Decentralized Robust Control for Interconnected Power System

*Wei Wang, Hiromitsu Ohmori (Keio Univ.)

Abstract—This paper proposes a robust decentralized output feedback control scheme for a two-area interconnected power system. The control objective is to reduce frequency and tie-line power deviations due to load variations. The decentralized control problem is converted to scaled $H_{\infty}$ control problem and an algorithm based on LMI is offered to obtain the decentralized controller. Further, parameter uncertainty is taken into account in the controller design to achieve robustness. Simulation results show the proposed method is effective.

Key Words: Interconnected power system, Decentralized control, Robustness, LMI

1 Introduction

For large-scale power system which normally consists of interconnected subsystems, centralized control is extremely difficult to design and very costly to implement. Contrary to centralized control, where only one controller takes all measurements and decides all actions, in decentralized control, each controller only takes partial measurements and decides partial actions. In addition, decentralized control is also effective to resolve issues of communication delay and data loss. As a result, the decentralized control problem for power system has been paid much attention.

Various control strategies have been proposed to solve the decentralized control problem. For instance, in Ref.1), this problem is reduced to the feasibility problem of non-linear matrix inequality (NMI). The NMI is solved iteratively by the idea of homotopy. In Ref.2), the decentralized control problem is solved using decentralized overlapping control which leads to a state feedback controller. This paper proposes a decentralized output feedback control scheme for two-area interconnected power system. The objective is to attenuate frequency and tie-line power deviations due to load variations. To this end, the decentralized control system will be equivalently represented as an interconnection of two closed-loop systems. Then, the decentralized control problem is converted to scaled $H_{\infty}$ control problem and an algorithm based on LMI is offered to obtain the decentralized controller. The effectiveness of proposed method is verified by simulation results.

2 Dynamic Model

The plant is two-area interconnected power system with two generators, two loads and a tie-line connecting them.

2.1 Dynamic Model of Area 1

The block diagram of area 1 is shown in Fig.1. The dynamic characteristics of generator 1 is described by the following differential equation:

$$\frac{d\Delta f_1}{dt} + [D_1 \cdot T_{12}(t) \cdot \Delta f_1(t)] = T_{12}(t) \cdot [\Delta P_{m1}(t) - \Delta P_{t1}(t) - 0.01 \cdot d_{g1}(t)] \quad (1)$$

where the time-varying parameter $T_{12} = \frac{2\pi}{2\pi F_1}$, the disturbance $d_{g1} = 100 \cdot \Delta P_{g1}$. Descriptions of all variables and parameters are given as follows:

![Fig. 1: Block diagram of area 1.](image)

$\Delta f_1$: the incremental frequency deviation of area 1;  
$\Delta P_{m1}$: the incremental change in mechanical input power of generator 1;  
$\Delta P_{t1}$: the incremental change in tie-line power;  
$\Delta P_{l1}$: the incremental change in load demand of area 1;  
$\delta_1$: the nominal frequency;  
$H_{1,0}$: the inertia constant of generator 1;  
$D_1$: the damping constant of generator 1.

It is assumed that $T_{12}$ takes value in $[\frac{1}{2\pi F_{1,0}}, \frac{1}{1.6\pi F_{1,0}}]$, where $H_{1,0}$ is the nominal value of $H_1$.

$\Delta P_{t1}$ is expressed approximately as

$$\Delta P_{t1}(t) \approx \frac{E_1 E_2}{X_{12} P_{rated,1}} \cos \left( \delta_{12} \right) \cdot \left[ \delta_{12}(t) - \delta_{12}^* \right] \quad (2)$$

where $E_1$ and $E_2$ are the terminal bus voltages, $X_{12}$ is the reactance of the tie-line, $P_{rated,1}$ is the rated power of area 1, $\delta_{12}$ is the power angle difference, $\delta_{12}^* = \frac{\pi}{6}$ is the nominal value of $\delta_{12}$.

Since $\frac{d\Delta f_1}{dt} = 2\pi (\Delta f_1 - \Delta f_2)$, the derivative of $\Delta P_{t1}$ is approximated as

$$\frac{d\Delta P_{t1}(t)}{dt} \approx T_{12} \cdot [\Delta f_1(t) - \Delta f_2(t)] \quad (3)$$

where the synchronous coefficient $T_{12} = 2\pi P_{t1,\text{max}} \cos \left( \delta_{12}^* \right)$ with $P_{t1,\text{max}} = \frac{E_1 E_2}{X_{12} P_{rated,1}}$. It is assumed that $P_{t1,\text{max}}$ takes value in $[0.08, 0.12]$. So the corresponding range of $T_{12}$ can be calculated as $[0.436, 0.654]$.

2.2 Dynamic Model of Area 2

The block diagram of area 2 is shown in Fig.2.
The dynamic characteristics of generator 2 is described by the following differential equation:

\[
\frac{d\Delta f_2(t)}{dt} + \left[ D_2 \cdot T_s(t) \right] \cdot \Delta f_2(t) = T_s(t) \cdot \left[ \Delta P_{\text{inj}}(t) - a_{12} \cdot \Delta P_{\text{inj},1}(t) - 0.01 \cdot d_{c}(t) \right]
\]

where the time-varying parameter \( T_s = \frac{1}{2H_s} \), the disturbance \( d_{c} = 100 \cdot \Delta P_{\text{inj},1} \).

It is assumed that \( T_s \) takes value in \[\frac{1}{2 H_{s,0}}, \frac{1}{1.6 H_{s,0}}\], where \( H_{s,0} \) is the nominal value of \( H_s \).

3 Generalized Plant

The decentralized control configuration shown in Fig.3 is adopted.

As shown in Fig.4, the decentralized system shown in Fig.3 can be expressed in a general form as the interconnection of closed-loop systems \( H_{d1}(\theta_i) \) and \( H_{d2}(\theta_i) \).

![Fig. 3: Decentralized control system.](image)

![Fig. 4: Representation of decentralized control system as interconnection of closed-loop systems.](image)

3.1 Generalized Plant of Area 1

The generalized plant used for the design of LTI controller \( K_1(\theta_i) \) is shown in Fig.5, where \( G_i(\theta_i) \) represents the plant corresponding to Fig.1.

![Fig. 5: Generalized plant \( G_{p1}(\theta_i) \).](image)

To attenuate low frequency components of \( \Delta P_{\text{inj},1} \) and \( \Delta f_1 \), we select weight functions \( W_{11}(s) \) and \( W_{1212}(s) \) as low-pass filters.

Meanwhile the weight function \( W_{31}(s) \) is selected as a high-pass filter to attenuate high frequency components of the control input \( u_i \).

