The parameterization of all robust stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic

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Abstract—This paper proposes the parameterization of all robust stabilizing simple multi-period repetitive controllers for multiple-input/multiple-output time-delay plants with specified input-output characteristic.

Key Words: Multi-period repetitive control, Finite number of poles, Robust Stability, Low-pass filter

1 Introduction

A simple multi-period-repetitive(MPR) control system 1) was proposed by Yamada et al. in order to specify the input-output characteristic and the disturbance attenuation characteristic more easily than MPR control system. Recently, the parameterization of all robust stabilizing simple MPR controllers for multiple-input/multiple-output (MIMO) time-delay plants was proposed 2). However, using this method, it is complex to specify the low-pass filter of which the role is to specify the input-output characteristic.

To specify the input-output characteristic easily, this paper proposes the parameterization of all robust stabilizing simple MPR controllers for MIMO time-delay plants such that the input-output characteristic can be specified beforehand.

2 Problem formulation

Consider the unity feedback control system in

\[
\begin{aligned}
& y = G(s)e^{-sT}u + d \\
& u = C(s)(r - y)
\end{aligned}
\]  

(1)

where \(G(s)e^{-sT}\) is the plant, \(G(s) \in \mathbb{R}^{m \times p}\), \(L > 0\) is the time-delay, \(C(s)\) is the controller, \(u \in \mathbb{R}^p\) is the control input, \(d \in \mathbb{R}^m\) is the disturbance, \(y \in \mathbb{R}^m\) is the output and \(r \in \mathbb{R}^m\) is the periodic reference input with period \(T > 0\) satisfying \(r(t) = r(t + T)\) \((\forall t \geq 0)\). It is assumed that \(m \leq p\) and rank \(G(s) = m\). The nominal plant of \(G(s)e^{-sT}\) is denoted by \(G_m(s)e^{-sTm}\), where \(G_m(s) \in \mathbb{R}^{m \times p}\). Both \(G(s)\) and \(G_m(s)\) are assumed to have no zero or pole on the imaginary axis.

In addition, it is assumed that the number of unstable poles of \(G(s)\) is equal to that of \(G_m(s)\). The relation between \(G(s)e^{-sT}\) and \(G_m(s)e^{-sTm}\) is written as

\[
G(s)e^{-sT} = (e^{-sTm}I + \Delta(s))G_m(s),
\]

(2)

where \(\Delta(s)\) is an uncertainty. The set of \(\Delta(s)\) is all functions satisfying

\[
\sigma\{\Delta(j\omega)\} < |W_T(j\omega)| \quad (\forall \omega \in \mathbb{R}_+),
\]

(3)

where \(W_T(s) \in \mathbb{R}(s)\) is a stable rational function.

The robust stability condition for the plant \(G(s)e^{-sT}\) with uncertainty \(\Delta(s)\) satisfying (3) is given by

\[
\|T(s)W_T(s)\|_\infty < 1,
\]

(4)

where \(T(s) = (I + G_m(s)e^{-sTm}C(s))^{-1}G_m(s)C(s)\).

According to 1), the general form of the MPR controller \(C(s)\) is written in the following form:

\[
C(s) = C_0(s) + \sum_{i=1}^{N} C_i(s)q_i(s)e^{-sT_i}\left(I - \sum_{i=1}^{N} q_i(s)e^{-sT_i}\right)^{-1},
\]

(5)

where \(N\) is an arbitrary positive integer, \(T_i > 0(T_i \in \mathbb{R})\), \(C_0(s) \in \mathbb{R}^{m \times m}\), \(C_i(s) \in \mathbb{R}^{m \times m}\) satisfying rank \(C_i(s) = m\) and \(q_i(s) \in \mathbb{R}^{m \times m}\) are low-pass filters satisfying \(\sum_{i=1}^{N} q_i(0) = I\).

