Transverse Reinforcement of RC Columns
Considering Effective Lateral Confining Reduction Factor

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Abstract
An experimental investigation was conducted to examine the hysteretic behavior of ultrahigh-strength concrete tied columns under stress to determine the effect of the volumetric ratio of transverse reinforcements on column deformability. Eight 1/3-scale columns were fabricated to simulate a half-story of actual structural members, and their axial load ratio, transverse reinforcement configuration, and transverse reinforcement volumetric ratio were changed during the simulation. The column deformability was found to be affected by the configurations and volumetric ratios of the transverse reinforcement. The column behavior was particularly affected by the axial load ratio as compared to the amount and configuration of the transverse reinforcement. To improve the ductility behavior of an RC column using ultrahigh-strength concrete in a seismic region, a volumetric ratio of transverse reinforcement was suggested for all data satisfying the required displacement ductility ratio of over 4. The results indicate that the effective lateral confining reduction factor \( \lambda_c \), calculated by considering the configuration and spacing of transverse reinforcement and the axial load ratio, is reflected in the volumetric ratio of transverse reinforcements.

Keywords: ultrahigh-strength concrete; tied column; configuration; transverse reinforcement; axial load ratio; ductility ratio; effective lateral confining reduction factor

1. Introduction
To design reinforced concrete structures for seismic loads safely and economically, absorbed seismic energy should be redistributed uniformly by plastic deformation of the structural member during a seismic event such as a large earthquake. To achieve ductile behavior, plastic hinges should be used in beams rather than columns for both safety and economic reasons. During a heavy earthquake, the plastic hinges in columns are subjected to a high shear stress at the bottom of the column. Therefore, it is very important to ensure the ductile behavior of columns to prevent a sudden collapse.

Previous research related to the design of transverse reinforcement for improving column strength and ductility demonstrated that proper confinement of the core concrete and the support of longitudinal bars using transverse reinforcement are the best ways to improve column ductile behavior. Even though mandatory building codes for transverse reinforcement exist in many countries to ensure ductile column behavior, transverse reinforcement for ultrahigh-strength concrete columns has not been thoroughly investigated. Ultrahigh-strength concrete is more brittle than normal-strength concrete, and therefore, the building codes for the latter may not be applicable to the former.

Previous studies revealed that ultrahigh-strength columns designed according to current building code recommendations showed ductile behavior when a low axial load and a small lateral force were applied. However, when a high axial load and large lateral force were applied, columns showed brittle behavior due to degradation of column strength and stiffness. To prevent the brittle failure of ultrahigh-strength concrete columns, transverse reinforcement should be carefully designed to ensure that the columns have sufficient ductility.

2. Objective and Scope
Although the ACI building code contains equations that determine the amount of transverse reinforced steel required by considering the compressive strength of the concrete and the yield strength of the transverse reinforcement, the column axial load has not been taken into consideration, as shown in Table 1. We suggest that the design considerations for transverse

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reinforcement of ultrahigh-strength concrete columns are different from those for normal-strength concrete columns.

To test this hypothesis, we examined the behavior of reinforced concrete columns under a variety of loading conditions to establish guidelines for the design and detailing of confining steel for ultrahigh-strength concrete columns.

We then developed equations to calculate the appropriate amounts of transverse reinforcement for ultrahigh-strength concrete columns by comparing our results with those of previous studies.