Interconnecting \( G_i(\theta_i) \) with weight functions, the generalized plant \( G_{p1}(\theta_i) \) can be described by the polytopic representation

\[
\tilde{x}(t) = A_{11}(\theta_i) x_1(t) + B_{11p}(\theta_i) \Delta f_1(t) + B_{11p}(\theta_i) u_1(t) \]

\[
\Delta P_{\text{inj}}(t) = C_{11} x_1(t),
\]

\[
z_{p1}(t) = C_{1212} x_1(t) + D_{1212} u_1(t),
\]

where \( x_1 \in \mathbb{R}^7 \) is the plant state and \( \theta_i = (T_i \ T_{i2})^T \) is the uncertain parameter vector.

\( A_{11}(\theta_i), B_{11p}(\theta_i) \) and \( B_{1212}(\theta_i) \) are affine in \( \theta_i \) which forms a polytope with four vertices

\[
\theta_{11} = \left( \frac{L_s}{2.4 H_{s,0}}, \ 0.436 \right)^T, \quad \theta_{12} = \left( \frac{L_s}{2.4 H_{s,0}}, \ 0.654 \right)^T,
\]

\[
\theta_{13} = \left( \frac{L_s}{1.6 H_{s,0}}, \ 0.436 \right)^T, \quad \theta_{14} = \left( \frac{L_s}{1.6 H_{s,0}}, \ 0.654 \right)^T.
\]

In this case, the two-dimensional vector \( \theta_i \) in the polytope can be written as \( \theta_i = \sum_{i=1}^{4} \alpha_i \theta_{1i} \) with \( \sum_{i=1}^{4} \alpha_i = 1 \) and \( 0 \leq \alpha_i \leq 1 \).

By normalization, \( T_i \) is replaced by \( s_i + r_i \delta_i(T_i) \) with \( s_i = \frac{5L_s}{9.6 H_{s,0}} \) and \( r_i = \frac{1}{9.6 H_{s,0}} \) such that \( |\delta_i(T_i)| \leq 1 \). Similarly, \( T_{i2} \) is replaced by \( s_{i2} + r_{i2} \delta_{i2}(T_{i2}) \) with \( s_{i2} = 0.545 \) and \( r_{i2} = 0.109 \) such that \( |\delta_{i2}(T_{i2})| \leq 1 \).

After simple mathematical manipulations, \( A_{11}(\theta_i) \) can be written in the upper LFT form

\[
A_{11}(\theta_i) = A_{11} + B_{11\Delta} \Delta_\theta(\theta_i(t)) C_{11\Delta}
\]

where \( A_{11} \) indicates the matrix with respect to the nominal values of uncertain parameters, \( B_{11\Delta} \) and \( C_{11\Delta} \) are constant matrices determined by the matrix decomposition.
\( \Delta_i (\theta_i) \) is the uncertainty operator which has the block diagonal structure
\[
\Delta_i (\theta_i) = \text{diag} \left[ \Delta_{i1} (\theta_i (t)), \Delta_{i2} (\theta_i (t)) \right]
\]

with
\[
\Delta_{i1} (\theta_i (t)) = \frac{\sqrt{2}}{2} \left[ \delta_i (T_i (t)) \quad \delta_{i2} (T_{i2}) \right]^T,
\]
\[
\Delta_{i2} (\theta_i (t)) = \delta_i (T_i (t)) I_{s \times s}.
\]

Based on the above discussion, it follows that

\[
A_{ii} (\theta_i) = A_{ii} + B_{ii1} \Delta_i (\theta_i) C_{ii2}, \\
\Gamma_i (\theta_i) : \Delta_i (\theta_i) = I_{s \times s}
\]

hold for all \( i = 1, \ldots, 4 \).

Substituting (5) into the 1st equation of (4) leads to
\[
\dot{z}_i (t) = A_{i1} z_i (t) + B_{i11} d_{i1} (t) + B_{i1p} (\theta_i (t)) \cdot \Delta f_i (t) + B_{i1p} (\theta_i (t)) d_{i1} (t) + B_{i1} u_i (t)
\]

where \( d_{i1} = \Delta_i (\theta_i) z_{i1} \) with \( z_{i1} = C_{i11} z_i \).

### 3.2 Generalized Plant of Area 2

The generalized plant used for the design of LTI controller \( K_e (s) \) is shown in Fig.6 where \( G_2 (\theta_2) \) represents the plant corresponding to Fig.2.

![Fig. 6: Generalized plant \( G_2 (\theta_2) \).](image)

Interconnecting \( G_2 (\theta_2) \) with weight functions \( W_{e1} (s) \) and \( W_{e2} (s) \), the generalized plant \( G_{p2} (\theta_2) \) can be described by the polytopic representation
\[
\begin{align*}
\dot{z}_2 (t) &= A_{p2} (\theta_2 (t)) z_2 (t) + B_{p2} (\theta_2 (t)) \Delta P_{p2} (t) \\
&+ B_{p2} (\theta_2 (t)) d_{p2} (t) + B_{p2} u_2 (t), \\
\Delta f_2 (t) &= C_{p2} (\theta_2 (t)) z_2 (t), \\
z_{p2} (t) &= C_{p2} z_2 (t) + D_{p2} u_2 (t), \\
y_2 (t) &= C_{p2} z_2 (t)
\end{align*}
\]

where \( z_2 = \begin{bmatrix} z_1^T & z_2^T \end{bmatrix} \) is the plant state and \( \theta_2 = T_2 \).

\( A_{p2} (\theta_2), B_{p2} (\theta_2) \) and \( B_{p2} (\theta_2) \) are affine in \( \theta_2 \) which forms a polytope with two vertices denoted by \( \theta_{21} = \frac{T_{21}}{T_{20}} H_{20} \) and \( \theta_{22} = \frac{T_{22}}{T_{20}} H_{20} \). In this case, \( \theta_2 \) in the polytope can be written as \( \theta_2 = \sum_{j=1}^{2} \alpha_j \theta_j \) with \( \sum_{j=1}^{2} \alpha_j = 1 \) and \( 0 \leq \alpha_j \leq 1, j = 1, 2 \).

By normalization, \( T_2 \) is replaced by \( s + r_2 \delta_2 (T_2) \) with \( s = \frac{\gamma_2 T_{20}}{T_{20}} \) and \( r_2 = \frac{\gamma_2 T_{20}}{T_{20}} \) such that \( \delta_2 (T_2) \leq 1 \).

