Active Vibration Control of a Flexible Beam with a Dynamic Damper

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Abstract—Vibration control of a flexible beam with a dynamic damper is investigated in this paper. The beam can rotate around the axis of a motor and active control is applied to the system. Lower two modes of vibration are considered in the dynamic model and the dynamic damper is tuned to suppress the first or second mode. Then an active controller based on the optimal servo system is constructed and the performance of the vibration control is studied.

Key Words: Vibration control, Flexible beam, Dynamic damper

1 Introduction

It is well known that a rotating hub-beam system has many applications in wide engineering fields. If a flexible beam is attached to a moving body, elastic deformations and vibrations may occur in the system. In this paper, active vibration control for the rotating hub-beam system attached to a movable base is studied. Effectiveness of dynamic damper with active control is also investigated.

For the purpose of this study, governing equations of the system are derived by using Hamilton’s principle. Neglecting the nonlinear terms from the equations, we can obtain the linearized equations. Free vibration of the beam is analyzed by using the finite element method (FEM), and the derived equations consist of an infinite number of modes of vibration; but we can’t deal with infinite modes. Therefore we deduce the infinite modes into finite modes.

2 Modeling

2.1 Derivation for the governing equations

Figure 1 shows the coordinate system in a horizontal plane. The local coordinates O-XY, O-x-y and O-xy are the coordinate fixed on the ground, moving body, and flexible beam, respectively. The displacement of the moving body is denoted by $y_a$. Angle $\theta$ is the rotating angle of the hub. The coordinate system in this moving and rotating hub-beam system. Neglecting the nonlinear terms, we can obtain the linearized equation of motion as follows. The equation of motion for the hub is

$$
\rho A \ddot{\theta} + \rho A \dot{y} \dot{\theta} + \rho A \dot{y}^2 \dot{\theta} + \rho \int_0^L (x + r_h) \dot{y}^2 dx + \rho \int_0^L (x + r_h) \dot{y}^2 dx + m_a \ddot{\theta} + m_a \dot{\theta} \dot{y} + m_a \dot{y}^2 \dot{\theta} + m_a \dot{y} \dot{\theta} \dot{y} + m_a \ddot{\theta}
$$

where $c_a$ is a viscous damping coefficient proportional to the angular velocity of the hub. The equation of motion for the beam is

$$
\rho A (x + r_h) \ddot{\theta} + \rho A \dot{y} \dot{\theta} + \rho A \dot{y}^2 \dot{\theta} + \rho \int_0^L (x + r_h) \dot{y}^2 dx + \rho \int_0^L (x + r_h) \dot{y}^2 dx + m_a \ddot{\theta} + m_a \dot{\theta} \dot{y} + m_a \dot{y}^2 \dot{\theta} + m_a \dot{y} \dot{\theta} \dot{y} + m_a \ddot{\theta}
$$

where $E$ is Young’s modulus and $c_h$ is a viscous damping coefficient proportional to the strain velocity of the beam. The motion equation for the dynamic damper is

$$
m_a \ddot{y} + [k_a (y_a - y) + c_a (\dot{y}_a - \dot{y})] \ddot{\theta} (x - l_p) = 0.
$$

The derived equations consist of an infinite number of modes of vibration; but we can’t deal with infinite modes. Therefore we deduce the infinite modes into finite modes. Displacement of the beam can be expressed by using the modal function $\phi_i(x)$ and time function $q(t)$ as

$$
y(x,t) = \sum_{i=1}^n \phi_i(x) q_i(t).
$$

Substituting Eq. (4) into Eq. (1)-(3) in consideration with
orthogonal relationship of the modal functions, following expressions are derived.

\[
\rho M \left\{ \left( x + r_h \right)^2 \ddot{x} + \rho M \sum_{i=1}^{L} \left( x + r_h \right) \phi_i(x) \ddot{q}_i dx \right. \\
\left. + \rho A \int_{0}^{L} \left( x + r_h \right) \cos \theta \cdot \dot{y}_d dx \right. \\
+ m_a \left( l_p + r_h \right)^2 \ddot{\theta} + m_a \left( l_p + r_h \right) \ddot{y}_a \right. \\
+ m_a \left( l_p + r_h \right) \cos \theta \cdot \dot{y}_d \\
+ J_i \ddot{\theta} + \rho A \sum_{i=1}^{L} \left( x + r_h \right) \phi_i(x) \ddot{q}_i dx = \tau,
\]

