Three-dimensional Hydrodynamic Analysis of Forelimb Propulsion of Sea Turtle With Prosthetic Flippers

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Abstract—This study is to develop prosthetic flippers for an injured sea turtle named “Yu” from the viewpoint of 3D (three-dimensional) hydrodynamic analysis of sea turtles’ forelimb propulsion. Firstly template matching method is used to compare the 3D movements of fore flippers in three cases respectively: those of a healthy turtle, those of Yu with and without prosthetic flippers. Secondly 3D hydrodynamic analyses for three cases based on quasi-steady wing element theory are carried out to investigate the hydrodynamic effects of prosthetic flippers on the swimming performance of sea turtles. Finally the hydrodynamic effects are clarified and some remarks for designing new prosthetic flippers in future are given.

Index Terms—forelimb propulsion of sea turtle, prosthetic flipper, template matching method, wing element theory

I. INTRODUCTION

An injured female loggerhead sea turtle (Caretta caretta) named Yu was found and rescued by Sea Turtle Association of Japan at Kiisuido in the summer of 2008. Her swimming speed was just 60% of that of healthy adult sea turtle because only a half of the left forelimb and two thirds of the right forelimb were left after being attacked by a shark. Realizing that we could not put her back into the sea under such a condition, “Yu Project” has begun since 2009 to develop prosthetic flippers forYu cooperated with veterinarians, a prosthetic company, aquariums, universities and a public administration for Yu. During this process studying the kinematics of sea turtles and evaluating the effects of prosthetic flippers on the swimming performance of Yu become an important job.

Turtle species exhibit a diversity of kinematic patterns in their forelimbs during swimming. Generally speaking, the flapping forelimb strokes are usually used by swimming marine turtles and the rowing forelimb strokes are usually used by most freshwater turtles, which have been documented in an extensive range of previous studies [e.g. 1, 2, 3, 4, 5]. Flapping strokes are characterized by predominantly drosorial forelimb movements, whereas rowing strokes are characterized by predominantly anteroposterior forelimb movements combined with rotation of the foot (perpendicular to flow during thrust and feathered during recovery) [6]. But specifically speaking, turtle species display considerable diversity in their styles of forelimb flapping or rowing. So quantifying the exact forelimb kinematics and the corresponding thrust forces during turtles’ swimming is a key, which is a significant challenge because direct measurements of force generated by the free turtles’ swimming are not feasible.

Davenport et al. estimated the thrust force by attaching a force transducer to the shells of turtles [2]. But this still puts some restriction on the free swimming of turtles. Walker et al. documented the changes in velocity and acceleration in aquatic locomotion by tracking the center of mass of an animal through an artificial locomotor cycle [7]. Some other researchers tried to obtain the thrust force by examining the properties of the flow field around the aquatic animals. For example Drucker et al. employed digital particle image velocimetry (DPIV) to examine the vortex wake shed by freely swimming fish and then evaluated the thrust forces [8]. Our team concentrated on directly observing the forelimb movements of sea turtles and calculating the corresponding hydrodynamic forces. Previously Isobe et al. [9] compared the 2D and 3D motions of fore flippers between Yu itself, Yu equipped with prosthetic flippers and “Sho” (a healthy sea turtle) in a pool of an aquarium, an artificial lagoon and a water circulating tank. At the same time assuming the flipper consist of a rigid wing and uniform flapping, rowing and feathering motion from root to tip, they analyzed 2D hydrodynamic characteristics of different flippers under uniform flow. But taking into account the 3D motion of fore flippers, we thought 2D analysis cannot accurately evaluate the swimming performance of sea turtles. In our following 3D analyses of forelimb motions, the flipper is treated as flexible in spanwise direction and consists of several wing segments with
different flapping, rowing and feathering motion from root to tip.

II. MOVEMENTS ANALYSIS

A. Sea Turtles

Table 1 and Fig.1 show the details of the sea turtle “Yu” and “Sho”.