After simple mathematical manipulations, \( A_{22} (\theta_2) \) can be written in the upper LFT form
\[
A_{22} (\theta_2 (t)) = A_{22} + B_{122} \Delta_2 (\theta_2 (t)) C_{122}
\]

where \( A_{22} \) indicates the matrix with respect to the nominal value of the uncertain parameter \( T_2, B_{122} \) and \( C_{122} \) are constant matrices determined by the matrix decomposition, \( \Delta_2 (\theta_2) = \delta_2 (T_2) I_{s \times s} \) is the uncertainty operator.

Substituting (7) into the 1st equation of (6) leads to
\[
\dot{z}_2 (t) = A_{22} z_2 (t) + B_{122} d_{22} (t) + B_{12p} (\theta_2 (t)) d_{22} (t) + B_{22} u_2 (t)
\]

where \( d_{22} = \Delta_2 (\theta_2) z_{22} \) with \( z_{22} = C_{122} z_2 \).

### 4 Decentralized Controller Design

#### 4.1 Controller Design for Area 1

Consider the LTI output feedback controller \( K_1 (s) \) with the state-space representation
\[
\begin{align*}
\dot{x}_1 (t) &= A_{11} x_1 (t) + B_{11} y_1 (t), \\
u_1 (t) &= C_{11} x_1 (t)
\end{align*}
\]

where \( x_1 \in \mathbb{R}^{n_x} \) is the controller state.

The closed-loop system \( H_{cl1} (\theta_1) \) can be expressed in a general form as the LFT interconnection of the nominal plant \( H_{cl1} (\theta_1) \) with \( \Delta_1 (\theta_1) \).

![Fig. 7: Representation of closed-loop system \( H_{cl1} (\theta_1) \) as the upper LFT interconnection.](image)

The nominal plant \( H_{cl1} (\theta_1) \) can be compactly described by the polytopic representation
\[
\begin{align*}
\dot{x}_{cl1} (t) &= A_{cl1} x_{cl1} (t) + B_{cl1} (\theta_1 (t)) d_{cl1} (t), \\
x_1 (t) &= C_{cl1} x_{cl1} (t)
\end{align*}
\]

where \( x_{cl1} = \begin{bmatrix} x_1^T & x_2^T \end{bmatrix} \) and \( d_{cl1} = \begin{bmatrix} d_{cl1} & \Delta P_{cl1} (T_{cl1}) \end{bmatrix} \) with \( \gamma_{cl1} = 0.62 \). The closed-loop matrices are given by
\[
\begin{align*}
A_{cl1} &= \left( \begin{array}{cc} A_{11} & B_{11} C_{11} \\ B_{11} & A_{11} \end{array} \right), \\
C_{cl1} &= \left( \begin{array}{cc} C_{11} & D_{12} C_{11} \end{array} \right), \\
B_{cl1} (\theta_1 (t)) &= \sum_{i=1}^{4} \alpha_i (t) B_{cl1} (\theta_{1i}) = \begin{bmatrix} B_{11} (\theta_1 (t)) \end{bmatrix}_\theta
\end{align*}
\]
where
\[ B_{11}(\theta_1(t)) = \begin{bmatrix} B_{11,1} & \varepsilon_1 B_{12,1}(\theta_1(t)) & B_{12,1}(\theta_1(t)) \end{bmatrix}, \]
\[ C_{11} = \begin{pmatrix} C_{11,11} & C_{11,12} \end{pmatrix}, \]
\[ D_{121} = \begin{pmatrix} 0_{1 \times 4} & \frac{1}{\gamma_1} D_{1121} \end{pmatrix}, \]

**Definition 1.** The $L_2$-norm of a signal is defined as
\[ \|g\|_2 = \sqrt{\int_0^\infty g^T g dt}. \]

**Lemma 1.** If there exists a symmetric positive definite matrix $Q_{k_1} \in \mathbb{R}^{7 \times 7}$ and a positive scalar $\gamma_1$ such that
\[ \begin{pmatrix} \text{He}(Q_{k_1} A_{cl_1}) & \ast & \ast \\ B_{'cl_1}(\theta_1(t)) Q_{k_1} & -\gamma_1 I_{6 \times 6} & \ast \\ C_{cl_1} & 0_{7 \times 6} & -\gamma_1 I_{7 \times 7} \end{pmatrix} < 0 \] (10)
for all $\theta_1$, $\|z_1\|_2 < \gamma_1$, $\|d_1\|_2 (\|d_1\|_2 \neq 0)$ is satisfied.

For simplicity, $\ast$ denotes the symmetric item in a symmetric matrix and $\text{He}(A) = A^T + A$.

Without loss of generality, suppose $K_1(s)$ is a full-order controller. Partition $Q_{k_1}$ and $Q_{k_1}^{-1}$ as
\[ Q_{k_1} = \begin{pmatrix} Y_{k_1} & N_{k_1} \\ N_{k_1}^T & Z_{k_1} \end{pmatrix}, \quad Q_{k_1}^{-1} = \begin{pmatrix} X_{k_1} & M_{k_1} \\ M_{k_1}^T & W_{k_1} \end{pmatrix} \]
where $X_{k_1}$ and $Y_{k_1}$ are $7 \times 7$ symmetric matrices.

It can be inferred from identity $Q_{k_1} Q_{k_1}^{-1} = I_{14 \times 14}$ that
\[ Q_{k_1} \begin{pmatrix} X_{k_1} \\ M_{k_1}^T \end{pmatrix} = \begin{pmatrix} I_{7 \times 7} \\ 0_{7 \times 7} \end{pmatrix} \]
which yields $Q_{k_1} \Pi_{X_1} = \Pi_{Y_1}$ with
\[ \Pi_{X_1} = \begin{pmatrix} X_{k_1} & I_{7 \times 7} \\ M_{k_1}^T & 0_{7 \times 7} \end{pmatrix}, \quad \Pi_{Y_1} = \begin{pmatrix} I_{7 \times 7} & Y_{k_1} \\ 0_{7 \times 7} & N_{k_1}^T \end{pmatrix}. \]

In the case of full-order controller design, it is always assumed that $N_{k_1}$ is an invertible matrix. By $Q_{k_1} \Pi_{X_1} = \Pi_{Y_1}$, and since $Q_{k_1}$ is nonsingular, $\Pi_{X_1}$ is nonsingular.

