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Scientific paper

Analysis of RC Structures Subjected to Multi-Directional Shear Loads

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Abstract

The true behavior of many large complex structures involves interaction between the in-plane and transverse shear loads acting on the reinforced concrete (RC) element. This paper presents a model using a fiber-based finite element formulation to predict the strength of reinforced concrete members subjected to multi-directional shear loads. The shear mechanism along the element is modeled by adopting a Timoshenko beam approach. The nonlinearity of the concrete and steel materials is accounted for at the fiber level through the use of proper constitutive laws. The concrete constitutive law is based on the Softened Membrane Model (SMM), which was modified to account for transverse shear load effects. The modification of the concrete model is derived based on the findings of an extensive experimental program. The validity of the finite element model is established by correlation of analytical results with experimental tests of RC specimens subjected to multi-directional loads and available in the literature. These numerical studies showed that the model can accurately predict the reduction in strength due to the effect of transverse loads.

1. Introduction

The evaluation of the constitutive behavior of reinforced concrete under three-dimensional loading conditions has not been previously properly established due to the lack of experimental data. As such, analysis of complex three-dimensional reinforced concrete structures is currently being conducted using simplified assumptions that do not capture the true behavior. The presence of transverse loads typically results in a different crack pattern that will alter the main characteristics of the concrete material. Specifically, the smeared stress-strain relation of concrete, the softening behavior of concrete under triaxial loading, the post-cracking Poisson ratio, and the smeared stress-strain behavior of reinforcement will be fundamentally different than their corresponding values under in-plane loadings only. In this case, different failure modes will be observed, which in turn will affect the element strength. As a result, a rational design cannot be accurately developed. Many trial studies attempted to develop a 3D model for RC structures. However, such model adds another dimension of complexity to both the constitutive modeling of concrete, its bond to steel reinforcement, as well as the pre and post-processing of data (Barzegar and Maddipudi, 1997). Rahal et al. (1995a) stated that 3D stresses on an element in the real structure are complex and cannot be determined easily with the current knowledge of cracked concrete behavior, while only 3D analysis can

fully represent all aspects of the response of concrete structures (Kwon et al. 2002). The lack of adequate test data on triaxial behavior of plain concrete still poses a major difficulty in reaching a consensus regarding the proper constitutive relations under complex loading histories (Barzegar and Maddipudi, 1997). Design provisions should be based on an understanding of the causes of the observed structural behavior and this can only be achieved by considering the triaxial stress conditions that always develop within a structure under complex loads (Kotsovos 1987).

Several numerical models of concrete subjected to multi-directional loads exist in the literature. Vecchio and Selby (1991) developed a finite element program for 3D analysis of concrete structures with an eight node regular hexahedral element. In their constitutive material model they used the Modified Compression Field Theory. Rahal and Collins (1995b) developed a simplified 3D truss model to analyze members subjected to combined loading also adopting the Modified Compression Field theory (MCFT). Cocchi and Volpi (1996) developed a nonlinear model of RC members subjected to combined axial, shear, bending and torsional loads, based on an extension of the Diagonal Compression Field theory. Maekawa et al. (1997) used the concept of stress decomposition/re-composition, in which the 3D stress field is decomposed into 3 in-plane sub-spaces. Gregori et al. (2007) analyzed the section of a concrete column subjected to biaxial bending, bidirectional shear and torsion by subdividing it into several regions that are subjected to either uniaxial, biaxial, or triaxial state of stress.

The objective of this paper is to develop an efficient model for analysis of concrete members subjected to multi-directional loads. The model is based on fiber beam-column element formulations. Fiber-based beam models are a simplified version of the continuum finite

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element approach that have the advantage of reduced computational cost. Plastic hinge beam models on the other hand are inferior to fiber models because the latter uses detailed geometry and material models with a good representation of the nonlinear behavior along the length of the member. The most promising model for nonlinear analysis of RC members is the fiber model because it is more sophisticated than the plastic hinge model, and slightly less detailed and thus computationally more efficient than continuum finite element models. A fiber model is developed by dividing each element into several sections along the member. The sections at each end of the element are further divided into several fibers which represent concrete and steel (Fig. 1). The strain in each fiber is calculated from the centroidal section strain and curvature with the help of the assumption that plane sections remain plane. Stresses and stiffnesses in the corresponding fibers are calculated from the fiber strain values. The constitutive relation of the section is derived by integration of the response of the fibers and the response of the element is derived by integration of the response of the sections along the length of the element. The development of fiber beam-column elements for analysis of concrete members subjected to in-plane and transverse shear is described in detail in the following sections.