<table>
<thead>
<tr>
<th>Name</th>
<th>Carapace length</th>
<th>Body mass</th>
<th>Area (left flipper)</th>
<th>Area (right flipper)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Yu</td>
<td>0.797 [m]</td>
<td>100.5 [kg]</td>
<td>0.039 [m²]</td>
<td>0.026 [m²]</td>
</tr>
<tr>
<td>Sho</td>
<td>0.751 [m]</td>
<td>84.8 [kg]</td>
<td>0.063 [m²]</td>
<td>0.067 [m²]</td>
</tr>
</tbody>
</table>

B. Prosthetic Flippers

The prosthetic flippers are made of a kind of copolymers by Kawamura Gishi Co. Ltd. The left and right flippers for “Yu” are shown in Fig.2. At first we let Yu put on the specially designed jacket (Fig.3). And then the prosthetic flippers are installed onto the corresponding forelimb of Yu. Finally Velcro tape is used to tighten the prosthetic flippers around each sleeve of the jacket. Honestly speaking, the present shape of flippers and the procedure of installing prosthetic flippers onto the forelimb are the results of many trials and errors in the past. The specially designed jacket has the function of avoiding the prosthetic flippers coming off the forelimb.

C. Background of Motion Analysis

A motion capturing software using template matching method was utilized to observe forelimb movements in the form of time variation of the prescribed points on the forelimbs. As Sho is healthy and possesses symmetric fore flippers, we will only analyze the movements of its left fore flipper during swimming. But as for the case of Yu, because its actual left flipper and actual right flipper show different shapes, we have to analyze the movements of both flippers in the following context. Locations of the targeted points on Sho’s left flipper, Yu’s flippers and the prosthetic flippers are shown in Fig.4, Fig.5 and Fig.6 respectively. Two targeted points located in the spanwise middle of the flipper are used to obtain the feathering motion of the flipper. The body fixed coordinate during motion analysis is defined as O-XYZ in Fig.17.

D. Experimental Condition
Three videos were taken simultaneously from the left side, the right side and the upper side of Sho, Yu with and without prosthetic flippers at the water tank of Suma Aqualife Park KOBE.

E. Results of Movement Analysis

Fig.7 shows the trajectories of the motion captured points on the left flipper of Sho in x-y plane of the body fixed coordinate. The period of motion is 3.0s. Fig.8 and Fig.9 show the trajectories of the motion captured points on the left and right flippers of Yu without prosthetic flippers in x-y plane of the body fixed coordinate separately. Fig.10 and Fig.11 show the trajectories of the motion captured points on the left and right prosthetic flippers of Yu in x-y plane of the body fixed coordinate separately. First of all it is observed that Yu swims with a bigger frequency of flipper movements than Sho does, but if the prosthetic flippers are installed, the flipper movement frequency of Yu decreases. From the five figures we can see that the flippers of Sho and Yu describe a circular arc with large curvature in both the power stroke (from anterior position to posterior position) and the recovery stroke (from posterior position to anterior position). On the other hand, in the case of Yu equipped with prosthetic flippers, the trajectory of the flippers in x-y plane is a circular arc with small curvature.

Fig.12 shows the trajectories of the motion captured points on the left flipper of Sho in x-z plane of the body fixed coordinate. Fig.13 and Fig.14 show the trajectories of the motion captured points on the left and right flippers of Yu in x-z plane of the body fixed coordinate. Fig.15 and Fig.16 show the trajectories of the motion captured points on the left and right prosthetic flippers of Yu in x-z plane of the body fixed coordinate. The trajectories of the flippers of Sho and Yu are ovals but in the case of Yu equipped with prosthetic flippers the trajectory of the flippers is similar to an oval with twist at the posterior position for the left prosthetic flipper and at the middle position for the right prosthetic flipper.

Fig.7 Trajectories of motion captured points on the left flipper of Sho in x-y plane (period: 3s)
(The arrows in Fig.7-16 denote trajectory direction with time variation)
III. METHOD FOR 3D ANALYSIS

A. Coordinates Definition

Altogether four coordinates are defined during hydrodynamic analysis (Fig. 17).

\( O_{XYZ} \) : We set the front point of the carapace as the origin and the centerline of the carapace as the X-axis.