Table 1. Code Requirements for Transverse Reinforcement

<table>
<thead>
<tr>
<th>Code</th>
<th>Equations used to calculate amount of transverse reinforcement required</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACI 318-05</td>
<td>$A_{tr} = 0.35d_t\frac{f_{c}}{f_{y,t}}(\frac{A_t}{A_c} - 1)$, $A_{tr} = 0.099d_t\frac{f_{c}}{f_{y,t}}$</td>
</tr>
<tr>
<td>SEAOCC</td>
<td>$A_{tr} = 0.35d_t\frac{f_{c}}{f_{y,t}}(\frac{A_t}{A_c} - 1)$, $A_{tr} = 0.12d_t\frac{f_{c}}{f_{y,t}}$</td>
</tr>
<tr>
<td>AASHTO</td>
<td>$A_{tr} = 0.35d_t\frac{f_{c}}{f_{y,t}}(\frac{A_t}{A_c} - 1)$, $A_{tr} = 0.12d_t\frac{f_{c}}{f_{y,t}}$</td>
</tr>
<tr>
<td>NZS 3101</td>
<td>$A_{tr} = \frac{\sum d_{tr} f_{tr}}{\sum d_{tr} f_{tr}} \frac{N}{A_c \phi_{dr}} \left( \frac{N}{A_c \phi_{dr}} \right)^{-\frac{0.006 d_t h}{A_c}}$</td>
</tr>
<tr>
<td>(SANZ)</td>
<td>$A_{tr} = \frac{\sum d_{tr} f_{tr}}{\sum d_{tr} f_{tr}} \frac{N}{A_c \phi_{dr}} \frac{N}{A_c \phi_{dr}}^{-\frac{0.006 d_t h}{A_c}}$</td>
</tr>
<tr>
<td>CALTRANS</td>
<td>$A_{tr} = 0.125 h f_{y,t} \frac{f_{c}}{f_{y,t}} (0.5 - f_{y,t} f_{c})^{-2} + 0.13 h f_{y,t} (h - 0.01)$</td>
</tr>
</tbody>
</table>

3. Experimental Program

3.1 Specimen details

A half-storied column with a stub was modeled to simulate the real column behavior. To prevent brittle failure and induce flexural failure, a column aspect ratio ($L/d$) of 4 was selected. Nineteen columns were fabricated on a 1/3 scale of the actual structural members. The concrete cover was 20 mm, and the ratio of the gross area of the concrete column to the net area of the core was designed to be 1.33 for all columns. The transverse reinforcement for the specimens had a 135° hook and an extended length of 6$d_t$, where $d_t$ is the diameter of the transverse reinforcement.

The dimensions used for the column specimens were 300 x 300 x 1200 mm and those of the stub were 500 x 600 x 700 mm. The test region was located at 2.5 times the column depth from the joint of the stub and column. The end region of the column section was manufactured with over-reinforced transverse reinforcement (1.5 times regular steel) to prevent localized failure and induce failure at the test region.

The main variables considered were the axial load ratio, configuration of the transverse reinforcement, and volumetric ratio of the transverse reinforcement. Five different configurations and three different volumetric ratios for the transverse reinforcement were considered (Fig. 1.), while two different axial load levels were applied.

The longitudinal bar ratio was generally the same among columns regardless of the configuration of the transverse reinforcement. 13mm-diameter transverse reinforcement was used in the Type A specimens, while 10mm-diameter transverse reinforcement was used in the other specimens. Fig. 1. and Table 4. show the details of the specimens and variants.

3.2 Material Tests

The concrete used had a compressive strength of 100 MPa, and a coarse aggregate with a maximum diameter of 13 mm was used for the cover depth and spacing. A deformed wire with a yield strength of SD400 was used for transverse reinforcement in all specimens except for Type A specimen. The material test results for the concrete and steel are shown in Tables 2. and 3.

Table 2. Material Properties of Concrete (Unit: MPa)

| Type | Modulus of  
| elasticity | Compression strength | Split strength | Poisson's Ratio |
|------|-------------------|-------------------|----------------|-----------------|
| 28 days | $3.82 \times 10^4$ | 105.64 | 4.32 | 0.1763 |
3.3 Test method

Testing was conducted using a universal test machine with 10,000 kN capacity. An axial load was applied in advance while lateral force devices were fixed, and a cyclic lateral force was applied to specimens using a 1000 kN actuator (Fig.2.). The yield displacement was estimated based on material tensile test results. One stage corresponds to two cyclic loadings, and the test was considered complete when maximum lateral forces were reduced to 50% of the maximum force of each specimen (see Fig.3.).