2. Incorporating transverse shear in membrane shear MODEL

Labib (2011) conducted an extensive experimental program in order to investigate the behavior of elements subjected to combination of transverse and in-plane shear stresses. The experimental investigation included constructing, instrumenting and testing several full scale reinforced concrete elements. Strong interaction between transverse and in-plane shear stresses was observed. Labib (2011) found that applying transverse shear caused reduction in the membrane shear capacity of the panels. This reduction in strength resulted in an additional softening parameter for the concrete com-

pression strut of the membrane element. Softening of concrete is defined as the decrease in the concrete compressive strength due to the applied lateral strains. Increasing the transverse shear loads caused reduction in that softening coefficient. Based on the test data, Labib (2011) modified the softening coefficient ζ proposed by Hsu and Zhu (2002) with an additional parameter $f(\Gamma)$ to account for the effect of transverse shear:

$$\zeta = \left(\frac{5.8}{\sqrt{f'_c} \text{ (MPa)}} \leq 0.9 \right) \left(\frac{1}{\sqrt{1 + 400 \frac{\epsilon_1}{\epsilon_2}}} \right) \left(1 - \frac{|\beta|}{24} \right) f(\Gamma) \quad (1a)$$

$$\beta = 0.5 \tan^{-1} \left(\frac{\gamma_{12}}{\epsilon_1 - \epsilon_2} \right),$$

$$f(\Gamma) = 1 - \left(0.0178\Gamma - \frac{\Gamma^2}{10000} \right), \Gamma = \frac{(\tau_o)_{total}}{\tau_{ou}} \times 100 \quad (1b)$$

where ζ is the softening coefficient, β is the deviation angle in degrees between the applied and principal stress angles, ϵ_1 is the lateral tensile strain, f'_c is the concrete compressive strength, $\zeta f'_c$ is the softened concrete compressive strength, $\epsilon_1, \epsilon_2, \gamma_{12}$ represent the strain state in the principal directions, τ_{ou} is the transverse shear strength, and $(\tau_o)_{total}$ is the total applied shear stress in the transverse direction.

In this paper, a fiber-based beam finite element is developed to simulate the behavior of RC structures subjected to multi-directional in-plane and transverse loads. To account for shear effect, fiber elements can be extended following a Timoshenko-based formulation (Mullanpudi and Ayoub 2010), and using an appropriate biaxial concrete constitutive law at the fiber level. Details of the implementation of shear-critical fiber elements are given in Mullanpudi (2010). Further, to account for the effect of transverse shear, the concrete constitutive model used in fiber elements can be modified to account for the strong interaction observed between in-plane and transverse shear loads, which was quantified experimentally by Labib (2011) and described by Eq. (1). The model assumes that concrete cracks are smeared, and neglect the shear deformations of the reinforcement.

The concrete constitutive model in the presence of transverse shear loads is described in detail in the following sections.

2.1 Concrete constitutive model

According to the ACI 318 (2008) building code, the ultimate strength of an RC beam is the combination of shear strength provided by the concrete (V_c) and shear strength provided by the transverse reinforcement (V_s). The Fixed Angle-Softened Truss Model (FA-STM), and Softened Membrane Model (SMM) theories are capable of accurately accounting for the effect of V_c because the crack angle is based on the principal coordinates of the applied stresses of the RC element with due consideration to the concrete shear stress term (τ_c). In this paper,

Fig. 1 Finite element model using fiber formulation.

the SMM model is formulated in the context of fiber-based beam-column finite elements, and accounting for the effect of transverse shear.

To formulate the SMM model, three coordinate systems are typically assumed as shown in Fig. 2. The first coordinate system (x, y) defines the local coordinate of the fiber element; the second coordinate system $(1, 2)$ represents the applied principal stresses; while the third coordinate system (r, d) represents the concrete principal coordinate system in which the concrete shear stress $\tau_{12}^c = 0$. In the figure, α_1 is the angle between the x -axis and 1 -axis, and α_r is the angle between the x -axis and r -axis.

The principal strains in the 1 - 2 coordinate system are derived from the x - y coordinate system as follows:

$$\varepsilon_1 = \varepsilon_x \cos^2 \alpha_1 + \varepsilon_y \sin^2 \alpha_1 + \gamma_{xy} \sin \alpha_1 \cos \alpha_1 \quad (2)$$

$$\varepsilon_2 = \varepsilon_x \sin^2 \alpha_1 + \varepsilon_y \cos^2 \alpha_1 - \gamma_{xy} \sin \alpha_1 \cos \alpha_1 \quad (3)$$

$$\gamma_{12} = 2(\varepsilon_y - \varepsilon_x) \sin \alpha_1 \cos \alpha_1 + \gamma_{xy} (\cos^2 \alpha_1 - \sin^2 \alpha_1) \quad (4)$$

The principal stresses in the 1 - 2 coordinate system are derived from the x - y coordinate system as follows:

$$\sigma_1 = \sigma_x^c \cos^2 \alpha_1 + \sigma_y^c \sin^2 \alpha_1 + \tau_{xy}^c 2 \sin \alpha_1 \cos \alpha_1 \quad (5)$$

$$\sigma_2 = \sigma_x^c \sin^2 \alpha_1 + \sigma_y^c \cos^2 \alpha_1 - \tau_{xy}^c 2 \sin \alpha_1 \cos \alpha_1 \quad (6)$$

$$\tau_{12} = (\sigma_y^c - \sigma_x^c) \sin \alpha_1 \cos \alpha_1 + \tau_{xy}^c (\cos^2 \alpha_1 - \sin^2 \alpha_1) \quad (7)$$

The transformation equations are graphically represented by Mohr's stress and strain circles in Fig. 3. In a fiber-based element formulation, the process of the state determination at the fiber level requires the calculation of the fiber stresses $\{\sigma_x, \sigma_y, \tau_{xy}\}$ from the strain state $\{\varepsilon_x, \varepsilon_y, \gamma_{xy}\}$. Because the SMM has been implemented in a Timoshenko type beam element, the values of $\varepsilon_x, \gamma_{xy}$ are typically known, while the lateral strain ε_y value needs to be evaluated from the equilibrium condition. A new process for evaluating these lateral strains is described in the next section.