Multiplying $Q_{k_1} > 0$ from the left by $\Pi_{X_1}^T$ and multiplying it from the right by $\Pi_{X_1}$, we have
\[ \Theta_{k_1}(X_{k_1}, Y_{k_1}) = \begin{pmatrix} X_{k_1} & I_{7 \times 7} \\ I_{7 \times 7} & Y_{k_1} \end{pmatrix} > 0. \] (11)

By pre-multiplying (10) by $\text{diag}(\Pi_{X_1}^T, I_{6 \times 6}, I_{7 \times 7})$ and post-multiplying it by $\text{diag}(\Pi_{X_1}, I_{6 \times 6}, I_{7 \times 7})$, we have
\[ \Psi_{k_1}(\theta_1(t), A_{k_1}, B_{k_1}, C_{k_1}, X_{k_1}, Y_{k_1}, \gamma_{k_1}) = \begin{pmatrix} \Psi_{k_1,11}(A_{k_1}, B_{k_1}, C_{k_1}, X_{k_1}, Y_{k_1}) & \ast \\ \Psi_{k_1,21}(\theta_1(t), C_{k_1}, X_{k_1}, Y_{k_1}) & \Psi_{k_1,22}(\gamma_{k_1}) \end{pmatrix} < 0 \] (12)

where
\[ \Psi_{k_1,11}(A_{k_1}, B_{k_1}, C_{k_1}, X_{k_1}, Y_{k_1}) = \]
\[ \begin{pmatrix} \text{He}(A_{k_1} X_{k_1} + B_{k_1} C_{k_1}) & A_{k_1}^T + A_{k_1} \\ A_{k_1} + A_{k_1}^T & \text{He}(Y_{k_1} A_{k_1} + B_{k_1} C_{k_1}) \end{pmatrix}, \]
\[ \Psi_{k_1,21}(\theta_1(t), C_{k_1}, X_{k_1}, Y_{k_1}) = \]
\[ \begin{pmatrix} B_{k_1}^T(\theta_1(t)) & B_{k_1}^T(\theta_1(t)) Y_{k_1} \\ C_{k_1} X_{k_1} + D_{121} C_{k_1} & C_{k_1} \end{pmatrix}, \]
\[ \Psi_{k_1,22}(\gamma_{k_1}) = \text{diag}(-\gamma_1 I_{6 \times 6}, -\gamma_1 I_{7 \times 7}). \]

The auxiliary variables are defined as
\[ A_{k_1} = Y_{k_1} A_{k_1} X_{k_1} + N_{k_1} A_{k_1} M_{k_1}^T + B_{k_1} (C_{k_1} X_{k_1}) + (Y_{k_1} B_{k_1}) C_{k_1}, \]
\[ B_{k_1} = N_{k_1} B_{k_1}, \]
\[ C_{k_1} = C_{k_1} M_{k_1}^T. \]

The left side of (12), which is clearly affine in $\theta_1$, can be expressed as
\[ \sum_{i=1}^4 \alpha_{k_1} (\Psi_{k_1}(\theta_1(t), A_{k_1}, B_{k_1}, C_{k_1}, X_{k_1}, Y_{k_1}, \gamma_{k_1})). \] (13)

In summary, the LMI optimization problem to be solved is
\[ \text{minimize} \quad \gamma_{k_1}, \]
subject to
\[ \begin{pmatrix} 11, 13 \end{pmatrix}, X_{k_1} = X_{k_1}^T, Y_{k_1} = Y_{k_1}^T. \]

A feasible solution set of the optimization problem is denoted by $(\hat{A}_{k_1}, \hat{B}_{k_1}, \hat{C}_{k_1}, \hat{X}_{k_1}, \hat{Y}_{k_1}, \hat{\gamma}_{k_1})$ where $\hat{\gamma}_{k_1} > 1$.

Compute $A_{k_1}, B_{k_1}, C_{k_1}$ by
\[ A_{k_1} = \hat{N}_{k_1}^{-1} \left( \hat{A}_{k_1} - \hat{Y}_{k_1} A_{k_1} \hat{X}_{k_1} \right) \left( \hat{M}_{k_1}^T \right)^{-1} - \hat{N}_{k_1}^{-1}, \]
\[ B_{k_1} = \hat{N}_{k_1}^{-1} \hat{B}_{k_1}, \]
\[ C_{k_1} = \hat{C}_{k_1} \left( \hat{M}_{k_1}^T \right)^{-1} = \hat{C}_{k_1}. \]

where $\hat{N}_{k_1}$ and $\hat{M}_{k_1}$ are nonsingular matrices such that
\[ \hat{N}_{k_1} \hat{M}_{k_1}^T = I_{7 \times 7} - \hat{Y}_{k_1} \hat{X}_{k_1}. \]

Consider scaling matrices
\[ L_{1\times} (s_{11}, S_{12}, l_1) = \begin{pmatrix} s_{11}, X_{1\times 3}, S_{12}, l_1, I_{1 \times 3} \end{pmatrix}, \]
\[ L_{1\times,1} (s_{11}, S_{12}, l_1) = \begin{pmatrix} s_{11}, S_{12}, l_1, I_{1 \times 3} \end{pmatrix}. \]
where \( S_{12} \in \mathbb{R}^{n \times n} \) is a symmetric positive definite matrix, \( s_{11} \) and \( I_1 \) are positive scalars.

Suppose there exists a matrix \( Q_{s_{11}} = Q_{s_{11}}^T > 0 \), a scalar \( \gamma_{s_{11}} > 0 \) and scaling variables \( s_{11}, S_{12}, I_1 \) such that

\[
\begin{align*}
\begin{pmatrix}
\text{He} \left( Q_{s_{11}} \hat{A}_{c_{11}} \right) & \ast \\
\hat{B}_{c_{11}}^T (\theta_{s_{11}}) Q_{s_{11}} & - \gamma_{s_{11}} L_{12} (s_{11}, S_{12}, I_1) \\
L_{12} (s_{11}, S_{12}, I_1) \hat{C}_{c_{11}} & 0_{n \times n}
\end{pmatrix} < 0
\end{align*}
\]

(14)

for all \( i = 1, \ldots, 4 \). The matrices \( \hat{A}_{c_{11}}, \hat{B}_{c_{11}} (\theta_{s_{11}}) \) and \( \hat{C}_{c_{11}} \) are given by

\[
\begin{align*}
\hat{A}_{c_{11}} &= \begin{pmatrix} A_{s_{11}} & B_{s_{11}} \hat{C}_{s_{11}} \end{pmatrix}, \\
\hat{B}_{c_{11}} (\theta_{s_{11}}) &= \begin{pmatrix} B_{s_{11}} (\theta_{s_{11}}) \\ 0_{n \times n} \end{pmatrix}, \\
\hat{C}_{c_{11}} &= \begin{pmatrix} C_{s_{11}} & D_{s_{12}} \hat{C}_{s_{11}} \end{pmatrix}.
\end{align*}
\]

