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Multi-scale based Simulation of Shear Critical Reinforced Concrete Beams Subjected to Drying

Esayas Gebreyouhannes¹, Taiju Yoneda², Tetsuya Ishida³ and Koichi Maekawa³

Abstract

The impact of drying at both material and structural levels is discussed based on the multi-scale thermo-mechanistic modeling. The effects of hygral states at nano to micro scales on the macro scale mechanics are focused on. It is quantitatively confirmed that the moisture state-dependent tensile strength and crack-dependent moisture diffusivity models play a key role in tackling with the responses of reinforced concrete (RC) members subjected to combined hygro-mechanical actions. Full 3D thermo-mechanics simulation of shear critical RC beams with effective depths, which ranges from 250mm to 1000mm, is carried out from the onset of casting. It is found that the impact of drying increases with the increasing effective depth at the constant beam width. Overall, it is emphasized that linkage of material science and structural mechanics is indispensable for holistic understanding of RC members under coupled mechanical loading and drying. The multi-scale analytical scheme is proved to be applicable for the performance assessment of RC members at ultimate states as well.

1. Introduction

Concrete structures are subjected to drying due to hygral imbalance between the pore spaces and ambient conditions. Drying of concrete is regarded as a quasi-equilibrated process of thermodynamics under normal atmospheric conditions. Although the outer layers reach hygral equilibrium with the humidity of environment quickly, due to its low permeability, the inner part may remain relatively high degree of saturation for decades. Thus, steep hygral gradients are created at the surface and the shrinkage of this infinitesimal thin surface layer is hindered by the inner layers. Consequently, shrinkage induced self-equilibrated stresses are created, leading to at least micro-cracking at the surface. As a matter of fact, the induced micro-cracking appears in any concrete structure (Bazant and Raffshol 1982). Drying not only causes surface cracking, but also can lead to penetrating cracks depending on its severity and restraint conditions. These effects of drying are generally related to durability and structural performance.

The variation in hygral states within the microstructure of cementitous composites has so much to do with the shrinkage, creep, chemical reactions and micro-cracking. Mechanical behavior of cement-based materials is strongly influenced by drying (Cook and Haque 1974; Okajima et al. 1980; Yurtadas et al. 2004; Skoczylas et al. 2005). Bazant et al. (1987) investigated the effect of cracking on the drying permeability and diffusivity of concrete. At structural level, Sato and Kawakane (2008) carried out extensive study on the effects of self-desiccation on shear performance of high strength RC beams. Recently, Mitani et al. (2011) investigated the influence of drying on the shear performance of reinforced normal strength concrete beams, showing significant influence. In this study, the authors take this contribution seriously in line with the future upgrading of design and maintenance specifications.

A multitude of approaches have been also proposed to simulate the effects of drying. At structural levels approaches principally based on the reductions of concrete tensile strength (Shima 1987; Tsuchiya et al. 2002) or approaches based on equivalent reinforcement ratio have been proposed (Kawakane and Sato 2008; Mitani et al. 2011). These methods are mainly of empirical type or sole mechanistic approaches with simplified assumptions. Doubtlessly, these approaches can provide helpful predictions about the impacts of drying for small number of cases. On the other hand, for holistic understanding of the realistic RC behaviors subjected to drying, the mutual linkage of material science and structural mechanics plays a key role. Under coupled hygro-mechanical loading, physico-chemical events of nano to micro levels occur with time inside concrete. These mutually interlinked multi-scale events are the driving forces to influence the macro scale behaviors. These are manifested in the mechanical properties, moisture transmission, shrinkage and the state of micro-cracks in concrete space.

The multi-scale analysis provides an integration of the thermo-hygro, physico-chemical models of cementitious composites (Maekawa et al. 1999) and the mechanical models of RC (Maekawa et al. 2003). The thermo-hygral model is responsible for computing ce-
ment hydration reactions in the composite mixture and the pore structure formation as well as moisture migration. The mechanistic models are responsible for predicting the material-structure responses from mm to meter-scale based on the space averaged constitutive laws on the fixed four way cracked concrete modeling. The applicability of this multi-scale platform is verified for the long-term excessive deflection of PC viaducts (Ohno et al. 2012). The major target of the current paper is to tackle with the modeling of drying problems and to demonstrate the applicability for ultimate limit states under coupled hygro-mechanical actions.