Fig. 2 Coordinates systems in fiber model formulation.

2.2 Evaluation of lateral strain

The equilibrium equations needed to evaluate the stresses in the x - y coordinate system $\{\sigma_x, \sigma_y, \tau_{xy}\}$ as a function of the principal stresses resisted by concrete $\{\sigma_1^c, \sigma_2^c, \tau_{12}^c\}$ and the reinforcing bar stresses f_{sx} and f_{sy} along the x and y directions respectively are:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \cos^2 \alpha_1 & \sin^2 \alpha_1 & -2 \cos \alpha_1 \sin \alpha_1 \\ \sin^2 \alpha_1 & \cos^2 \alpha_1 & 2 \cos \alpha_1 \sin \alpha_1 \\ \cos \alpha_1 \sin \alpha_1 & -\cos \alpha_1 \sin \alpha_1 & \cos^2 \alpha_1 - \sin^2 \alpha_1 \end{bmatrix} \begin{Bmatrix} \sigma_1^c \\ \sigma_2^c \\ \tau_{12}^c \end{Bmatrix} + \begin{Bmatrix} \rho_{sx} f_{sx} \\ \rho_{sy} f_{sy} \\ 0 \end{Bmatrix} \quad (8)$$

where ρ_{sx} and ρ_{sy} are the smeared steel ratio in the direction of x and y respectively. The stresses in the x - y coordinate system can be written as a combination of concrete and steel stresses as follows:

$$\begin{aligned} \sigma_x &= \sigma_{cx} + \sigma_{sx} \\ \sigma_y &= \sigma_{cy} + \sigma_{sy} \\ \tau_{xy} &= \tau_{cxy} \end{aligned} \quad (9)$$

where superscript c stands for concrete and s stands for steel.

The second of the equilibrium equations in (8) is used to evaluate the lateral strain ε_y in fiber i owing to the fact that the value of σ_y equals zero:

$$\sigma_1^{c,i} \sin^2 \alpha_1 + \sigma_2^{c,i} \cos^2 \alpha_1 + \tau_{12}^{c,i} 2 \sin \alpha_1 \cos \alpha_1 + \rho_{sy}^i f_{sy}^i = 0 \quad (10)$$

Fig. 3 Stress and strain mohr circles.

which can also be written as:

$$\sigma_{cy}^i A_{cy}^i + \sigma_{sy}^i A_{sy}^i = 0 \quad (11)$$

where σ_{cy}^i is the concrete stress in the transverse direction of fiber i and is equal to the sum of the first three terms in Eq. (8), σ_{sy}^i is the steel stress in the transverse direction of fiber i , A_{cy}^i and A_{sy}^i are respectively the areas of concrete and steel within the spacing S of the transverse reinforcement as shown in Fig. 4, ρ_{sy}^i is the ratio of steel to concrete area in the transverse direction of fiber i and f_{sy}^i is the transverse steel bar stress which equals σ_{sy}^i .

Due to the nonlinear behavior of the concrete and steel, an iterative procedure is needed to determine the lateral strain ε_y that satisfies Eq. (8) (Petrangeli 1999). An initial value for ε_y is assumed at each fiber, and the iterations proceed until Eq. (8) is satisfied. The process for evaluating the concrete stress from the fiber strain through the concrete constitutive model is described next.

2.3 Evaluation of concrete and reinforcement stresses

The typical concrete stress-strain curves are derived from uni-axial tests, and hence the biaxial strains in the x - y direction $\{\varepsilon_x \ \varepsilon_y \ \gamma_{xy}\}$ need to be converted to equivalent uni-axial strains in the 1-2 direction $\{\bar{\varepsilon}_1 \ \bar{\varepsilon}_2 \ \gamma_{12}\}^T$ to calculate the concrete stresses.

Initially, the direction of cracks is calculated based on a rotating angle α_r derived from the strain state. At a rotating principal angle α_r , the concrete shear stress $\tau_{12}^c = 0$ (Fig. 3) and the value of α_r is:

$$\tan 2\alpha_r = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \quad (12)$$

Calculation of the rotating angle α_r is evaluated by assuming first a trial rotating angle α_r^* from Mohr circle (Fig. 3), that is later modified based on the current strain state to derive the correct rotating angle α_r :

$$\alpha_r^* = 0.5 \tan^{-1} \left(\left| \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} \right| \right) \quad (13)$$

After evaluating the ε_y term that satisfies the equilib-

rium conditions as described in the previous section, the calculated principal angle α_1 from the known stress state is evaluated as follows:

$$\tan 2\alpha_1 = \frac{2\tau_{xy}}{\sigma_x - \sigma_y} \quad (14)$$

The biaxial principal strains are then evaluated as follows:

$$\begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ 0.5\gamma_{12} \end{Bmatrix} = [R(\alpha_1)] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ 0.5\gamma_{xy} \end{Bmatrix} \quad (15)$$