Bisection method is taken to solve the optimization problem

\[
\begin{align*}
\text{minimize} & \quad \gamma_{s_{11}} \\
\text{subject to} & \quad (14), Q_{s_{11}} - Q_{s_{11}}^T > 0, S_{12} - S_{12}^T.
\end{align*}
\]

A feasible solution set of the optimization problem is denoted by \( \hat{\Phi}_{s_{11}} (\theta_{s_{11}}, \hat{s}_{11}, \hat{S}_{12}, I_1, \hat{Q}_{s_{11}}, \hat{\gamma}_{s_{11}}) < 0, i = 1, \ldots, 4 \) (15)

**Corollary 1.** By Schur complement, (15) is transformed equivalently to the following inequality:

\[
\begin{align*}
\text{He} \left( \tilde{Q}'_{s_{11}} \tilde{A}_{c_{11}} \right) + \tilde{C}_{c_{11}}^T \cdot \tilde{L}_{12} \cdot \tilde{C}_{c_{11}} + \tilde{Q}'_{s_{11}} \hat{B}_{c_{11}} (\theta_{s_{11}}) \\
\frac{1}{2} \tilde{L}_{12}^{-1} \cdot \hat{B}_{c_{11}}^T (\theta_{s_{11}}) \tilde{Q}'_{s_{11}} < 0, i = 1, \ldots, 4
\end{align*}
\]

where \( \tilde{Q}'_{s_{11}} = \hat{\gamma}_{s_{11}} \hat{Q}_{s_{11}}, \tilde{L}_{12} = L_{12} (\hat{s}_{11}, \hat{S}_{12}, I_1) \) and \( \tilde{L}_{12} = L_{12} (\hat{s}_{11}, \hat{S}_{12}, I_1) \). Since \( \hat{\gamma}_{s_{11}}^2 < 1 \), it follows that

\[
\begin{align*}
\text{He} \left( \tilde{Q}'_{s_{11}} \tilde{A}_{c_{11}} \right) + \tilde{C}_{c_{11}}^T \cdot \tilde{L}_{12} \cdot \tilde{C}_{c_{11}} + \tilde{Q}'_{s_{11}} \hat{B}_{c_{11}} (\theta_{s_{11}}) \\
\frac{1}{2} \tilde{L}_{12}^{-1} \cdot \hat{B}_{c_{11}}^T (\theta_{s_{11}}) \tilde{Q}'_{s_{11}} < 0, i = 1, \ldots, 4.
\end{align*}
\]

Replacing \( \hat{C}_{c_{11}}, \hat{B}_{c_{11}} (\theta_{s_{11}}), \hat{L}_{12}, \) and \( \hat{L}_{12} \) by their explicit expressions leads to

\[
\begin{align*}
\text{He} \left( \tilde{Q}'_{s_{11}} \tilde{A}_{c_{11}} \right) + \tilde{C}_{c_{11}}^T \cdot \tilde{L}_{12} \cdot \tilde{C}_{c_{11}} + \tilde{Q}'_{s_{11}} \hat{B}_{c_{11}} (\theta_{s_{11}}) \\
\frac{1}{2} \tilde{L}_{12}^{-1} \cdot \hat{B}_{c_{11}}^T (\theta_{s_{11}}) \tilde{Q}'_{s_{11}} < 0, i = 1, \ldots, 4.
\end{align*}
\]

where

\[
\begin{align*}
\tilde{S}_{12} = \text{diag} \left( \tilde{s}_{11} I_{n \times n}, \tilde{s}_{12} \right), \tilde{S}_{12} = \text{diag} \left( \hat{s}_{11}, \hat{S}_{12} \right), \\
\hat{C}_{c_{11}} = \left( C_{s_{11}} \hat{C}_{s_{11}} \right), \hat{C}_{c_{11}} = \left( C_{s_{11}} \hat{C}_{s_{11}} \right), \\
\hat{B}_{c_{11}} (\theta_{s_{11}}) = \left( B_{s_{11}} (\theta_{s_{11}}) \right), \hat{B}_{c_{11}} (\theta_{s_{11}}) = \left( B_{s_{11}} (\theta_{s_{11}}) \right).
\end{align*}
\]

The scaling matrix \( \tilde{S}_{12} \) has a decomposition \( \tilde{S}_{12} = \left( \tilde{S}_{12}' \right)^2 \) where \( \tilde{S}_{12}' = \text{diag} \left( \sqrt{\tilde{s}_{11}}, I_{n \times n}, \sqrt{\tilde{s}_{12}} \right) \). Meanwhile, the scaling matrix \( \hat{S}_{12} \) has a decomposition \( \hat{S}_{12} = \left( \hat{S}_{12}' \right)^2 \) where \( \hat{S}_{12}' = \text{diag} \left( \sqrt{\hat{s}_{11}}, \sqrt{\hat{s}_{12}} \right) \).

By means of \( \Delta_{s_{11}} (\theta_{s_{11}}) \), \( \Delta_{s_{12}} (\theta_{s_{11}}) \), we obtain

\[
\begin{align*}
\hat{C}_{c_{11}} \cdot \hat{S}_{12} \cdot \hat{C}_{c_{11}} + \hat{Q}_s' \cdot \hat{B}_{c_{11}} \cdot \hat{S}_{12} \cdot \hat{Q}_s' = \\
H_{s_{11}} (\theta_{s_{11}}, H_s (\theta_{s_{11}})) + G_s G_s^T.
\end{align*}
\]

where

\[
\begin{align*}
H_s (\theta_{s_{11}}) = \Delta_{s_{11}} (\theta_{s_{11}}) \hat{S}_{12}' \cdot \hat{C}_{c_{11}} \cdot \hat{C}_{c_{11}} \cdot \hat{B}_{c_{11}} \cdot \hat{S}_{12}' \cdot \hat{C}_{c_{11}}, \\
G_s G_s^T = \text{He} \left( G_s H_s (\theta_{s_{11}}) \right).
\end{align*}
\]

Replacing \( \Delta_{s_{11}} (\theta_{s_{11}}) \), and \( \hat{S}_{12}' \) by their explicit expressions leads to

\[
\begin{align*}
\Delta_{s_{11}} (\theta_{s_{11}}) \hat{S}_{12}' = \text{diag} \left( \Delta_{s_{11}} (\theta_{s_{11}}) \sqrt{\tilde{s}_{11}}, \Delta_{s_{12}} (\theta_{s_{11}}) \sqrt{\tilde{s}_{12}} \right).
\end{align*}
\]