\( o_{xyz} \) : The origin is located at the leading point of one cross-section of the forelimb. The three axes are parallel to the axes of \( O_{XYZ} \) respectively.

\( o_{xyz'} \) : The origin is located at the leading point of the same cross-section of the forelimb. \( o_{xyz'} \) can be obtained by rotating \( o_{xyz} \) around axis \( o_{z} \) with \( \gamma \).

\( o_{xyz''} \) : The origin is also located at the leading point of the same cross-section. Plane \( o_{xyz''} \) coincides with the cross-section. \( o_{xyz''} \) can be got by rotating \( o_{xyz'} \) around \( o_{x} \) with angle \( \phi \).

\[
\begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} = \begin{pmatrix}
  X \\
  Y \\
  Z
\end{pmatrix} = \begin{pmatrix}
  X_o \\
  Y_o \\
  Z_o
\end{pmatrix} \quad \ldots \text{(Eq. 1)}
\]

\[
\begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix} = R_x(\gamma) \begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} = \begin{pmatrix}
  \cos \gamma & \sin \gamma & 0 \\
  -\sin \gamma & \cos \gamma & 0 \\
  0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
  x \\
  y \\
  z
\end{pmatrix} \quad \ldots \text{(Eq. 2)}
\]

\[
\begin{pmatrix}
  x'' \\
  y'' \\
  z''
\end{pmatrix} = R_x(\phi) \begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix} = \begin{pmatrix}
  1 & 0 & 0 \\
  0 & \cos \phi & \sin \phi \\
  0 & -\sin \phi & \cos \phi
\end{pmatrix} \begin{pmatrix}
  x' \\
  y' \\
  z'
\end{pmatrix} \quad \ldots \text{(Eq. 3)}
\]
are uniquely determined separately. Let us take the division of Sho’s left flipper as example (Fig.18).

The values of \( \gamma \) and \( \varphi \) of Section Lroot-LI2-LI1 can be calculated by considering the direction of Triangle Lroot-LI2-LI1, which can be assumed to be included in the representative surface of this section. For the other three sections the same method is adopted.

Next we concentrate ourselves on only one section to get the values \( \gamma \) and \( \varphi \) for that section (Fig.17). Line segment AB denotes the chord of one cross-section (L11-L12, L1M-LM2 or L01-LO2 in Fig.18) that lies in the plane \( o^* - x^* z^* \), and that C denotes one point (Lroot, LIM, LOM or Ltip in Fig.18) of the triangles above.

In coordinate \( O - XYZ \), the values of \( A(X_A, Y_A, Z_A) \), \( B(X_B, Y_B, Z_B) \) and \( C(X_C, Y_C, Z_C) \) can be obtained by movement analysis. Then the expression of surface ABC can be given as follows:

\[
X - X_A \quad Y - Y_A \quad Z - Z_A \\
X_B - X_A \quad Y_B - Y_A \quad Z_B - Z_A \\
X_C - X_A \quad Y_C - Y_A \quad Z_C - Z_A
\]

This determinant can be expanded as follows:

\[
(y_B z_C - y_C z_B) x - (x_B z_C - x_C z_B) y + (x_B y_C - x_C y_B) z = 0
\]

Therefore \( \mathbf{m} \), a normal vector to surface ABC should have the following expression:

\[
\mathbf{m} = (y_B z_C - y_C z_B, x_B z_C - x_C z_B, x_B y_C - x_C y_B)
\]

On the other hand, the unit vector of axis \( o^* - y^* \) is:

\[
\hat{n}(o^{*} - y^{*}) = (0, 1, 0)
\]

The expression of this unit vector in coordinate \( o - xyz \) can be got by using (Eq.2) and (Eq.3).

\[
\hat{n}(o - yz) = R_{z}^{-1}(\varphi) R_{y}(\gamma) \begin{pmatrix}
0 \\
1 \\
0
\end{pmatrix}
\]

If we assume \( o^* - y^* \) is in the plane ABC, we have that \( \hat{n} \) is perpendicular to \( \mathbf{n} \):

\[
\hat{n} \cdot \mathbf{n} = 0
\]