(5)

\[
\rho A \int_{0}^{L} \left( x + r_h \right) \phi_i(x) dx \ddot{\phi}_i + \rho A \int_{0}^{L} \phi_2 L \phi_i(x) dx i \ddot{q}_i \\
+ \rho A \int_{0}^{L} \phi_1 L \phi_i(x) dx \phi_i(x) \ddot{\phi}_i \\
+ \rho A \int_{0}^{L} \phi_2 L \phi_i(x) dx \phi_i(x) \ddot{\phi}_i \\
+ \rho A \int_{0}^{L} \phi_1 L \phi_i(x) dx \phi_i(x) \ddot{\phi}_i \\
= k_a(y_a \phi_i(l_p) - \phi_i^2(l_p) q_i) \\
+ c_a(y_a \phi_i(l_p) - \phi_i^2(l_p) q_i), \quad (i = 1, 2)
\]

(6)

\[
m_a \left( l_p + r_h \right) \ddot{\theta} + m_a \ddot{y}_a + m_a \ddot{y}_a \cos \theta \\
+ \left[ (k_a y_a - k_n \sum_{i=1}^{L} \phi_i(l_p) q_i) \\
+ (c_a y_a - c_n \sum_{i=1}^{L} \phi_i(l_p) q_i) \right] = 0.
\]

(7)

2.2 Free vibration analysis of the hub-beam system

It is difficult to calculate the model of actual structure of the hub-beam system, because of some parts, like bolts and nuts. Therefore we employ the finite element method (FEM) and simplify each part of them to lumped mass, which attached to the node. The model is shown in Fig.2. The length of the beam from the center of the axis to the free end is 492mm. We divide it into 492 elements with equal length for FEM model. The boundary condition of the beam is shown as follows:

Fig. 2: FEM model for experimental setup

Table 1 the natural frequencies of the experimental beam

<table>
<thead>
<tr>
<th>(i)-th mode</th>
<th>1st</th>
<th>2nd</th>
<th>3rd</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega_0) [rad/s]</td>
<td>7.4808</td>
<td>23.15</td>
<td>55.58</td>
</tr>
</tbody>
</table>

Natural frequencies obtained by FEM are shown in Table 1.

3.2 Motion equation in state space

The equations of motion (5)-(7) can be expressed in the following form

\[
\{ M \} \ddot{z} + \{ C \} \dot{z} + \{ K \} z + \{ F \} = \{ D \} \tau,
\]

(10) where

\[
\{ z \} = [\theta \quad q_1 \quad q_2 \quad y_a \quad \ddot{\theta} \quad \dot{q}_1 \quad \dot{y}_a]^T,
\]

\[
\{ x \} = [\theta \quad q_1 \quad q_2 \quad y_a \quad \ddot{\theta} \quad \dot{q}_1 \quad \dot{y}_a]^T,
\]

If we define

\[
\{ x \} = [A_x] \{ x \} + [B_c] \tau + [F_c] \{ F \},
\]

(12) where

\[
[A_x] = \begin{bmatrix} 0 & 0 \\ -[M]^{-1}[-K] & -[M]^{-1}[-C] \end{bmatrix},
\]

\[
[B_c] = \begin{bmatrix} 0 \\ [M]^{-1}[D] \end{bmatrix},
\]

(13) Suppose the external force \( F_e(x,t) = F_0 e^{j\omega_0 t} \delta(x - l_p) \), we can obtain

\[
y(l_p,t) = \sum_{i=1}^{L} \frac{s_i}{\rho A l_p^2} \int_{0}^{L} \phi_i^2(l_p) dx + \frac{s_i}{\rho A l_p^2} \int_{0}^{L} \phi_i^2(l_p) dx \\
+ \int_{0}^{L} \phi_i^2(l_p) dx + \int_{0}^{L} \phi_i^2(l_p) dx \]

where

\[
s_i = \frac{\phi_i^2(l_p)}{\rho A l_p^2}.
\]