4. Experimental Results and Discussion

4.1 General behavior and failure

Most of the column specimens showed very similar behavior at an early loading stage. The following sequential events until failure were observed (Fig.4.): (1) flexural cracks occurred along the length of the column section \(d\) from the stub front and back surface; (2) flexural cracks propagated toward the side; (3) additional flexural cracks occurred on the front side, and diagonal side cracks occurred at \(d/4\)–\(2d\); (4) spalling of the concrete cover was observed at 0–2.5d, and concrete fracture occurred at the jointed area; (5) the core concrete was crushed, and resistance strength was reduced; (6) transverse reinforcement bending occurred with failure of the core concrete; (7) and the longitudinal bars failed. Sudden spalling of the concrete cover occurred and a large number of cracks appeared just before or at maximum loading. As the displacement increased, the column specimens behaved differently depending on their axial load ratio and the volumetric ratio of transverse reinforcement.

In particular, there was a marked difference in the axial load ratio between the 3N and 5N series. In the 3N series, most failures were concentrated at a distance of \(d/2\) from the joint face of the stub, and flexural cracks and spalling of the concrete occurred at a column length of 2.5\(d\). In the 5N series, most failures occurred at a distance of 3\(d/4\) from the joint face of the stub, and although the failure areas were not much wider than those seen in the 3N series, the cracks were more severe.

4.2 Hysteretic response

Table 4. shows the test and analysis results. Hysteretic curves for each specimen are presented in Fig.5. Under the same conditions, as the axial load ratio increased, the lateral force capacity increased, but lateral deformability decreased. Furthermore, as the volumetric ratio of the transverse reinforcement increased, the columns behaved in a more ductile and therefore safe manner. In particular, columns with an inner hoop for the circular shape were the most ductile.

4.3 Discussion on axial load effect

Figs.5.(d), (e), (g), and (h) show the lateral force-displacement curves for different transverse reinforcement configurations (Types D and E).
The amount of transverse reinforcement was about 1.0 times $\rho_{ACI}$ based on the ACI code, but the axial load ratios were about 0.3 and 0.5 times that of $P_0$, respectively. The specimens presented in Figs.5.(d) and (g) showed ductile behavior with large lateral deformation until failure under low axial load (3 N), even though the transverse reinforcement did yield. However, under a high axial load (5 N), the specimens in Figs.5.(e) and (h) showed sudden degradation in the lateral force, and the resistance dropped to about 50% of the maximum force with a small lateral deformation.

These results indicate that the axial load level should be taken into consideration when calculating the amount of transverse reinforcement required for a particular transverse reinforcement configuration.

4.4 Volumetric ratio of transverse reinforcement

Increasing the volumetric ratio of the transverse reinforcement resulted in more ductile behavior. As shown in Fig.5.(f), specimen D-13-3N with 1.3 $\rho_{ACI}$ showed the most ductile behavior without any strength degradation compared to D-10-3N and E-10-3N (Figs.5.(d) and (e)), both of which had 1.0 $\rho_{ACI}$.

These results were due to the fact that when lateral force is applied, if the volumetric ratio of the transverse reinforcement is sufficient to support the longitudinal bar in the transverse direction and confine the core concrete, the confinement effect of the core concrete is increased, which results in more ductile behavior.

4.5 Transverse reinforcement configurations

Column behavior can also be influenced by the configuration, diameter, spacing, and yield strength of the transverse reinforcement when the volumetric ratio is held constant. In this study, when columns were designed with the same transverse reinforcement volumetric ratio, specimen A-07-3N, which had no inner hoop (Fig.5.(a)), behaved unsafely, while specimen B-07-3N (Fig.5.(b)), which did, behaved in a ductile manner. For specimens with a similar volumetric ratio of transverse reinforcement, specimen D-10-3N (Fig.5.(d)), which had an octagonal-shaped inner hoop, showed more ductile behavior than C-10-3N, which had a diamond-shaped inner hoop. Thus, ductility is also dependent on the shape of the inner hoop and arrangement of the longitudinal bars, both of which may function to prevent stress concentration. Therefore, the configuration of transverse reinforcement should also be considered when designing them for ultrahigh-strength concrete columns.