Because AB is also perpendicular to \( \hat{n} \), we also have

\[
\mathbf{AB} \cdot \mathbf{n} = 0
\]

Substituting (Eq.7) and (Eq.8) into (Eq.9) and (Eq.10), finally we can obtain that

\[
\tan \gamma = \frac{y_B (x_B y_C - x_C y_B) - z_B (x_B z_C - x_C z_B)}{x_B (y_B y_C - y_C y_B) - z_B (y_B z_C - y_C z_B)}
\]

\[
\tan \varphi = \frac{x_B \sin \gamma - y_B \cos \gamma}{z_B}
\]

Up to here, the values of \( \gamma \) and \( \varphi \) can be obtained.

C. Hydrodynamic Force Calculation

In the water the fore flippers serve as wings. The movement of forelimbs is unstable, but here we use two-point hinge oscillating wing theory, which is a quasi-steady theory developed by Nagai [10].

There are altogether three important forces during hydrodynamic analysis: lift force, drag force and added mass force. During the calculation of lift and drag force we modify 2D wing theory by using lifting line theory. The vortex wake consists of streamwise vortices due to the spanwise circulation gradient and transverse vortices due to the variation of the circulation with time. Given the large aspect ratio of flippers and the low frequency of flipper motion, we assume the transverse vortices are very small, the flow within each chord-wise cross-section of the flipper can be dealt with 2D wing theory and 3D effect is only considered by the difference of induced velocities and induced angles of attack. As for the calculation of added mass, given the density of water is much larger than that of air, we chose to obtain the corresponding value of each segment along the spanwise direction without any assumption on the flow because of its simplicity. By this way we simplified the hydrodynamic calculation of sea turtles’ flippers.

Because the chord length of cross-sections vary greatly in different position of the forelimbs, in order to reflect this variation we subdivide each section of the flipper in the previous part of this article into three segments separately, which is shown in (Fig.18).

Considering each subsection as a regular 3-D wing, each subsection can be represented with the middle chord length of each subsection as the representative chord length of the 3-D wing, with the width of each subsection as the span length of the 3-D wing.

For each segment the two-point hinge oscillating wing theory is adopted to simulate crosssection motion, which is coupled with heaving and pitching motions. Then we can calculate the corresponding thrust in each segment. Finally the resultant force can be obtained by adding all the thrust forces in each subsection.

In one cross-section of the fore flipper, schematic compositions of velocities are given in Fig.19.

![Fig.19 Velocity compositions in one cross-section](image-url)

Here, the origin \( o^* \) is actually the leading point of each cross-section. \( U_x^* \) and \( W_x^* \) are the incoming flow velocity components in the \( o^* - x^* \) and \( o^* - y^* \) direction.

Here we use the motion of one quarter point on the chord of wing to represent wing movement. Assuming that \( (x_q^*, z_q^*) \) is the coordinate of the leading point in \( o^* - x^* z^* \), the coordinate of the one-quarter point on the chord of wing can be given as:

\[
A_{q4} = x_q^* + \frac{c}{4} \cos \beta \\
z_q^* = z_q^* + \frac{c}{4} \sin \beta
\]

Then the velocity at the one-quarter point can be got:
\[ u_{t1}^* = \frac{dx_{t1}^*}{dt} = u_t^* + d \left( \frac{c_0}{4} \cos \beta \right) \]

\[ w_{t1}^* = \frac{dz_{t1}^*}{dt} = w_t^* + d \left( \frac{c_0}{4} \sin \beta \right) \]

\[ T^* = -L^* \sin \left( \alpha_i \right)_i/4 + D^* \cos \left( \alpha_i \right)_i/4 - F_n^* \sin \beta \]  \( \text{(Eq.21)} \)

\[ Z^* = L^* \cos \left( \alpha_i \right)_i/4 + D^* \sin \left( \alpha_i \right)_i/4 + F_n^* \cos \beta \]

According to (Eq.1) and (Eq.4), the expressions of lift force and vertical force in coordinate \( o-xyz \) and \( O-XYZ \) can be obtained:

\[ T = T^* \cos \gamma + Z^* \sin \gamma \sin \phi, \quad Z = Z^* \cos \phi \]  \( \text{(Eq.22)} \)

T is thrust force along O-X direction and Z is vertical force along O-Z direction.