2. Simulation of concrete under drying

2.1 Analytical framework - Three-dimensional multi-scale thermo-hygro-mechanical modeling

DuCOM-COM3 is a multi-scale analysis program that mutually links chemo-physics platforms Du-COM and the nonlinear structural analysis coded as COM3 (Maekawa et al. 2008). The former is an integrated thermo-hygral analysis platform that includes multi-component hydration, micro-pore structure and mass transport models for concrete ranging from 10^{-3} to 10^{-9} meter scale, while the latter is a mechanistic 3D finite-element analysis platform with uncracked and cracked concrete constitutive laws rooted in the multi-directional fixed crack model. The DuCOM-COM3 is capable of tracing the micro to nano-scale hygral states and mechanical events of concrete over its life. This special attribute makes it a tool for simulating concrete responses under coupled environmental and mechanical loading. Figure 1 shows the scheme of the multi-scale thermo-hygro-mechanical analytical framework.

2.2 Local mechanical properties

Moisture migration in pores is the key driving force for shrinkage in concrete. This moisture migration is triggered by hygral gradients arising from external drying and/or self-desiccation due to hydration reaction. For low water to cement ratio, the autogenous shrinkage brought by self-desiccation is predominant and its distribution is spatially uniform in concrete. Gebreyouhannes and Maekawa (2012) successfully simulated the shear capacity and post-peak ductility of reinforced concrete beams, which are greatly affected by autogenous shrinkage using the multi-scale analytical framework (Maekawa et al. 2008). Subsequently, the target of this paper is to tackle with the externally driven drying, using this same analytical framework. When the water to cement ratio is not so much low, drying shrinkage becomes predominant rather than self-desiccation. The unstable hygral equilibrium in concrete can easily be disrupted due to external drying, creating hygral gradients. The outermost layer reaches local hygral equilibrium with the relative humidity (RH) of the ambient condition quickly with the inner part remaining at higher saturation degree for long time. This leads to non-uniform shrinkage. Shrinkage of the infinitesimal thin surface near layers, however, is hindered by inner...
core part. As a result, large stress associated strain is imposed on these outer layers, causing micro-cracking. This process continues to the subsequent layers up to a certain limit of depth, which depends on the drying rate, specimen size, surface area, exposure condition, creep, elastic modulus, tensile strength and strain softening.

Several reports indicated that drying influences the tensile strength of concrete. Ueda et al. (1997) reported that drying causes an initial decrease in the tensile strength of specimens followed by progressive increase up to 12% increment. Hansen (1968) also indicated initial decrement followed by a progressive increase up to zero percent in bending and slight increment in splitting test. Cook (1974) experimentally observed a reduction up to 50% of the tensile strength for water soaked specimens after oven drying. These reports are made based on the tensile strength of the specimens. Inherently, the specimens include micro-cracks at their surface. The micro-cracks have a negative impact to the specimen tensile strength because they reduce the effective resisting section of the specimen. On the other hand, the near surface parts of the specimens are at reduced hygral state than the inner part. This reduced hygral state in the pores of the surface concrete leads to increment in local tensile strength. Therefore, the specimens exhibit like a structure and the reported tensile strength data are the resultant of the mentioned two actions with minus and plus roles. From the above discussion, it can be inferred that the actual increase in local tensile strength upon drying must be higher than what is reported at macro levels. Increase in capillary pressure brought by drying is one possible mechanism that can lead to an increase in the mortar strength. Thus, it is crucial to consider the interdependence of hygral state and the uniaxial tensile strength of concrete.

In consideration of the varying hygral states along the depth of members, concrete under drying conditions cannot be regarded as uniform. Depending on the hygral states, the spatial distribution of mechanical properties is different. Therefore, cracking criterion shall be set based on the local tensile strength rather than by the nominal strength obtained from uniaxial tests. Based on Biot’s theory (1941), Yoneda et al. (2012) proposed a local tensile strength model by taking into account the shrinkage stress caused by negative capillary pressure.