Biaxial principal strains are needed to evaluate the equivalent uni-axial strains. The equivalent uni-axial strains are derived from the biaxial strains with the help of the modified Poisson ratios of cracked concrete, also called the Hsu/Zhu ratios (μ_{12}, μ_{21}) (Zhu and Hsu 2002). Based on test data, the following equations are suggested by Zhu and Hsu (2002):

$$\mu_{12} = 0.2 + 850\varepsilon_{sf} \quad \varepsilon_{sf} \leq \varepsilon_y \quad (16)$$

$$\mu_{12} = 1.9 \quad \varepsilon_{sf} > \varepsilon_y \quad (17)$$

where μ_{12} is the ratio of the resulting tensile strain increment in the principal 1-direction to the source compressive strain increment in the principal 2-direction, μ_{21} is the ratio of the resulting compressive strain increment in the principal 2-direction to the tensile source strain increment in the principal 1-direction, ε_{sf} is defined as the strain in the reinforcement that yield first, and ε_y is the yield strain of the reinforcing steel.

After cracking, the Hsu/Zhu ratio μ_{12} lies outside the typical range of zero to 0.5 for Poisson ratios of continuous materials. Before cracking the Hsu/Zhu ratio $\mu_{21} = 0.2$ and, after cracking the Hsu/Zhu ratio $\mu_{21} = 0$, indicating that the tensile strain does not have any effect on the compressive strain.

The equivalent uni-axial strains are derived from the biaxial principal strains with Hsu/Zhu ratios (μ_{12}, μ_{21}) as follows:

$$\begin{Bmatrix} \bar{\varepsilon}_1 \\ \bar{\varepsilon}_2 \\ 0.5\gamma_{12} \end{Bmatrix} = [\mu] \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ 0.5\gamma_{12} \end{Bmatrix} \quad (18)$$

Fig. 4 Global and principal stresses.

where

$$[\mu] = \begin{bmatrix} \frac{1}{1-\mu_{12}\mu_{21}} & \frac{\mu_{12}}{1-\mu_{12}\mu_{21}} & 0 \\ \frac{\mu_{21}}{1-\mu_{12}\mu_{21}} & \frac{1}{1-\mu_{12}\mu_{21}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (19)$$

The equivalent uni-axial principal strain in the longitudinal reinforcement is given by:

$$\begin{aligned} \bar{\varepsilon}_{sx} = & \left(\frac{1}{1-\mu_{12}\mu_{21}} \varepsilon_1 + \frac{\mu_{12}}{1-\mu_{12}\mu_{21}} \varepsilon_2 \right) \cos^2(\alpha_1) \\ & + \left(\frac{\mu_{21}}{1-\mu_{12}\mu_{21}} \varepsilon_1 + \frac{1}{1-\mu_{12}\mu_{21}} \varepsilon_2 \right) \sin^2(\alpha_1) - \gamma_{12} \sin(\alpha_1) \cos(\alpha_1) \end{aligned} \quad (20)$$

The equivalent uni-axial principal strain in the transverse reinforcement is given by:

$$\begin{aligned} \bar{\varepsilon}_{sy} = & \left(\frac{1}{1-\mu_{12}\mu_{21}} \varepsilon_1 + \frac{\mu_{12}}{1-\mu_{12}\mu_{21}} \varepsilon_2 \right) \sin^2(\alpha_1) \\ & + \left(\frac{\mu_{21}}{1-\mu_{12}\mu_{21}} \varepsilon_1 + \frac{1}{1-\mu_{12}\mu_{21}} \varepsilon_2 \right) \cos^2(\alpha_1) + \gamma_{12} \sin(\alpha_1) \cos(\alpha_1) \end{aligned} \quad (21)$$

The equivalent uni-axial longitudinal steel stress f_{sx} and transverse steel stresses f_{sy} are calculated from the steel strains $\bar{\varepsilon}_{sx}$ and $\bar{\varepsilon}_{sy}$ respectively through a steel material stress-strain relationship.

The current equivalent uni-axial strains $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$ are individually used to calculate the concrete stresses σ_1^c and σ_2^c respectively in the principal direction from the stress f_c - strain ε_c relation of the uni-axial concrete material. Equivalent uni-axial steel strains $\bar{\varepsilon}_{sx}$ and $\bar{\varepsilon}_{sy}$ are used to calculate the steel stresses f_{sx} and f_{sy} , respectively from the stress f - strain ε relation of the uni-axial concrete material as described in the next section.

2.4 Uni-axial constitutive relationships of concrete and steel

Concrete model

The envelope curve of the concrete in compression follows the modified Kent and Park model (Park et al. 1982), which offers a balance between simplicity and accuracy. The model implemented in this study has the

Fig. 5 Monotonic stress-strain envelope of concrete.

following characteristics: First, the effect of softening in both the stress and strain curves was considered; second, the successive degradation of stiffness of both the unloading and reloading curves for increasing values of compressive strain was considered; and third, the effect of tension-stiffening is considered (Belarbi and Hsu 1994).