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**D. The Determination of \( \beta \)**

During previous part, parameters except \( \beta \) were introduced. So in this part the process of how to get the value of \( \beta \) will be explained.

In Fig.21, we assume that \( AB \) is the chord of one cross-section. \( BE \) and \( BF \) are perpendicular to plane \( o-xy \) and \( o-xz \) respectively. \( G \) is the intersecting point between plane \( BEF \) and line \( o-x \). We assume \( \angle FAG = \delta \) and \( \angle EAG = \psi \). \( \delta \) and \( \psi \) are defined as the upper view projecting value and the side view projecting value separately of the angle between the line of the flipper located at the spanwise middle (line segment \( AB \) in Fig.4) and the carapace line (line segment \( OX \) in Fig.4). The values of \( \delta \) and \( \psi \) can be obtained by movement analysis.

Assuming that the coordinates of point \( B \) is \( (x_B, y_B, z_B) \), we can get the relations below:

\[ y_B = x_B \tan \psi, \quad z_B = x_B \tan \delta \]

Then the vector parallel to \( OB \) can be written as:

\[ \vec{c}_{o-oxyz} = (1, \tan \psi, \tan \delta) \]

According to (Eq.2) and (Eq.3), the expression of \( \vec{c} \) at coordinate \( o' - x'y'z' \) can be calculated:

\[
\begin{align*}
\hat{c}_{o' - x' y' z'} &= \begin{pmatrix}
\cos \gamma + \sin \gamma \tan \psi \\
-\sin \gamma \cos \phi + \cos \gamma \cos \phi \tan \psi + \sin \phi \tan \delta \\
\sin \gamma \sin \phi - \cos \gamma \sin \phi \tan \delta + \cos \phi \tan \delta
\end{pmatrix} \\
&= \begin{pmatrix}
\cos \gamma \cos \phi + \sin \gamma \sin \phi \tan \psi \\
\sin \gamma \sin \phi - \cos \gamma \cos \phi \tan \psi - \sin \phi \tan \delta \\
\sin \gamma \cos \phi - \cos \gamma \sin \phi \tan \psi - \cos \phi \tan \delta
\end{pmatrix}
\end{align*}
\]  \( \text{(Eq.23)} \)

\( \beta \) can be considered as the angle between \( \vec{c}_{o' - x' y' z'} \) and \( o' - x' \). And then the value of \( \beta \) can be obtained:

\[ \beta = \arccos \left( c_{x'} / \sqrt{c_{x'}^2 + c_{y'}^2 + c_{z'}^2} \right) \]  \( \text{(Eq.24)} \)
IV. RESULTS AND DISCUSSIONS

A. Hydrodynamic Analysis of Sho’s left flipper

Using the theoretical analysis mentioned above, we discuss here the relationship between drag force acting on the body of sea turtle and thrust force produced by the left forelimb of Sho.

Fig.22 shows time variation of swimming velocity components of Sho. Fig.23 gives the time variation of angle of attacks of the 7th segment of Sho’s left flipper in Fig.18. On the other hand, hydrodynamic drag coefficients on turtle body were referred from the paper by Watson and Granger [13]. We plotted time variation of total thrust force generated by left flipper and body drag force in Fig.24. The average thrust force generated by the left flipper is -0.9873N, the total thrust force can be got as twice of the thrust generated by left flipper and the average body drag force is 2.0114N. Consequently the total thrust force and body drag force can be balanced.

In order to verify the validity of our method in carrying out the hydrodynamic analysis of sea turtle, some further calculations were performed. The total thrust force (twice of the thrust force generated by the left flipper) and the body drag force in several swimming velocities were calculated and then the tendency of both forces were compared (Fig.25). The velocity of the intersecting point in Fig.25 is 0.269m/s and the actual swimming velocity during Sho’s calculation is 0.278m/s. So we can say our method can almost predict the hydrodynamic analysis of sea turtles’ forelimb movements.