4.6 Evaluation of ductility capacity

The ductility ratios of the columns showed a linear trend when plotted (Fig.6.). Hence, the displacement ductility ratio increased as the configuration of the inner hoops became more complicated and the volume ratio increased. However, the displacement ductility ratio for the 5N series with high axial load ratio appeared to decrease even though the volume ratio of the transverse reinforcement was as specified by the ACI codes.

Therefore, when the axial load ratio is 50%, transverse reinforcement 30% more than currently specified in the ACI codes is required to ensure the same ductile behavior as seen when a 30% axial load ratio is applied. Furthermore, these results indicate that
an appropriate volume ratio of transverse reinforcement is very effective at conferring sufficient displacement ductility to ultrahigh-strength concrete when appropriate configurations of transverse reinforcement and axial load ratios are used (see Fig.6.).

5. Evaluation of Volumetric Ratio of Transverse Reinforcement

In this study, our goal was to identify experimental variables affecting the ductile behavior of reinforced concrete column members; we considered various axial load ratios and configurations of transverse reinforcement based on those specified by building codes and previous studies.

By analyzing ultrahigh-strength concrete specimens with a displacement ductility ratio greater than 4, which is equivalent to a strength of greater than 70 MPa, we provide equations for each axial load ratio. The capacity of transverse reinforcement \( (\rho_{sh} \cdot f_{hy}/f_{ck}) \) to withstand stress according to each axial load ratio was calculated, and reduction factors based upon the number of effective legs of transverse reinforcement were calculated to take the configuration of the transverse reinforcement into consideration.

Furthermore, we suggest amounts of transverse reinforcement that should be used to ensure sufficient ductility; these are based upon an analysis of the ductile capacity according to axial load level (divided into low and high axial loads based on the criterion of a 0.4 axial load ratio).

5.1 Equivalent lateral confining pressure

Based upon the static equilibrium state of lateral confining pressure, the nominal lateral confining pressure applied to the core concrete through transverse reinforcement can be described as the following lateral confining pressure \( f_{l} \) (Eq. 1). In addition, as shown in Fig.7, the lateral confining pressure, which is affected by the configuration of transverse reinforcement confining the core concrete, varies depending on the effective legs of transverse reinforcement.

\[
f_{l}^{'} = f_{cc}^{'} + k_{1} f_{1}
\]

where \( f_{cc}^{'} \) = Concrete strength confined by members
\( f_{cc}^{'} \) = Concrete strength not confined by members
\( k_{1} \) = Factor corresponding to lateral confining pressure

As described in Figs.7 and 8., the distribution of lateral confining pressure experienced by the actual confined concrete differs according to the cross-sectional shape, distribution of longitudinal bars, and distances between longitudinal bars; these are dependent on whether the bars are supported laterally.

In particular, the distribution of the lateral confining pressure is characterized by non-linear unequal distribution between laterally supported longitudinal bars and transverse reinforcement spacing.

To calculate the equivalent lateral confining pressure \( f_{le} \), the average confining pressure \( f_{l} \) was assumed to have an equivalent distribution in the transverse
reinforcement level, as shown in Fig. 8. The actual lateral confining pressure has an irregular distribution, thus reducing the average pressure applied to the critical section in the vertical column direction, i.e., the critical section to which the least lateral confining pressure is applied. The equation to calculate the equivalent lateral confining pressure is as follows (Eq. 2):

\[ f_{le} = \lambda_0 f_l \]  

where  
\[ f_{le} = \text{Equivalent lateral confining pressure} \]  
\[ f_l = \text{Average lateral confining pressure} \]  
\[ \lambda_0 = \text{Lateral confining reduction factor} \]