The monotonic stress-strain envelope of concrete is represented by a parabolic curve (Fig. 5):

$$f_c = f'_c \left[2 \left(\frac{\varepsilon_c}{\varepsilon_o} \right) - \left(\frac{\varepsilon_c}{\varepsilon_o} \right)^2 \right] \quad (22)$$

The compressive stress-strain curve of concrete typically softens due to the perpendicular tensile strains that reduce the compressive stress and strain of concrete with an appropriate softening coefficient ζ . When the softened stress-strain curve is modeled, it is assumed that the slope of the line connecting the origin and peaks of the non-softened and softened curve is the same. The pre-peak ascending curve and the post-peak descending curve of the softened stress-strain curve are assumed to adhere to a parabolic shape. The descending section of the softened parabolic curve is gently sloped until the stress reaches 20 percent of maximum stress $\zeta f'_c$ at a strain of ε_{20} . The parabolic softened stress-strain curve follows the relationships below, which was validated by Mullapudi (2010) and implemented in this work:

$$\text{Region OA, } \varepsilon_c \leq \zeta \varepsilon_o, f_c = \zeta f'_c \left[2 \left(\frac{\varepsilon_c}{\zeta \varepsilon_o} \right) - \left(\frac{\varepsilon_c}{\zeta \varepsilon_o} \right)^2 \right] \quad (23)$$

Region AB,

$$\zeta \varepsilon_o < \varepsilon_c \leq \varepsilon_{20}, f_c = \zeta f'_c \left[1 - 0.8 \left(\frac{\varepsilon_c - \zeta \varepsilon_o}{\varepsilon_{20} - \zeta \varepsilon_o} \right)^2 \right] \quad (24)$$

$$\text{Region BC, } \varepsilon_c > \varepsilon_{20}, f_c = 0.2 \zeta f'_c \quad (25)$$

The tangent modulus is evaluated as follow:

$$\text{Region OA, } \varepsilon_c \leq \zeta \varepsilon_o, E_t = \frac{2f'_c}{\varepsilon_o} \left(1 - \frac{\varepsilon_c}{\zeta \varepsilon_o} \right) \quad (26)$$

Region AB,

$$\zeta \varepsilon_o < \varepsilon_c \leq \varepsilon_{20}, E_t = -1.6 \zeta f'_c \left(\frac{\varepsilon_c - \zeta \varepsilon_o}{(\varepsilon_{20} - \zeta \varepsilon_o)^2} \right) \quad (27)$$

$$\text{Region BC, } \varepsilon_c \leq \varepsilon_{20}, E_t = 0 \quad (28)$$

Steel model

The hysteretic stress-strain relationship of reinforcing steel is described by the nonlinear model of Menegotto and Pinto (1973), as modified by Filippou et al. (1983) to include isotropic strain hardening (Fig. 6). The model accounts for the Bauschinger effect under cyclic loads and agrees very well with experimental results. The as-

Fig. 6 Stress-strain relationship of reinforcing steel.

cending curve follows a straight line with the slope of an initial elastic modulus E_0 , then joins another straight line with a slope of $E_1 = bE_0$ where, the parameter b is the strain hardening ratio. The curvature of the transition between the two curves is governed by the parameter R , which permits a good correlation of the Bauschinger effect. The curvature parameter R depends on the absolute strain difference between the current curve intersection point and the previous maximum or minimum strain reversal point, yield strain, and the parameter R_0 which is the parameter R at virgin loading. The experimentally determined parameters a_1 and a_2 are calibrated to represent the degradation of curvature with each of the successive cycles.

2.5 Concrete biaxial constitutive relations

The biaxial concrete constitutive models in the principal directions 1-2 is evaluated based on the following three strain conditions:

Case1: 1-Tension, 2-Compression

In this case, the equivalent uni-axial strain of concrete $\bar{\varepsilon}_1$ in principal direction 1 is in tension, and the equivalent uni-axial strain $\bar{\varepsilon}_2$ in principal direction 2 is in compression. Due to this condition, the uni-axial concrete stress σ_1^c in direction 1 is calculated from $\bar{\varepsilon}_1$, and is not a function of the perpendicular concrete strain $\bar{\varepsilon}_2$. The compressive strength in principal direction 2 σ_2^c will soften due to the tension in the orthogonal direction. Zhu and Hsu (2002) derived a softening equation in the tension-compression region (Fig. 5), and is based on panel testing as proposed in Belarbi and Hsu (1995). The equation for compressive strength and strain reduction factor ζ in the presence of transverse shear was given earlier in Eq. (1).

The softening coefficient ζ value is limited to 0.9, because the uni-axial concrete compressive strength f_c' is calculated from standard cylinder tests, while it was observed from the panel experiments at the University

of Houston, that the concrete strength does not reach f_c' . The reason is due to size effect, loading rate effect and shape factor which have ample effects on the concrete compressive strength f_c' . The ultimate stress in the orthogonal directions is therefore $\zeta f_c'$ at a softened strain $\zeta \varepsilon_0$ (Fig. 5).

Case2: 1-Tension, 2-Tension

The equivalent uni-axial strain of concrete $\bar{\varepsilon}_1$ in direction 1 is in tension, and the equivalent uni-axial strain $\bar{\varepsilon}_2$ of concrete in direction 2 is also in tension. In this case, the uni-axial concrete stress σ_1^c in direction 1 is evaluated from $\bar{\varepsilon}_1$, and σ_2^c in direction 2 is evaluated from $\bar{\varepsilon}_2$. Both σ_1^c and σ_2^c are functions of the orthogonal concrete strains $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$, respectively.