And the equation of motion of the damper system is derived
as

\[ m\ddot{y}_p(t) = -k[y_p(t) - y(l_p, t)] - c[\dot{y}_p(t) - \dot{y}(l_p, t)], \quad (16) \]

The particular solutions of \( y(l_p, t) \) and \( y_d(t) \) are expected to be harmonic, then we assume them in the form

\[
\begin{align*}
\dot{y}(l_p, t) &= \tilde{A}e^{iot + \varphi} \\
y_d(t) &= \tilde{B}e^{iot + \varphi}
\end{align*}
\]

(17)

where \( \tilde{A}, \tilde{B} \) and \( \varphi \) denote the amplitudes and phase angle of the response, respectively. By substituting Eq.(17) into Eqs.(15) and (16), we arrive at

\[
\tilde{A} = F_0 \frac{(k - m\omega^2) + (c\omega)^2}{\left(\frac{k - m\omega^2}{S} - m\omega^2k\right) + (c\omega)^2\left(\frac{1}{S} - m\omega^2\right)^2} \]

(18)

\[
\tilde{B} = F_0 \frac{k^2 + (c\omega)^2}{\left(\frac{k - m\omega^2}{S} - m\omega^2k\right)^2 + (c\omega)^2\left(\frac{1}{S} - m\omega^2\right)^2} \]

(19)

\[
S = \sum_{i=1}^{\infty} \frac{s_i}{\omega_i^2 - \omega^2} \]

(20)

By introducing following expressions,

\[
\omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2\sqrt{mck}},
\]

(21)

Eq.(19) is rewritten as

\[
\tilde{A} = F_0 \frac{(\omega_n^2 - \omega^2)^2 + (2\zeta\omega_n\omega)^2}{\left(\frac{\omega_n^2 - \omega^2}{S} - m\omega^2\omega_n^2\right)^2 + (2\zeta\omega_n\omega)^2\left(\frac{1}{S} - m\omega^2\right)^2} \]

(22)

It can be seen from Eq.(22) that the damping ratio \( \zeta \) only appears in two terms. When \( \omega \) satisfies the following conditional equation

\[
\frac{(\omega_n^2 - \omega^2)^2}{\left(\frac{\omega_n^2 - \omega^2}{S} - m\omega^2\omega_n^2\right)^2} = \frac{1}{\left(\frac{1}{S} - m\omega^2\right)^2},
\]

(23)

the corresponding amplitude \( \tilde{A} \) in Eq.(22) has no relationship with the damping ratio \( \zeta \). We call these points “fixed point”.

For the beam-absorber system, it is considered that the dynamic absorber most efficiently suppresses the \( k_{th} \) resonance when the ordinates of the two successive points of intersection are equal\(^5,6\). The tuning ratio is then chosen so that the two fixed points lie on the same amplitude. When dynamic damper suppresses the first mode, we obtain optimal tuning curve as Fig. 4 which shows the relationship between damper’s optimal tuning and mass of damper.

There may be several ways to give the optimal procedure for damping ratio. For example, one of them is to find the smoothest curve around the peak; another is to find the curve with minimum peak in a series of curves. This study chooses the later procedure. When we suppress the vibration of the first mode, we obtain the curve as Fig.5, which shows the relationship between damper’s optimal damping ratio and mass of damper.