The shrinkage stress depends on local hygral state. Based on the poro-mechanics theory (Biot 1941), the deformation of isotropic poro-elastic materials can be approximately regarded as independent from the pore water pressure. Accordingly, the volumetric stress of water-solid system $\sigma'_{ij}$ can be expressed by,

$$\sigma'_{ij} = \sigma_{ij} + \alpha p \delta_{ij}$$  \hspace{1cm} (1)

where, $\sigma_{ij}$ is stress tensor of solid skeleton, $p$ is pore water pressure, $\alpha$ is coefficient, $\delta_{ij}$ is Kronecker delta. $\alpha$ is usually set as 1.0. Unlike the water-soil system, the pore water pressure in concrete is below the atmospheric one (i.e., negative pressure). The negative water pressure acts as pre-stress and enhances the local resistance against tensile stress. This effect can be regarded as enhancement of the local tensile strength and its formulation can be expressed by,

$$f_i = f_{she} + \sigma_{ps}$$  \hspace{1cm} (2)

where $f_{she}$ is the skeleton strength of concrete, $\sigma_{ps}$ is shrinkage stress which works as pre-stress. When concrete is saturated, shrinkage stress becomes zero and the effect of pre-stress disappears. In this case, the local tensile strength equals to the equivalent strength of saturated concrete specimens.

**Figure 2** shows the comparison of direct tension experimental results (Ueda et al. 1997) and analytical one by incorporating the local hygral-state dependent tensile strength model. As can be seen both results indicate an initial drop in tensile strength followed by gradual increase in strength. The initial drop is attributed to micro-cracking effect at the surface of the specimen which reduces the effective resisting section. The difference between the experiment and analysis is mainly attributed to the slight overestimation of the micro-cracking extent by the analysis. The depth of micro-cracking from the surface is about 0.5-0.9 times the maximum aggregate size after 14 days of drying. Similar results were also observed by Yoneda et al. (2012). On the other hand, as drying continues, the local strength also increases thereby indicating recovery in tensile strength. The results show a clear necessity for the consideration of hygral-state dependent tensile strength, without which the tensile strength will continuously degrade and the cracking zone depth from the surface will highly be overestimated. Disregarding the dependency of local tensile strength on the hygral-state not only underestimates the tensile strength but also leads to underestimation of shrinkage due to relaxation caused by continued micro-cracking. Increase in capillary pressure brought by drying is one possible mechanism that can lead to an increase in the mortar strength, Yoneda et al. (2012).
2.3 Finite element discretization: referential volume

In some cases, small-scale finite elements are needed in order to capture a sharp moisture gradient developing close to outer surfaces of structures subjected to severe drying. Using such fine elements is ideal from the perspective of hygral analysis. However, the space-averaged constitutive models for concrete have been developed with regard to a referential size of 5-20cm. Therefore, finite element sizes less than the coarse aggregate shall be treated in consideration of the aggregate locality in space. One plausible approach is to apply a probabilistic process with concrete treated as a two phase; i.e., assembly of mortar and coarse aggregate. In consideration of the spatial dispersion of aggregates inside each finite element, an element size dependent fictitious cracking criteria is proposed when the size of elements less than the mean coarse aggregate is defined. Figure 3 illustrates the schematic concept of the fictitious cracking criteria. Here, a potential function for cracking criteria is proposed as,

