bar after the drift angle of \(R=1/100\). Considerable deterioration of the shear transfer by the truss action of the first layer main re-bars especially, can be seen in the figures. The decrease in the shear force transfer...
by the partial truss action of the second layer cut-off main re-bars was not remarkable as the cut-off length decreased. As a result, although a rapid decrease in the shear transfer of all the specimens is not observed, the reduction of the shear capacity of the specimen was increased as the cut-off length increased. This phenomenon was observed in the experiment, and a comparison between the test and theoretical results for the shear force versus displacement response showed excellent agreement. Therefore, the proposed method in this study can be reasonably used for predicting the shear response of the beams with cut-off main re-bars.

6. Conclusions

The partial truss action formed by the transverse confinement of the shear reinforcements and the bond strength of the main re-bars in a beam member with cut-off steel bars was clearly examined and explained by experimental and analytical investigations. In addition, the fundamental data to reasonably determine the location of cut-off on the main re-bar and an analytical method to predict the shear force considering the bond characteristic were proposed. The following conclusions are drawn from this research:

(1) The test specimens with cut-off re-bars exhibited a lower shear strength than the specimen without cut-off re-bar, and a reduction of 10% for the peak load was observed as the cut-off length increased by a quarter of the whole span.

(2) The bond stress of the second layer cut-off main re-bars of the beam with two layers of the longitudinal reinforcements decreased after the drift angle of $R=1/100$ regardless of whether the main re-bar was cut off or not. However, when the second layer main re-bars were not cut off, the bond stress of the first layer cut-off main re-bar maintained the uniform bond stress from the drift angle of $R=1/100$ up to the maximum displacement. While on the contrary, when the second layer main re-bar was cut off, the bond stress of the first layer cut-off main re-bar decreased after the drift angle of $R=1/100$.

(3) Considering the tension shift based on the bond stress versus displacement relationships of the beam members with cut-off re-bars, the partial truss action formed by the transverse confinement of the shear reinforcements and the bond strength of the main re-bars can be clearly analyzed.

(4) A method based on the lower bound theory was proposed to predict the shear strength of the beams with the cut-off main re-bars. In particular, theoretical results calculated from the proposed method were shown to be in good agreement with the experimental results.

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