B. Hydrodynamic Analysis of Yu’s Forelimbs Without Prosthetic Flippers

Fig.26 shows the time variation of swimming velocity components of Yu with actual flippers. Fig.27 and Fig.28 give the time variation of angle of attacks of the 8th segment for Yu’s actual left flipper and Yu’s actual right flipper. Fig.29 is the time variation of total thrust force generated by both actual flippers and body drag force. The average thrust force generated by left flipper is -0.3789N, the average thrust force generated by right flipper is -0.5381 , and the average body drag force is 0.9228N. Consequently the total thrust force and body drag force can be almost balanced.

The verification calculations in different swimming velocities were also done (Fig.30). The total thrust force is the sum of the thrust force generated by the left and right actual flippers. The velocity of the intersecting point in Fig.30 is 0.203m/s and the actual swimming velocity during Yu’s calculation is 0.187m/s. Although there exist some discrepancies, our method can predict the hydrodynamic analysis of Yu’ forelimb movements.
average thrust force generated by right prosthetic flipper is -1.1661N, and the average body drag is 2.573N. The average thrust and body drag can be balanced.

The verification in different swimming velocities is shown in Fig.35. The velocity of the intersecting point in Fig.35 is 0.268m/s and the actual swimming velocity is 0.293m/s. Although the discrepancy is a little bigger, almost we can say that if installing prosthetic flippers our method can also predict well the hydrodynamic analysis of Yu’s forelimb movements.

C. Hydrodynamic Analysis of Yu’s Forelimbs With Prosthetic Flippers

Fig.31 shows the time variation of swimming velocity components of Yu with prosthetic flippers. Fig.32 and Fig.33 give the time variation of angle of attack of the 7th segment of Yu’s left and right prosthetic flippers. Fig.34 gives the time variation of total thrust force generated by Yu’s both flippers and body drag force with prosthetic flippers. The average thrust force generated by left prosthetic flipper is -1.0399N, the
D. Comparison Among Three Cases

1) Our method can predict well the hydrodynamic characteristics in the case of Sho’s flippers, Yu’s actual flippers and Yu’s prosthetic flippers.
2) Prosthetic flippers play a positive role in generating thrust force. But the thrust generated by right prosthetic flipper is larger than that generated by the left one.
3) The angle of attack of Sho’s left flipper stays in the large lift coefficient area for a long time in one period. But the angle of attack of Yu’s actual left flipper varies largely, and much of it is not in the large lift coefficients area. After installing the prosthetic flippers, the situation of angle of attack was improved.
4) From validation results, we can see for Sho’s case it has good agreement but for Yu’s cases with and especially without prosthetic flippers, there exist some discrepancies. The first reason is from the validation method itself: sea turtles’ swimming velocity is not constant, but during our validation we adopted the relationship of average velocity and average thrust to inspect the validity of our predicting method. And the asymmetry of Yu’s real and prosthetic flippers aggravates this inconstancy of swimming velocities. The second reason may come from that when Yu swims with prosthetic flippers, it cannot suit itself with the appendage and therefore cannot swim regularly, which increase the difficulty of taking good videos.

V. CONCLUSION

To contribute to design and development of prosthetic flippers, we contradistinguished the 3D kinematics and hydrodynamics of three cases and validated that our proposed method can predict well the hydrodynamics of sea turtles’ forelimb motions.

We also tried finding some bijective correspondence between kinematics and propulsive force generation. One is that flipper flexibility plays a great role during the generation of thrust force. Flexible Sho’s flippers and Yu’s actual flippers behave curvilinear motions and can bend actively in both chordwise and spanwise directions, which correspondingly produce effective thrust force. But Yu’s prosthetic flippers behave nearly linear motions and can only passively utilize the flexibility, which to some extent affect the effective generation of thrust.

From the viewpoint of making prosthetic flippers, we can see the thrust generated by right prosthetic flipper is larger than that generated by the left one. So in the future we think it is better to develop new prosthetic flippers or only install the left prosthetic flipper, which can make both flippers generate equal thrust and therefore Yu’s swimming motion will become smooth.

REFERENCE