In particular, \( \Delta_{s_{12}} (\theta_{s_{11}}) \) commutes with \( \hat{S}_{12}' \). Thus, we infer

\[
\Delta_{s_{11}} (\theta_{s_{11}}) \hat{S}_{12}' = \Delta_{s_{11}} (\theta_{s_{11}}) \hat{S}_{12}'.
\]

which leads to

\[
\hat{S}_{12}' \Delta_{s_{11}} (\theta_{s_{11}}) \hat{S}_{12}' = \Delta_{s_{11}} (\theta_{s_{11}}).
\]
By means of $A_{11}$ ($\theta_{11}$) = $A_{11} + B_{11\Delta} \Delta_1$ ($\theta_{11}$) $C_{11\Delta}$, we obtain

\[
\dot{Q}'_{11} \dot{A}_{cl11} + C_1 \cdot H_1 (\theta_{11}) = \dot{Q}'_{11} \dot{A}_{cl11} (\theta_{11})
\]

where $\dot{A}_{cl11} (\theta_{11}) = \begin{pmatrix} A_{11} (\theta_{11}) & B_{21} \hat{C}_{k1} \\ \hat{B}_{k1} C_{21} & \dot{A}_{k1} \end{pmatrix}$. Based on the above discussion, we have

\[
\text{He} \begin{pmatrix} \dot{Q}'_{11} \dot{A}_{cl11} (\theta_{11}) \end{pmatrix} + \frac{\varepsilon_2^2}{l_1} \dot{Q}'_{11} \dot{B}^{T}_{cl1} (\theta_{11}) \cdot \dot{B}^{T}_{cl1} (\theta_{11}) \dot{Q}'_{11} + \frac{1}{\gamma^2_{11}} \dot{C}^{T}_{cl1} \dot{C}_{cl1} < 0, i = 1, \ldots, 4.
\]

(16)

4.2 Controller Design for Area 2

Consider the LTI output feedback controller $K_2 (s)$ with the state-space representation

\[
\begin{align*}
\dot{x}_{cl2} (t) &= A_{cl2} x_{cl2} (t) + B_{cl2} (\theta_2) d_2 (t), \\
u_2 (t) &= C_{cl2} x_{cl2} (t).
\end{align*}
\]

The closed-loop system $H_{cl2} (\theta_2)$ can be expressed in a general form as the upper LFT interconnection of the nominal plant $H_{cl2\Delta} (\theta_2)$ with $\Delta_2 (\theta_2)$.

The nominal plant $H_{cl2\Delta} (\theta_2)$ can be described by the polytopic representation

\[
\begin{align*}
\dot{x}_{cl2} (t) &= A_{cl2} x_{cl2} (t) + B_{cl2} (\theta_2) d_2 (t), \\
z_2 (t) &= C_{cl2} x_{cl2} (t).
\end{align*}
\]

where $x_{cl2} = (x_{cl2}^T \ x_{cl2}^{T})^T$, $d_2 = (d_{21}^T \ \Delta P_{ncl}^T \ d_{22})^T$, $z_2 = (z_{cl2}^T \ \varepsilon_2 \Delta f_2 \ \frac{1}{\gamma^2_{cl2}} x_{cl2}^T)^T$ with $\varepsilon_2 = \frac{1}{\upsilon_5} = 0.5$, $\gamma_{cl2} = 0.75$.

**Lemma 2.** If there exists a positive scalar $\gamma_{k2}$ and a symmetric positive definite matrix $Q_{k2} \in \mathbb{R}^{x_{k2} \times x_{k2}}$ such that

\[
\begin{pmatrix}
\text{He} \begin{pmatrix} Q_{k2} A_{cl22} \end{pmatrix} & \ast & \ast \\
B_{cl2}^T (\theta_2) Q_{k2} & - \gamma_{k2} I_{x_{k2} \times x_{k2}} & \ast \\
C_{cl2} & 0_{x_{k2} \times x_{k2}} & - \gamma_{k2} I_{x_{k2} \times x_{k2}}
\end{pmatrix} < 0
\]

(19)

for all $\theta_2$, $\|z_2\|_2 < \gamma_{k2} \|d_2\|_2 (\|d_2\|_2 \neq 0)$ is satisfied.

The optimal solution set of (19) and $Q_{k2} > 0$ is denoted by $(\hat{A}_{k2}, \hat{B}_{k2}, \hat{C}_{k2}, \hat{Q}_{k2}, \hat{\gamma}_{k2})$ where $\hat{\gamma}_{k2} = 3.67$. Consider scaling matrices

\[
\begin{align*}
L_{22} (S_2, l_2) &= \text{diag} (S_2, l_2, I_{x_{k2} \times x_{k2}}), \\
L_{23} (S_2, l_2) &= \text{diag} (S_3, l_2, I_{x_{k2} \times x_{k2}})
\end{align*}
\]

where $S_3 \in \mathbb{R}^{x_{k2} \times x_{k2}}$ is a symmetric positive definite matrix, $l_2$ is a positive scalar.

Suppose there exists a matrix $Q_{k2} = Q_{k2}^T > 0$, a scalar $\gamma_{k2} > 0$ and scaling variables $S_2, l_2$ such that

\[
\begin{pmatrix}
\text{He} \begin{pmatrix} Q_{k2} \hat{A}_{cl22} \end{pmatrix} & \ast & \ast \\
\hat{B}_{cl2}^T (\theta_2) Q_{k2} & - \gamma_{k2} L_{2k2} (S_2, l_2) & 0_{5 \times 5} \\
L_{2k2} (S_2, l_2) \hat{C}_{cl2} & 0_{5 \times 5}
\end{pmatrix} < 0
\]

(20)

for all $j = 1, 2$. The matrices $\hat{A}_{cl2j}, \hat{B}_{cl2} (\theta_{2j})$ and $\hat{C}_{cl2}$ are given by

\[
\hat{A}_{cl2j} = \begin{pmatrix} A_{2j} & B_{2j} \hat{C}_{k2} \\ \hat{B}_{k2} C_{2j} & \hat{A}_{k2} \end{pmatrix},
\]

\[
\hat{B}_{cl2} (\theta_{2j}) = \begin{pmatrix} B_{1j} (\theta_{2j}) \\ 0_{5 \times 4} \end{pmatrix}, \quad \hat{C}_{cl2} = \begin{pmatrix} C_{12} & D_{122} \hat{C}_{k2} \end{pmatrix}
\]

where

\[
\begin{align*}
B_{1j} (\theta_{2j}) &= (B_{12\Delta} \ B_{12p} (\theta_2) \ B_{12p} (\theta_2)), \\
C_{12} &= \begin{pmatrix} C_{12\Delta} & \varepsilon_2 C_{12p} & -\frac{1}{\gamma_2} C_{12p}^T \end{pmatrix}^T, \\
D_{122} &= \begin{pmatrix} 0_{1 \times 3} & \frac{1}{\gamma_{k2}} D_{12p}^T \end{pmatrix}^T.
\end{align*}
\]

The optimal solution set of (20) and $Q_{k2} > 0$ is denoted by $(\hat{S}_2, \hat{I}_2, \hat{Q}_{k2}, \hat{\gamma}_{k2})$ where $\hat{\gamma}_{k2} = 0.9312 < 1$.