Using the MPR controller \(C(s)\) in (5), transfer functions from \(r\) to \(y\) and from \(d\) to \(y\) in (1) have infinite numbers of poles, even if \(\Delta(s) = 0\). As noted above, the transfer functions need finite numbers of poles to make the input-output and disturbance attenuation characteristics easy to specify.

On the other hand, according to 1), it is note that if low-pass filters \(q_i(s) (i = 1, \ldots, N)\) satisfy

\[
\sigma\left\{I - \sum_{i=1}^{N} q_i(j\omega_k)e^{-j\omega_k T_i}\right\} \approx 0 \quad (\forall \omega_k = \frac{2\pi k}{T}, k = 0, 1, \ldots, n),
\]

(6)

where \(\omega_n\) is the maximum frequency component of \(r\), then \(y\) in (1) follows \(r\) with small steady-state error.

Using the result in 2), in order for \(q_i(s)\) to satisfy (6) in wide frequency range, we must design \(q_i(s)\) to be stable and of minimum phase. To do this, \(q_i(s)\) are desirable to be specified beforehand.

From above practical requirement, we clarify the parameterization of all controllers satisfying following Definition 1.

Definition 1 We call the controller \(C(s)\) a “robust stabilizing simple MPR controller for MIMO time-delay plants with specified input-output characteristic”, if following expressions hold true:

1. \(q_i(s) \in \mathbb{R}^{m \times m}\) in (5) are set beforehand.
2. The controller \(C(s)\) is described by (5).
3. When \(\Delta(s) = 0\), the controller \(C(s)\) ensures that transfer functions from \(r\) to \(y\) in (1) and from \(d\) to \(y\) in (1) have finite numbers of poles.
4. \(C(s)\) satisfies the robust stability condition in (4).
3 The parameterization

To obtain the controller $C(s)$ satisfying (4), we consider the control system shown in Fig. 1. $P(s)$ is

\[ P(s) \]

\[ \begin{array}{c}
    \text{w} \\
    \text{u} \\
    \text{z} \\
    \text{y} \\
    \text{C(s)}
\end{array} \]

Fig. 1: Block diagram of $H_\infty$ control problem

selected such that the transfer function from $w$ to $z$ in Fig. 1 is equal to $T(s)W_T(s)$. The state space description of $P(s)$ is, in general,

\[
\begin{align*}
\dot{x}(t) &= Ax(t) + B_1w(t) + B_2u(t-L_m) \\
\dot{z}(t) &= C_1x(t) + D_{12}u(t) \\
y(t) &= C_2x(t) + D_{21}w(t)
\end{align*}
\]

where $A \in R^{n \times n}$, $B_1 \in R^{n \times m}$, $B_2 \in R^{n \times p}$, $C_1 \in R^{m \times n}$, $C_2 \in R^{m \times p}$, $D_{12} \in R^{m \times p}$, $D_{21} \in R^{m \times m}$, $x(t) \in R^n$, $w(t) \in R^m$, $z(t) \in R^m$, $u(t) \in R^p$ and $y(t) \in R^m$. $P(s)$ is called the generalized plant. $P(s)$ is assumed to satisfy assumptions of standard problem in (3) and $C_1A^TB_2 = 0 \ (i = 0, 1, 2, \ldots)$. Under these assumptions, from (4), following lemma holds true.

**Lemma 1** There exists an $H_\infty$ controller $C(s)$ for the generalized plant $P(s)$ in (7) if and only if there exists an $H_\infty$ controller $C(s)$ for the generalized plant $\hat{P}(s)$ written by

\[
\begin{align*}
\dot{\hat{q}}(t) &= A\hat{q}(t) + B_1w(t) + \hat{B}_2u(t) \\
\dot{\hat{z}}(t) &= C_1\hat{q}(t) + D_{12}u(t) \\
\hat{y}(t) &= C_2\hat{q}(t) + D_{21}w(t)
\end{align*}
\]

where $\hat{B}_2 = e^{-AL_m}B_2$. When $u(s) = C(s)\hat{y}(s)$ is an $H_\infty$ control input for the generalized plant $\hat{P}(s)$ in (8),

\[
u(t) = L^{-1}\{C(s)\hat{y}(s)\}
\]

is an $H_\infty$ control input for the generalized plant $P(s)$ in (7), where

\[
\hat{y}(s) = \mathcal{L}\left\{y(t) + C_2\int_0^t e^{-A(t+\tau)}B_2u(t+\tau)d\tau\right\}.
\]

From Lemma 1 and (3), following lemma holds true.