Case3: 1-Compression, 2-Compression

The equivalent uni-axial strains of concrete in principal directions 1 and 2 are both in compression. The current research uses the Vecchio's (1992) simplified version of Kupfer et al. (1969) biaxial compression strength equation. The concrete compressive strength increase in one direction depends on the confining stress in the orthogonal direction. The strength enhancement and increase in ductility are dependent on the biaxial compressive stresses. Concrete in compression exhibits lateral expansion and an increase in the value of Poisson ratio. An upper limit of 0.5 has been considered for Poisson ratio. The principal stresses in the two orthogonal directions are denoted by σ_1^c , σ_2^c and their corresponding strains are referred by $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$. The equations for the biaxial compression failure surface are:

$$K_{c1} = 1 + 0.92 \left(\frac{-\sigma_2^c}{f_c'} \right) - 0.76 \left(\frac{-\sigma_2^c}{f_c'} \right)^2$$

$$K_{c2} = 1 + 0.92 \left(\frac{-\sigma_1^c}{f_c'} \right) - 0.76 \left(\frac{-\sigma_1^c}{f_c'} \right)^2 \quad (29)$$

where K_{c1} and K_{c2} are the bi-axial strength magnification factors. The ultimate stresses in the orthogonal directions are:

$$\sigma_{1p} = K_{c1} f_c'$$

$$\sigma_{2p} = K_{c2} f_c' \quad (30)$$

The ultimate strains in the orthogonal directions are:

$$\varepsilon_{1p} = K_{c1} \varepsilon_0$$

$$\varepsilon_{2p} = K_{c2} \varepsilon_0 \quad (31)$$

where ε_0 is the uni-axial compressive strain of the concrete at the peak stress.

Case 4: 1-Compression, 2-Tension

In this case, the equivalent uni-axial strain of concrete $\bar{\varepsilon}_1$ in principal direction 1 is in compression, and the equivalent uni-axial strain $\bar{\varepsilon}_2$ in principal direction 2 is in tension. Due to this condition, the uni-axial concrete stress and strain in direction 1 softens similar to the case

1 condition. The uni-axial concrete stress σ_2^c in direction 2 is calculated from $\bar{\epsilon}_2$, and is not a function of the perpendicular concrete strain $\bar{\epsilon}_1$.

3. Validation of finite element MODEL

In this section, implementation of the previously described model to predict the behavior of different structural elements is presented.

3.1 Columns tested by Kobayashi *et al.* (1986)

Kobayashi *et al.* (1986) performed an experimental program to obtain the interaction diagram of ultimate shear strength of columns subjected to bi-directional horizontal forces. All specimens had the same cross section 150 mm x 150 mm (5.9"x5.9") as shown in Fig. 7. The shear span to depth ratio (a/d) was fixed as 2.0 for all specimens in order to avoid the occurrence of diagonal shear cracks from the compressive zone of the bottom to that of the top of the specimen for every loading path. To increase the bending strength of specimens without in-

creasing the amount of longitudinal reinforcement, all specimens were longitudinally reinforced by eight deformed bars of 10.2 mm (0.4") diameter for which the yield stress was increased by hardening. The lateral stirrups were 4.1 mm (0.16") diameter and with a spacing of 50 mm (1.97"). The yield stress for the longitudinal steel was 841.4 MPa (122 ksi) and the concrete compressive strength was 23.4 MPa (3.4 ksi). The column section was subdivided into 12 concrete fibers and 3 steel fibers, and modeled with only one element along the length. A Gauss-Lobatto integration scheme with five integration points is used in the analysis. These numbers of sections and fibers proved to be sufficient to reach a converged solution. The model is analyzed with a displacement-control strategy with an appropriate time variant lateral displacement.

In this experiment, two types of loading paths were adopted. In the first, the angle between the displacement and the direction of the principle axis of the cross section was kept constant. The number at the end of the specimen name specifies this angle (e.g. BS15D,

Fig. 7 Columns tested by Kobayashi *et al.* (1986).

Fig. 8 Load-deflection behavior of Kobayashi *et al.* Specimen BS00.

Fig. 9 Load-deflection behavior of Kobayashi *et al.* Specimen BS02.

Fig. 10 Load-deflection behavior of Kobayashi *et al.* Specimen BS06.

Fig. 11 Load-Deflection Behavior of Kobayashi *et al.* Specimen BS08.

Fig. 12 Load-deflection behavior of Kobayashi *et al.* Specimen BS30D.

Fig. 13 Load-Deflection Behavior of Kobayashi *et al.* Specimen BS45D.

BS30D, BS45D). In the second, the horizontal force in one direction was kept constant, and the force in the orthogonal direction was increased until failure. The number associating the specimen name in this case shows the rate of shear in the first direction to the ultimate shear strength under unidirectional horizontal force (BS00, BS02, BS04, BS06, BS08). All specimens exhibited a shear failure mode. Yielding of longitudinal reinforcement was not observed and there were no splitting cracks due to bond effect along the longitudinal

reinforcement. At the top and bottom of the specimen, crushing of the concrete in the compression zone occurred at the maximum strength.