5 Performance of Decentralized System

The schematic diagram of decentralized control system $\hat{H}_{cl} (\theta)$ is shown in Fig.8. The output feedback controller $\hat{K} (s) = \text{diag} (\hat{K}_1 (s), \hat{K}_2 (s))$ is used as the decentralized controller.

Fig. 8: Decentralized control system $\hat{H}_{cl} (\theta)$.

In particular, the decentralized control system $\hat{H}_{cl} (\theta)$ can be expressed as the interconnection of closed-loop systems.
Interconnecting the generalized plant $G_{P1} (\theta_i)$ with LTI controller $K_1 (s) = \hat{C}_{k1} (s I - \hat{A}_{k1})^{-1} \hat{B}_{k1}$, the closed-loop system $\bar{H}_{cl} (\theta_i)$ can be described by the polytopic representation

$$\dot{x}_{cl1} (t) = \hat{A}_{cl1} (\theta_i) x_{cl1} (t) + \hat{B}_{clp} (\theta_i) d_{p1} (t),$$

$$\Delta P_{t1} (t) = \hat{C}_{clp1} x_{cl1} (t),$$

$$z_{cl1} (t) = \hat{C}_{clx1} x_{cl1} (t). \quad (21)$$

**Theorem 1.** The inequality

$$\Psi_{x1} (\theta_i) = \text{He} \left( \hat{P}_{x1} \hat{A}_{cl1} (\theta_i), \hat{P}_{x1} \right) + \hat{C}_{clp1}^T \hat{C}_{clp1},$$

$$+ \sum_{i=1}^{2} \alpha_i (t) \alpha_{j} (t) \hat{A}_{cl1} (\theta_i) \hat{A}_{cl1} (\theta_i),$$

$$+ \frac{1}{\gamma_{cl1}^{2}} \hat{P}_{x1} \hat{B}_{clp1} (\theta_i) \hat{B}_{clp1} (\theta_i) \hat{P}_{x1} > 0 \quad (22)$$

holds for all $i = 1, ..., 4$. Here, $\hat{P}_{x1} = \hat{P}_{x1}^{H} > 0$.

**Proof.** Dividing both sides of (16) by $I_{i1} > 0$ leads to (22).

Interconnecting the generalized plant $G_{P2} (\theta_i)$ with LTI controller $K_2 (s) = \hat{C}_{k2} (s I - \hat{A}_{k2})^{-1} \hat{B}_{k2}$, the closed-loop system $\bar{H}_{cl2} (\theta_i)$ can be described by the polytopic representation

$$\dot{x}_{cl2} (t) = \hat{A}_{cl2} (\theta_i) x_{cl2} (t) + \hat{B}_{clp2} (\theta_i) d_{p2} (t),$$

$$\Delta f_{2} (t) = \hat{C}_{clp2} x_{cl2} (t),$$

$$z_{cl2} (t) = \hat{C}_{clx2} x_{cl2} (t). \quad (23)$$

**Theorem 2.** The inequality

$$\Psi_{x2} (\theta_i) = \text{He} \left( \hat{P}_{x2} \hat{A}_{cl2} (\theta_i), \hat{P}_{x2} \right) + \frac{1}{\gamma_{cl2}^{2}} \hat{C}_{clp2}^T \hat{C}_{clp2},$$

$$+ \hat{P}_{x2} \hat{B}_{clp2} (\theta_i) \hat{B}_{clp2} (\theta_i) \hat{P}_{x2} + \frac{1}{\gamma_{cl2}^{2}} \hat{C}_{clp2}^T \hat{C}_{clp2},$$

$$+ \hat{I}_{j} \hat{P}_{x2} \hat{B}_{clp2} (\theta_i) \hat{B}_{clp2} (\theta_i) \hat{P}_{x2} < 0 \quad (24)$$

holds for all $j = 1, 2$. Here, $\hat{P}_{x2} = \frac{\gamma_{x2}}{\gamma_{x2}} \hat{P}_{x2} = \hat{P}_{x2}^H > 0$.

**Proof.** Omitted for brevity.

Combining (21) and (23), the decentralized control system $\bar{H}_{cl} (\theta)$ can be described by the polytopic representation

$$\dot{x}_{cl} (t) = \hat{A}_{cl} (\theta) x_{cl} (t) + \hat{B}_{clp} (\theta) d_{p} (t),$$

$$z_{r} (t) = \hat{C}_{clr} x_{cl} (t) \quad (25)$$

in which $x_{cl} = (x_{cl1}^T, x_{cl2}^T)^T, d_{p} = (d_{p1}^T, d_{p2}^T)^T, z_{r} = (z_{r1}^T, z_{r2}^T)^T, \theta = (\theta_{i}^T, \theta_{j}^T)^T$ and

$$\hat{A}_{cl} (\theta) = \sum_{i=1}^{2} \sum_{j=1}^{2} \alpha_{i} (t) \alpha_{j} (t) \hat{A}_{cl1} (\theta_i, \theta_j),$$

$$\hat{A}_{cl} (\theta_i, \theta_j) = \left( \begin{array}{cc} \hat{A}_{cl1} (\theta_i) & \hat{B}_{clp1} (\theta_i) \hat{C}_{clp1}^T \\ \hat{B}_{clp2} (\theta_j) & \hat{A}_{cl22} (\theta_j) \end{array} \right),$$

$$\hat{C}_{clr} = \text{diag} \left( \hat{C}_{clx1}, \hat{C}_{clx2} \right),$$

$$\hat{B}_{clp} (\theta) = \sum_{i=1}^{2} \sum_{j=1}^{2} \alpha_{i} (t) \alpha_{j} (t) \hat{B}_{clp} (\theta_i, \theta_j),$$

$$\hat{B}_{clr} (\theta_i, \theta_j) = \text{diag} \left( \hat{B}_{clp1} (\theta_i), \hat{B}_{clp2} (\theta_j) \right).$$

with $\sum_{i=1}^{2} \sum_{j=1}^{2} \alpha_{i} \alpha_{j} = 1$ and $0 < \alpha_{i} \leq 1, i = 1, ..., 4$.