5.2 Definition of effective lateral confining pressure reduction factor

The reduction factor in Eq. 2 is the ratio of the effective confined area to the confined core area, and it can be calculated from the compressive stress under axial loads. Effective confined areas were first proposed by Sheikh and Uzumeri\(^8\) as shown in Eq. 3, and they were then applied by Mander and Cusson-Paultre\(^11\).

\[ \lambda^e = \left(1 - \frac{n c^2}{6 B^2} \right) \left(1 - \frac{s}{3 B} \right)^2 \]  

where  
\[ n = \text{number of longitudinal bar supported laterally} \]  
\[ c = \text{center-to-center distance between longitudinal bars} \]  
\[ B = \text{size of core concrete section} \]  
\[ s = \text{spacing of transverse reinforcement} \]

However, the lateral confining reduction factor (\(\lambda^o\)) was reset in this study to evaluate the confinement effects of transverse reinforcement under lateral force since this is the only case where axial loads are applied. Previous studies investigated the effects of different configurations of transverse reinforcement on the lateral confining pressure of transverse reinforcement confining core concrete (Fig. 9.).

To evaluate the degree of lateral confining pressure according to the configuration of the transverse reinforcement, the effective legs were taken into consideration, and a new effective lateral confining reduction factor \(\lambda^c\) was deduced from the experimental results. These results are presented in Table 5. The number of effective legs based upon the configuration of the transverse reinforcement was applied to the reduction factor under axial loads using the basic configuration (Type A), as indicated in Table 5. Furthermore, Table 5 shows the average values of the effective lateral confining reduction factor \(\lambda^c\). These were calculated based on the values of reduction factors corresponding to Categories 1–4. In this calculation, a regression analysis of the ductility ratio based upon the configurations of transverse reinforcement was performed. Tge Specimen A-07-3N had an effective lateral confining reduction factor and configuration of transverse reinforcement that yielded a displacement ductility ratio close to 4 (3.79). The effective reduction factors for confining the pressure experienced by the core concrete of columns subjected to an axial load and lateral force are presented in Table 5.

<table>
<thead>
<tr>
<th>Specify</th>
<th>Category 1</th>
<th>Category 2</th>
<th>Category 3</th>
<th>Category 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective legs</td>
<td>2</td>
<td>3</td>
<td>3.414</td>
<td>3.61</td>
</tr>
<tr>
<td>Reduction factor ((\lambda^c))</td>
<td>0.842</td>
<td>0.741</td>
<td>0.732</td>
<td>0.776</td>
</tr>
<tr>
<td>(\lambda^c = \lambda^0 \times)</td>
<td>1.684</td>
<td>2.222</td>
<td>2.499</td>
<td>2.802</td>
</tr>
<tr>
<td>Effective legs</td>
<td>1</td>
<td>1.320</td>
<td>1.484</td>
<td>1.664</td>
</tr>
</tbody>
</table>

5.3 Volumetric ratio of transverse reinforcement

Next, we derived equations for the transverse reinforcement amounts required to satisfy a displacement ductility ratio of greater than 4 for reinforced concrete columns subjected to axial loads and lateral forces. Furthermore, the maximum and minimum values of the relationship between the axial load \(\eta\) and lateral confining pressure \(\rho_{\sigma} f_{hy} / f_{ck}\) were input into a selected database. Specimens with a
displacement ductility ratio of greater than or equal to 4 and a concrete compression strength of 70 MPa are shown in Fig.10. The relationship between the axial load ratio and amount of transverse reinforcement can be calculated according to Eqs. 4–6:

\[
\rho_{sh} = (0.385\eta + 0.162)f_{ck}/f_{yh} \tag{4}
\]
\[
\rho_{sh} = (0.202\eta + 0.129)f_{ck}/f_{yh} \tag{5}
\]
\[
\rho_{sh} = (0.108\eta + 0.050)f_{ck}/f_{yh} \tag{6}
\]

According to our experiment results, the factor with the greatest effect on the behavior of columns was the axial load ratio. Approximately 30% more transverse reinforcement was required to obtain the same ductility in reinforced concrete columns subjected to axial loads greater than \(0.4f_{ck} A_g\) compared to axial loads less than \(0.4f_{ck} A_g\). This result indicates that the current ACI Code equation for calculating transverse reinforcement amounts, which is indiscriminately applied to all cases without considering the axial load level, does not ensure sufficient ductility in columns.