**Lemma 2** The parameterization of all controllers satisfying (4) is given by

\[
C(s) = C_{11}(s) + C_{12}(s)Q(s)(I - C_{22}(s)Q(s))^{-1}C_{21}(s),
\]

where $C_{ij}(s)(i = 1, 2; j = 1, 2)$ are given by using the method in (3) and $Q(s) \in H_\infty^{m \times m}$ is any function satisfying $\|Q(s)\|_\infty < 1$.

Using Lemma 1 and Lemma 2, the parameterization of all controllers satisfying Definition 1 is given by following theorem.

**Theorem 1** The parameterization of all control laws satisfying Definition 1 is given by (9), where $\hat{y}(s)$ is (10) and

\[
C(s) = (Z_{11}(s)Q(s) + Z_{12}(s))(Z_{21}(s)Q(s) + Z_{22}(s))^{-1},
\]

where $Z_{ij}(s)(i = 1, 2; j = 1, 2)$ are defined by

\[
\begin{bmatrix}
Z_{11}(s) & Z_{12}(s) \\
Z_{21}(s) & Z_{22}(s)
\end{bmatrix} = \begin{bmatrix}
C_{12}(s) - C_{11}(s)C_{21}(s)^{-1}C_{22}(s) & C_{11}(s)C_{21}(s)^{-1} \\
-C_{21}(s)C_{22}(s) & C_{21}(s)^{-1}
\end{bmatrix}.
\]

\[
C_{ij}(s)(i = 1, 2; j = 1, 2)
\]

are given by using the method in (3) and $Q(s) \in H_\infty^{m \times m}$ is any function satisfying $\|Q(s)\|_\infty < 1$ and written by

\[
Q(s) = \left(Q_{00}(s) + \sum_{i=1}^{N}Q_{ni}(s)q_i(s)e^{-sT_i}\right)^{-1}
\]

\[
\begin{bmatrix}
Q_{00}(s) + \sum_{i=1}^{N}Q_{ni}(s)q_i(s)e^{-sT_i} \\
Q_{0i}(s) + \sum_{i=1}^{N}Q_{di}(s)q_i(s)e^{-sT_i}
\end{bmatrix}.
\]

\[
Q_{ni}(s) = G_{2i}(s)Q_i(s) \in RH_\infty^{m \times m},
\]

Here, $G_{1i}(s) \in RH_\infty^{m \times m}$, $G_{1i}(s) \in RH_\infty^{m \times m}$, $G_{2i}(s) \in RH_\infty^{m \times p}$ and $G_{2i}(s) \in RH_\infty^{m \times p}$ are coprime factors satisfying

\[
Z_{22}(s) + G_{m}(s)Z_{12}(s) = G_{11}(s)G_{12}(s),
\]

\[
G_{11}(s)(Z_{21}(s) + G_{m}(s)Z_{11}(s)) = G_{2n}(s)G_{21}(s).
\]

\[
Q_{an}(s) \in RH_\infty^{m \times m},
\]

\[
\{Z_{11}(s)Q_{00}(s) + Q_{ni}(s)\} + Z_{12}(s)
\]

\[
(Q_{00}(s) + Q_{di}(s)) = m \ (i = 1, \ldots, N).
\]

4 Conclusions

We proposed the parameterization of all robust stabilizing simple MPR controllers for MIMO time-delay plants with specified input-output characteristic.

References