**Theorem 3.** Since $\Psi_{x1} (\theta_i) < 0$ holds for $i = 1, ..., 4$ as well as $\Psi_{x2} (\theta_i) < 0$ holds for $j = 1, 2$, the decentralized control system $\bar{H}_{cl} (\theta)$ is internally stable and the $L_2$-gain from $d_{p}$ to $z_{r}$ is smaller than a given value $\bar{\gamma}$.

**Proof.** It follows that the inequality

$$\text{diag} (\Psi_{x1} (\theta_i), \Psi_{x2} (\theta_j)) < 0 \quad (26)$$

holds for all $i = 1, ..., 4$ and $j = 1, 2$. By matrix decomposition, (26) can be expressed as

$$\text{He} \left( \hat{P}_{x1} Q (\theta_i, \theta_j), \hat{P}_{x1} \right) + G (\theta_i, \theta_j) \hat{C}_{x2}^T (\theta_i, \theta_j),$$

$$+ \hat{C}_{clp}^T \hat{I}_{j} \hat{C}_{clp} < 0, i = 1, ..., 4, j = 1, 2$$

where $\hat{I}_{j} = \text{diag} \left( \frac{I_{x2}}{\gamma_{x2}}, \frac{I_{x2}}{\gamma_{x2}} \right), \hat{I}_{j} = \text{diag} \left( \frac{I_{1} I_{2}}{\gamma_{x2}}, \frac{I_{1} I_{2}}{\gamma_{x2}} \right),$$

$$\hat{P}_{x} = \text{diag} \left( \hat{P}_{x1}, \hat{P}_{x2} \right) = \hat{P}_{x2}^H > 0 \quad \text{and}$$

$$Q (\theta_i, \theta_j) = \text{diag} \left( \hat{A}_{cl1} (\theta_i), \hat{A}_{cl22} (\theta_j) \right),$$

$$G (\theta_i, \theta_j) = \hat{P}_{x1} \text{diag} \left( \frac{I_{x2}}{\gamma_{x2}}, \frac{I_{x2}}{\gamma_{x2}} \right),$$

$$H = \text{diag} \left( \frac{1}{\gamma_{x2}}, \frac{1}{\gamma_{x2}} \right).$$

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By He \((G(\theta_{ix}, \theta_{iz}) H) \leq G(\theta_{ix}, \theta_{iz}) G^T(\theta_{ix}, \theta_{iz}) + H^T H\), we infer
\[
\begin{align*}
\text{He} \left( \dot{P}' Q \left( \theta_{ix}, \theta_{iz} \right) + G \left( \theta_{ix}, \theta_{iz} \right) H \right) & \leq H^T H \\
+ G \left( \theta_{ix}, \theta_{iz} \right) G^T \left( \theta_{ix}, \theta_{iz} \right) + \text{He} \left( \dot{P}' Q \left( \theta_{ix}, \theta_{iz} \right) \right) \\
\end{align*}
\]
where
\[
\dot{P}' Q \left( \theta_{ix}, \theta_{iz} \right) + G \left( \theta_{ix}, \theta_{iz} \right) H = \dot{P}' \dot{\hat{A}}_{cl} \left( \theta_{ix}, \theta_{iz} \right).
\]
Based on the above discussion, it follows that
\[
\begin{align*}
\text{He} \left( \dot{\hat{B}}_{cl}^T \left( \theta_{ix}, \theta_{iz} \right) \hat{P} \right) + \dot{\hat{C}}_{cl}^T \dot{\hat{C}}_{cl} \theta_{i,j} \leq 0
\end{align*}
\]
holds for all \(i = 1, \ldots, 4\) and \(j = 1, 2\). Here, \(\hat{\gamma} = \sqrt{\frac{l_{\max}}{l_{\min}}}, \hat{P} = \frac{1}{l_{\min}} \dot{P}' = \hat{P}^T > 0\) with \(l_{\min} = \min \left( \frac{1}{l_{cl} l_{fg}}, \frac{1}{l_{cl} l_{tt}} \right)\)
and \(l_{\max} = \max \left( \frac{1}{l_{cl} l_{fg}}, \frac{1}{l_{cl} l_{tt}} \right)\).
From (27), we deduce that the inequality
\[
\begin{align*}
\text{He} \left( \dot{\hat{B}}_{cl} \left( \theta \left( t \right) \right) \right) + \dot{\hat{C}}_{cl}^T \dot{\hat{C}}_{cl} \theta_{i,j} \leq 0
\end{align*}
\]
holds for all \(\theta\).

It implies that the decentralized control system \(\hat{H}_{cl}(\theta)\)
is internally stable and the \(L_2\)-gain from \(d_{ce}^{e+}\) to \(z_{ce}^{e+}\) is smaller than \(\hat{\gamma}\).

6 Simulation Results

The simulations are carried out using Matlab R2007b. The date of power system is taken from Ref.5). The time-varying parameter \(H_s\) is modeled as
\[
H_s = \left( 1 + \frac{\Delta f}{f_s} \right) H_{1,0}
\]
while \(H_s\) is modeled as
\[
H_s = \left( 1 + \frac{\Delta f}{f_s} \right) H_{2,0}.
\]

In simulations, \(\Delta P_{lc}\) is modeled as a rapidly changing signal with
\[
\left| \frac{d\Delta P_{lc}}{dt} \right| \leq 1.3 \times 10^{-3} \text{ [pu MW/s]}
\]
while \(\Delta P_{lc}\) is modeled as a slowly changing signal with
\[
\left| \frac{d\Delta P_{lc}}{dt} \right| \leq 0.8 \times 10^{-3} \text{ [pu MW/s]}
\]

In the case where \(P_{\text{ref, max}} = 0.1\), the simulation results are shown in Fig.10 and Fig.11.

It can be easily confirmed that area frequencies are almost not influenced by irregular load variations and have been maintained close to the nominal frequency, while the tie-line power deviation has been also effectively attenuated.

**Fig. 10:** Incremental frequency deviations of area 1 (top) and area 2 (bottom).

**Fig. 11:** Incremental change in power angle difference (top) and incremental change in tie-line power (bottom).

7 Conclusion

This paper has proposed a decentralized output feedback control scheme for a two-area interconnected power system. First, the decentralized control system is equivalently represented as the interconnection of two closed-loop systems. Then, the decentralized control problem is converted into the scaled \(H_{\infty}\) control problem and an algorithm based on LMI is offered to obtain the decentralized controller. It is confirmed from simulation results that frequency and tie-line power deviations can be effectively attenuated.

References