$$\sigma_i > k_i f_i + k_i n_i$$

where, $k_i$ is a probabilistic value dependent on the normalized element size as indicated in Fig. 3. First, the normalized dimensions of an element with respect to the mean aggregates size in the x, y and z axes are denoted by $D_x$, $D_y$, and $D_z$, respectively. If $D_i$ is greater than 1.0, (i.e when more than one aggregate can exist in the $i^{th}$ direction) a crack plane perpendicular to the $i^{th}$ direction can definitely take place and the value of $k_i$ is set as 1.0. This means that the tensile strength in the $i^{th}$ direction is $f_i$. If $D_i$ is less than 1.0, (i.e when only an aggregate can exists in the $i^{th}$ direction) a crack plane perpendicular to the $i^{th}$ direction is less likely to occur. Here, the authors set up the digit value of 1.0 or 3.0 for $k_i$, assuming that the aggregate is three times stronger than the mortar in tension. For such a case, the digit value of $k_i$ is selected by using Monte Carlo method, i.e., if $D_i$ is less than the generated random number in between 0.0 and unity, $k_i = 3.0$, otherwise $k_i = 1.0$. Thus, the local tensile strength in the $i^{th}$ direction has the probability of being $3.0 f_i$ or $1.0 f_i$, depending on the normalized size of the element.

As mentioned above, at the meso-scale level, concrete can no more be regarded as isotropic. Thus, the cracking criteria for an element is decided according to Equation (3) on the basis of Rankine theory. The values of $l$, $m$, and $n$ are directional cosines for the principal stress vector. With this consideration, the stochastic strength of an element becomes variable in different directions. The above consideration ensures a minimum crack spacing equivalent to the mean size of aggregates at the surface layers. If the element is not smaller than the coarse aggregate, $\sigma_i > f_i$ is the cracking criterion as usual.

2.4 Cracking and drying permeability

The increase of overall drying permeability and diffusivity of concrete due to cracking is reported both theoretically and experimentally. Based on the assumptions of viscous laminar flow through continuous planar cracks of constant thickness, Bazant and Raftshol (1982) reported three order increase in moisture diffusivity for crack width of 0.3mm and crack spacing of 300mm. Bazant et al. (1987) experimentally found an increase in moisture diffusivity of 2.25 times for crack width of 0.1mm and crack spacing of 70mm. Gerard and Marchand (2000) also reported that cracks can increase the diffusivity of concrete by a factor of 2 to 10, depending on width, spacing, orientation and connectivity of the cracks.

For the permeability and diffusivity of concrete under drying, Bazant et al. (1987) carried out experiments on C-shaped specimens with crack producing tie rod, as shown in Fig. 4. The specimens were cast in the labora-
tory with the mix proportion of cement: sand: gravel: water = 1:2:2:0.5 (by weight). The maximum size of the coarse aggregate is 9.5mm. Two sets of specimens were cast: an initial set of two specimens with No. 3 bars (diameter 9.5 mm), and a second set of six specimens with No. 2 bars (diameter 6.4mm). Each set consists of uncracked and mechanically cracked specimens both being subjected to drying. The details of the experiment can be referred in the original paper (Bazant et al. 1987), and the summary is presented in Table 1. The geometry of the specimen is shown in Fig. 4.

To analytically determine the overall effect of cracks, which may serve as conduits to enhance moisture transmission, the cracked and uncracked specimens for each set are simulated using the multi-scale thermo-hygro model. To incorporate the effect of cracking on moisture conductivity of concrete, a tentative linear diffusivity equation is proposed as,

$$ J_i = -k_i (D_{ij} \nabla p_i + D_{ij} \nabla T) $$

where,

- $J_i$ – Total moisture flux
- $D_{ij}$ – the liquid and gas conductivity in concrete
- $D_{ij}$ – the liquid and gas conductivity in concrete in terms of temperature gradient
- $\nabla p_j$ – the absolute gradient of pore pressure
- $\nabla T$ – the temperature gradient
- $D$ – acceleration factor for cracked concrete
- $A_{ij}$ – the equivalent aerial strain

In this study $D$, the acceleration factor for cracked concrete, is tentatively estimated from the experimental facts (Bazant et al. 1987) of moisture loss in cracked concrete. The detailed discussion on the value of $D$ is required in future.

Figure 6a shows the finite element mesh of the specimen used. Flexural cracks are introduced by a displacement controlled loading up to an equivalent strain corresponding to the prescribed average crack spacing and average crack width. The corresponding distribution of strain is indicated in Fig. 6b. Note that in the current analysis, assembly of cracks is treated using smeared strain in the compatibility field equations rather than the discretely running ligaments and the color contour indicate the space averaged strain field. The bond between concrete and reinforcing bar is considered by tension stiffness model (Shima 1987) where integral of local bond behaviors are built according to experiments in

$$ k_i = \begin{cases} 1 + D \cdot (\epsilon_{ii} - 300 \times 10^{-6}) \times 10^3 & \text{for cracked concrete} \\ 1.0 & \text{for uncracked concrete} \end{cases} $$

Table 1 Experimental details (Bazant et al. 1987).