Figures 8-11 show the experimental and analytical load-deformation behavior of specimens BS00, BS02, BS06 and BS08. A good match was observed between the analysis and experiments. In addition, the figures revealed the model was able to accurately capture the reduction in strength due to the presence of the transverse load. The same plots are shown for specimens BS30D and BS45D in Figs. 12-13. The same conclusions were observed for this series of specimens. The

Table 1 Summary of FE results.

paper	Specimen	$(F_{in})_{max,exp}$ kN (kip)	$(F_{out})_{max,exp}$ kN (kip)	(F_{out}) (%)	$(F_{in})_{model}$ kN (kip)	$(F_{in})_{model} / (F_{in})_{max,exp}$ %
Columns by Yoshimura	S00	415 (93.4)	0	0	418 (94.1)	-0.72
	S15	385 (86.6)	147 (33.1)	35.4	403 (90.8)	-4.85
	S25	350 (78.8)	245 (55.1)	59.0	341 (76.9)	2.35
	S35	250 (56.3)	343 (77.2)	82.7	251 (56.5)	-0.4
	D00	464 (104.6)	0	0	503 (113.4)	-8.43
	D15	425 (95.6)	147 (33.1)	31.6	438 (98.7)	-3.22
	D25	410 (92.3)	245 (55.1)	52.7	361 (81.2)	11.98
	D35	310 (69.8)	343 (77.2)	73.8	320 (72.1)	-3.37
Columns by Kobayashi <i>et al.</i>	BS00	56 (12.6)	0	0	52 (11.6)	7.72
	BS02	49 (11.0)	11 (2.5)	20	49 (11.0)	0.4
	BS04	49 (11.0)	22 (5.0)	40	44 (9.9)	10.00
	BS06	34 (7.7)	33 (7.5)	60	34 (7.7)	0.7
	BS08	20 (4.4)	45 (10.1)	80	20 (4.6)	-4.58
	BS15D	52 (11.7)	17 (3.9)	30.7	52 (11.6)	1.13
	BS30D	44 (9.9)	27 (6.1)	48.2	44 (9.9)	0.00
	BS45D	37 (8.3)	44 (9.9)	78.9	43 (9.6)	-2.35
Tilted beams by Hansapinyo <i>et al.</i>	B0-1	216 (48.6)	0	0	218 (49.1)	-0.97
	B45_1	165 (37.2)	165 (37.2)	76.47	164 (36.9)	0.67
	B45W_1	104 (23.4)	104 (23.4)	80.31	95 (21.4)	8.54
	B0_2	165 (37.1)	0	0	163 (36.7)	1.03
	B20_2	147 (33.1)	53 (12.0)	68.98	147 (33.1)	0.04
	B20W_2	101 (22.8)	37 (8.3)	81.18	105 (23.6)	-3.50
	B45-2	80 (18.0)	80 (18.0)	103.33	79 (17.8)	1.35
	B45_2	57 (12.9)	57 (12.9)	126.35	58 (13.1)	-1.08
	B90-2		78 (17.5)	0	77 (17.3)	0.78
	B0-3	250 (56.3)	250 (56.3)	0	255 (57.4)	-1.92
	B25-3	195 (44.0)	91 (20.5)	58.14	202 (45.5)	-3.33
	B45-3	141 (31.8)	141 (31.8)	90.06	139 (31.3)	1.56
	B90-3	157 (35.3)	157 (35.3)	0	154 (34.7)	1.79

initial slope was slightly stiffer than the experimental one due to the assumption of smeared crack behavior, and possibly due to slippage of the test set-up. **Table 1** shows the FE results for all specimens. The FE model was in general able to predict the strength of all specimens rather accurately.

3.2 Columns tested by Yoshimura (1996)

Yoshimura (1996) tested a group of short RC columns subjected to combination of bi-directional shear loads. First, a constant axial compressive load of 1470 kN (330.5 kip) and a fixed horizontal load in the y-direction of 0, 147, 245, and 343 kN (0, 33, 55, and 77.1 kip) were applied to the columns and kept constant. A horizontal load was then monotonically applied in the x-direction until failure. Under varying loading directions, the principle stresses and inclined cracks rotated in a three-dimensional manner. Specimens with two different reinforcement arrangements were tested. The cross section of the column was kept constant as 400 mm by 400 mm (15.75" by 15.75") for all specimens as shown in **Fig. 14**. For the S series specimen, only stirrups at spacing of 178 mm (7") were used, while the D series specimens contained additional hoops in the x-direction

Table 2 Mechanical properties of reinforcement for Yoshimura columns.

Bars	σ_y MPa (ksi)	σ_u MPa (ksi)	Elongation (%)	E_s MPa (ksi)
D10	372 (54.0)	536 (77.7)	0.205	181001 (26252)
D22	405 (58.7)	592 (85.9)	0.205	197997 (28717)

Table 3 Mechanical properties of concrete for Yoshimura columns.

$C \sigma_B$ MPa (psi)	26 (3713)
$C \sigma_t$ MPa (psi)	2.3 (331)
$C E(1/3)$ MPa (ksi)	21098 (3060)

at spacing of 89 mm (3.5"). The mechanical properties of materials are presented in **Tables 2** and **3**. The column section is subdivided into 12 concrete fibers and 5 steel fibers, and modeled with only one element along the length. A Gauss-Lobatto integration scheme with five integration points is used in the analysis. These

Fig. 14 Details of columns D and S series tested by Yoshimura (1996).