However, if the amount of transverse reinforcement to be used is calculated by taking the axial load level into consideration based on low and high axial loads, then the resulting columns will have sufficient ductility capability.

To derive more appropriate equations, the relationship between the axial load ratio and transverse reinforcement amount was analyzed by classifying axial loads as low or high axial loads. The results are presented in Eqs. 7 and 8:

- When the axial load ratio is greater than 0.4:
  \[
  \rho_{sh} = (0.38\eta + 0.05)f_{ck}/f_{yh} \tag{7}
  \]
- When the axial load ratio is less than 0.4:
  \[
  \rho_{sh} = (0.21\eta + 0.1)f_{ck}/f_{yh} \tag{8}
  \]

After using Eqs. 7 and 8 and plotting the results (Fig.11.), the distribution of trend lines is clearly not consistent at an axial load ratio 0.4. Therefore, these equations were revised by adding the effective reduction factor \(\lambda^c\) to better represent the lateral confining pressure \(\rho_{sh} f_{cy}/f_{ck}\), leading to Eqs. 9–11 (Fig.12.).

\[
\rho_{sh} = (0.246\eta + 0.135)f_{ck}/f_{yh} \cdot 1/\lambda^c \tag{9}
\]
\[
\rho_{sh} = (0.138\eta + 0.110)f_{ck}/f_{yh} \cdot 1/\lambda^c \tag{10}
\]
\[
\rho_{sh} = (0.077\eta + 0.062)f_{ck}/f_{yh} \cdot 1/\lambda^c \tag{11}
\]

Equations 12 and 13 were derived from the results of regression analyses of the relationship between the transverse reinforcement amount and different axial load ratios, which were divided into low and high axial load based on an axial load ratio of 0.4\(f_{ck} A_g\).

- When the axial load ratio is greater than 0.4:
  \[
  \rho_{sh} = (0.55\eta + 0.05)f_{ck}/f_{yh} \cdot 1/\lambda^c \tag{12}
  \]
- When the axial load ratio is less than 0.4:
  \[
  \rho_{sh} = (0.36\eta + 0.12)f_{ck}/f_{yh} \cdot 1/\lambda^c \tag{13}
  \]

Thus, there are multiple ways to derive transverse reinforcement amounts that meet the safety requirement of a displacement ductility ratio greater than 4. To more accurately calculate the confining pressure of the

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core concrete, the effective lateral confining pressure \( \lambda_c \cdot \rho_{sh} \cdot f_{hy} / f_{ck} \) was used. The configuration and spacing of the transverse reinforcement were also considered by incorporating the effective reduction factor \( \lambda_c \) and axial load ratio into calculations.

Equations 12 and 13 indicate that a minimum displacement ductility capacity greater than 4 ensures ductile behavior for reinforced concrete columns. Furthermore, our equations for calculating the amount of transverse reinforcement required are likely to be more accurate because they take into consideration factors such as the increase in the axial load ratio, the configuration and spacing of transverse reinforcement, and the distribution of longitudinal bars.

6. Conclusions

The major findings of this study are as follows:

1) A reasonable method to enhance the strength and ductility of tied columns is to increase the amount of transverse reinforcement confining the core concrete and to take the transverse reinforcement configuration into account when designing the columns.

2) In order to achieve a ductile behavior similar to that of specimens under an axial load ratio of 30% with transverse reinforcement amount meeting ACI codes requirements, 30% more transverse reinforcement amount was required than that specified by the ACI codes when the axial load ratio was 50%.

3) We propose equations to calculate the amount of transverse reinforcement required for a tied column that take the axial load ratio and transverse reinforcement configuration into account and should ensure the ductile behavior of the reinforced concrete columns under seismic stresses.

Acknowledgements

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