<table>
<thead>
<tr>
<th>Sample Id.</th>
<th>Cracking condition</th>
<th>Average crack spacing / width</th>
<th>Environmental Exposure</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 2</td>
<td>Uncracked</td>
<td></td>
<td>Six-weeks (Submerged at 80F)</td>
</tr>
<tr>
<td></td>
<td>Cracked</td>
<td>60mm/0.1mm</td>
<td>Two-Weeks (Rh=98 at 80F)</td>
</tr>
<tr>
<td>No. 3</td>
<td>Uncracked</td>
<td></td>
<td>Drying at Rh = 50 at 76F</td>
</tr>
<tr>
<td></td>
<td>Cracked</td>
<td>80mm/0.1mm</td>
<td></td>
</tr>
</tbody>
</table>
dry/wet conditions. Therefore, drying effect is inherently included without the need for explicit bond-slip model.

The computed water loss for the uncracked and cracked specimens are indicated in Figure 7, both for No. 2 and No.3 specimens. There exists a considerable difference (20 – 50%) in the normalized weight loss between the analysis and experiment. In the analysis high quality normal weight aggregate with a specific gravity of 2.7 and zero porosity is assumed. Apart from the weight ratio of cement:sand:gravel no details on the property of the ingredient materials is indicated in the original paper. Aggregate porosity has a remarkable influence on the water loss of concrete due to drying and differences up to two times has been observed both experimentally and analytically (Kokubu et al. 1969; Asamoto et al. 2008). In addition, the efficiency of sealing in the analysis is assumed to be perfect whereas it may not be easy to have a perfect sealing condition in the experiment. In fact, it is possible to achieve comparable results by taking several assumptions in the analysis. However, the authors call direct attention to the relative difference in weight loss between the cracked and uncracked specimens. Computationally, the difference of water loss between the cracked and uncracked specimens, which is rooted in the parameter $D$ in Equation...
analyzed and the results are indicated in the experiment by Ueda et al. (1997) experiment and DuCOM-COM3 analysis. The relative difference in water loss between the pseudo-uncracked and perfectly uncracked specimens is approximately 15% at 40 days and it gradually reduces with the increase of drying time. This indicates that moisture transmission through micro-cracks (caused by drying) affects the rate of water loss. For longer period of drying, the influence of these micro-cracks to the total water loss becomes less evident and similar results were also observed by Gerard and Marchand (2000) due to the discontinuous aggregate dispersion (see Fig. 8). Noteworthy, the specimen (Bazant et al. 1987) defined as “uncracked” means the one free from structural cracking. As a matter of fact, the specimens just subjected to drying will have some micro and dispersed cracks just close to the surfaces and do not reach the core of the members. As a trial, the authors conducted the analysis where no surface cracking is allowed by increasing the tensile strength and checked the moisture loss computed. The progress of water loss for this ideal specimen is also indicated in Fig. 7. The relative difference in water loss between the pseudo-uncracked and perfectly uncracked specimens is approximately 15% at 40 days and it gradually reduces with the increase of drying time. This indicates that moisture transmission through micro-cracks (caused by drying) affects the rate of water loss. For longer period of drying, the influence of these micro-cracks to the total water loss becomes less evident and similar results were also observed by Gerard and Marchand (2000) due to the discontinuous nature of these micro-cracks. Here it must be noted that, the moisture transmission through micro-cracks after the rate of water loss and this is important in the prediction of drying shrinkage at the initial periods of drying (Yoneda et al. 2012).