4 Active controller

As described in Section 2, we have the state equation (12) for continuous system. We transform the equation into digital system as follows:

\[
\begin{align*}
\{x(k+1)\} &= [A_d]\{x(k)\} + [B_d]\tau(k) + [F_d]\{F(k)\}, \\
\{y(k)\} &= [C_d]\{x(k)\}.
\end{align*}
\]

(24)

(25)

![Fig.4: Relationship between damper’s optimal natural frequency and mass](image1)

![Fig.5: Relationship between damper’s optimal damping ratio and mass](image2)
Output equation (25) consists of angle of rotation and stain of the beam. We can measure the angle and strain by using rotary encoder and the strain gauge, respectively. In this regard, a state observer can be utilized to estimate the states of the controlled system. The design for observer gain is based on the method of optimal regulator\(^7\). We obtain the feedback torque \(\tau(k)\) by the linear quadric regulator (LQR). The object of LQR is to determine an optimal control minimizing the quadratic performance measure

\[
J = \frac{1}{2} \sum_{k=1}^{\infty} [x^T(k)Qx(k) + \tau^T(k)R \tau(k)],
\]

where \(Q\) is the semi-positive definite matrix.

5 Simulation

The control strategy of this study is based on active and passive control working on the first and second modes, respectively; the active control is based on the method of linear quadratic regulator (LQR) and a dynamic damper is used for the passive control.

5.1 Passive control

Firstly we let the passive control work to the first mode. The mass of the dynamic damper is set as \(m=0.4\)kg. With the study in Section 3, we obtain all the optimal parameters in the Table 2. The response for the first and second modes is shown in Fig.6 as a bode diagram. With passive control, we suppress the peak by 1.9454 times.

Next we let the passive control work to the second mode. In this case, the mass of the damper is set as \(m=0.1\)kg. From Section 3, we obtain the parameters in Table 3. The response for the first and second modes is shown in Fig.7. With passive control, we suppress the peak by 2.316 times.

5.2 Active control

Firstly, we show the results when the active control works to the first mode under steady-state input for the moving body

\[
\dot{y}_d = 1.5 \sin(\omega \ast t), \quad \omega = 48.4 \text{rad/s}.
\]

We set the coefficients in Eq.(26) as

\[
Q = \text{diag}(0.1 \quad 10000000 \quad 1 \quad 0.01 \quad 0.1 \quad 0.1),
\]

\[
R = \text{diag}(10).
\]

Table 2 Optimal parameters of dynamic damper for suppress the first mode

<table>
<thead>
<tr>
<th>Mass of damper</th>
<th>Stiffness of Spring</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4kg</td>
<td>43.681N/m</td>
<td>0.6585</td>
</tr>
</tbody>
</table>

Table 3 Optimal parameters of dynamic damper for suppress the second mode

<table>
<thead>
<tr>
<th>Mass of damper</th>
<th>Stiffness of Spring</th>
<th>Damping ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1kg</td>
<td>254.52N/m</td>
<td>0.35</td>
</tr>
</tbody>
</table>
We compare the results of both first and second mode responses between the systems without and with active control in time history in Fig. 8. We can find that the active control suppresses the first mode well, and also doesn’t give bad effect to the second mode.

Next we show the results when the active control works to the second mode under steady-state input for the moving body

$$\ddot{y}_d = 1.5 \sin(\omega_2 \cdot t), \quad \omega_2 = 299.8 \text{rad/s}.$$
We set the coefficients in Eq.(26) as
\[ Q = \text{diag}(10, 1, 10000000, 0.01, 0, 10), \]
\[ R = \text{diag}(100). \]

The results are shown in Fig. 9. We can find that the active control suppresses the second mode well.

5.3 System controlled by both active and passive control

Our control strategy here is that active control works to the first mode, and passive control works to the second mode. External input to the moving body is
\[ y_d = 1.5 \sin(\omega_2 t), \quad \omega_2 = 299.8 \text{rad/s}. \]

We compare the four cases as follows: ① Without any control; ② With active control for first mode; ③ With passive control for the second mode; ④ With both active and passive control. The results of the first mode are shown in Fig.10, and the results of the second mode are shown in Fig.11.

6 Conclusions

Vibration control of the hub-beam system that is attached to a moving body has been studied in this paper. Combination of the passive control by the dynamic damper and the active control was considered to suppress the first and second vibration modes of the flexible beam. For the purpose of this study, governing equations of the system were derived by using Hamilton’s principle. The active control is based on the method of linear quadratic regulator (LQR). The performance of the passive and active control system was shown by the computer simulation.

References