Fig. 15 Load-Deflection Behavior of Yoshimura Specimen D00.

Fig. 16 Load-Deflection Behavior of Yoshimura Specimen D35.

Fig. 17 Load-Deflection Behavior of Yoshimura Specimen S25.

Fig. 18 Load-Deflection Behavior of Yoshimura Specimen S35.

numbers of sections and fibers proved to be sufficient to reach a converged solution. The model is analyzed with a displacement-control strategy by applying a constant force at the top of the column with an appropriate time variant lateral displacement. The columns were tested in a single curvature set up.

Figures 15-18 show the experimental and analytical

load-deformation plots of several test specimens. The vertical axis represents the applied load in the x-direction, while the horizontal axis represents the corresponding deflection of the column. The figures show that providing the hoop stirrups in the x-direction increased the strength of specimens for the D-series compared to those of the S-series. The experimentally ob-

tained initial stiffness was quite similar in all specimens, but the point where this stiffness noticeably decreased is different and depended on the magnitude of pre-loading in the y -direction. Note that the load applied is a mixed force- and displacement-control loading. In force-controlled loading, the post-peak response cannot be obtained. The analytical model was able to capture the effect of transverse load on the capacity of the column in the in-plane direction. Increasing the lateral loads caused reduction in the in-plane strength of the columns. The model slightly overestimated the shear capacity due to the assumption of neglecting the shear deformations of the reinforcement. **Table 1** shows a comparison between the experimental results and analytical predictions for all tested specimens.

3.3 Beams tested by Hansapinyo *et al.* (2003)

Hansapinyo *et al.* (2003) tested thirteen reinforced concrete beams with square and rectangular cross sections to investigate the ultimate capacity under bi-axial shear loading. Tests of tilted simply-supported reinforced concrete beams under a concentrated load at midspan were performed in this study as shown in **Fig. 19**. The vertical force P is applied monotonically to the specimen with one loading stub at midspan and two support stubs at both ends. These loading and support stubs are made of reinforced concrete, which are cast together with the specimens. The reinforcement inside the stubs is arranged to prevent bearing failure. The applied vertical force is resolved into two forces acting in the direction of principal axes of the beam section. Hence, two shears are induced simultaneously. The ratio of shear loads in the x and y directions can be changed according to the inclination angle of the beam (β) between the principal axes and the line of load P . This experimental set-up requires only one hydraulic jack to apply bi-axial shear to the beam specimen; the test procedure, therefore, is quite simple but the results obtained are informative and similar to the set-up using multiple jacks. The beam section is subdivided into 10 concrete fibers and 7 steel fibers, and modeled with four elements along the length. A Gauss-Lobatto integration scheme with five integration points is used in the analysis. These numbers of sections and fibers proved to be sufficient to reach a converged solution. The model is analyzed with a displacement-control strategy with an appropriate time variant lateral displacement.

Fig. 19 Test set-up by Hansapinyo *et al.* (2003).

Fig. 20 Load-deflection behavior of Hansapinyo *et al.* Specimen B0_1.

Three series of reinforced concrete beams were designed to investigate the bi-axial shear capacity. Series I contains three reinforced concrete beams having square cross section with the inclination angle of 0° and 45° . Series II contains 6 specimens, while 4 specimens belong to series III. The concrete compressive strength of the specimens was in the range of 28 and 32 MPa (4 and 4.6 ksi). Deformed bars with diameter of 25 mm (0.98 in.) are used for the longitudinal reinforcement. Two types of stirrups used for all specimens were 6 and 9 mm (0.24 in. and 0.36 in.) diameter with a yield strength of 28 MPa (54 ksi). Adequate longitudinal reinforcement was provided in all specimens to prevent flexural failure. For the shear span-to-depth ratio (a/d) of the specimens subjected to vertical loads, series I and II have $a/d = 2.6$ and series III has $a/d = 2.68$. These ratios are in the range classified for slender beams, which should fail in a diagonal mode. **Figures 20-22** show the analytical load-deformation behavior of specimens B0_1, B45_1, and B45W_1. The figures confirmed that the model can accurately predict the ultimate shear capacity of the beams. **Table 1** summarizes the analytical prediction for all beam specimens. As shown, the FE model predicted rather well the shear capacity.

4. Conclusions

This paper presents a new fiber-based beam-column finite element that account for the effect of transverse shear loads. The derivation of the element assumes that concrete cracks are smeared, and neglect the shear deformations of the reinforcement. The concrete constitutive law was based on the recently developed Softened Membrane Model (SMM), with a new softening coefficient derived from experimental tests on concrete panels that account for transverse shear. The new finite element model was used to conduct a correlation study with a large number of experimentally tested RC columns subjected to multi-directional shear loads. The model in general provided very good predictions of the experimental results, and successfully simulated the reduction in strength due to the effect of multi-directional loads.

Fig. 21 Load-deflection behavior of Hansapinyo *et al.* Specimen B45_1.

Fig. 22 Load-deflection behavior of Hansapinyo *et al.* Specimen B45W_1.

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