To further verify the proposed moisture diffusivity, the experiment by Ueda et al. (1997) on water loss is analyzed and the results are indicated in Fig. 8. In the experiment, details of aggregate and cement properties are provided and hence it is possible to make direct comparison in absolute terms between the experiment and analysis. The size of the specimen is 100 mm x 100 mm x 400 mm and is subjected to drying at a relative humidity of 60 ± 5% at T = 20 °C, after sealing for 28 days. It can be clearly seen that the analytical results is consistent with the experiment. By integrating the local tensile strength and moisture transmission models into the multi-scale analysis and in consideration of the aggregate dispersion (see Section 2.3), the following chapter further assesses the applicability of the multi-scale analysis for shear critical RC beams subjected to drying.

### 3. Shear critical RC Beams subjected to drying

To date numerous experiments on the determination of shear capacity have been carried out and several empirical formulae have been proposed. While the scatter of shear capacity in experiments of similar material and geometric conditions are usually explained by the heterogeneous nature of concrete, the rest may be attributed to some real physical cause. One potential reason is drying. Design codes consider the impact of drying for the serviceability limit states. But to date, there is not explicit consideration about the impacts of drying for the ultimate limit state of RC members. The background could be due to the common understanding that the impact of drying is prominent on local responses such as crack spacing rather than the macro responses like the ultimate limit states. As a matter of fact, drying effect does not reach the core of large sectional members and the impact of drying is limited just close to the surface area. But, it is not the case when we deal with RC members with thin sections. Recently, Mitani et al. (2011) experimentally investigated the effect of drying on the shear capacity of RC beams with small sectional sizes, and their result shows some disparity from the current understanding.

Unlike that of autogenous shrinkage, simulating the impact of drying on shear capacity of beams cannot simply be tackled by sole mechanistic approach. This is because the effects of drying are not uniform throughout the beam space. Therefore, it is less realistic to address the shear response of RC beams subjected to drying by using equivalent factors unlike the beams subjected to autogenous shrinkage of rather uniformity. To this end, the authors use the multi-scale approach, which is able to treat cement-chemical events as well as mechanistic processes without the need for any equivalent factors. Analysis can be carried out from the onset (post-casting) up to cement hydration in space. The microstructure, solid strength, elastic modulus, internal relative humidity, shrinkage, stress/strain, and cracking can be computed for specified ambient and loading conditions.

#### 3.1 Experimental details

Mitani et al. (2011) experimentally investigated the effect of drying on the shear capacity of RC beams. This section outlines the experimental details on physical properties of ingredients used, mixture proportion, geometry of specimens and environmental exposure, based on which the applicability of the multi-scale thermo-hygro-mechanistic based model for concrete is examined. Table 2 indicates the physical properties of ingredients used and the mixture proportion of the concrete for producing the RC beams. Table 3 indicates the dimensions and detailing of the specimens. The tensile
reinforcement ratio of the beams is 1.03% - 1.06% with no web reinforcement and compression bars. High strength steel with a yield strength of 1080MPa (Ultimate strength 1230MPa) is used to ensure that the beams definitely fail in the mode of shear. Three different beams’ sizes, with effective depths of 250mm, 500mm and 100mm are investigated.

For each size, two conditions are examined. One subjected to drying at a relative humidity of 46.1% and 15.2°C, and its companion sealed to prevent moisture flux. For the two different environmental conditions, a control prism test showing the evolution of shrinkage with time was reported.

Table 4 indicates the recorded test results of compressive strength, elastic modulus, tensile strength and fracture energy of concrete samples corresponding to the different exposure conditions. Detailed information on the experimental program is available in the original paper (Mitani et al. 2011).

### Table 3 Details of experiment by Mitani et al. (2011).

<table>
<thead>
<tr>
<th>Designation of Specimen</th>
<th>Initial Wet curing time (days)</th>
<th>Drying/sealed time (days)</th>
<th>$F_c$ (MPa)</th>
<th>$E_{qc}$ (%)</th>
<th>$F_t$ (MPa)</th>
<th>$G_t$ (N/mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-250</td>
<td>7 days</td>
<td>91</td>
<td>38.6</td>
<td>23.9</td>
<td>3.2</td>
<td>0.15</td>
</tr>
<tr>
<td>S-500</td>
<td>7 days</td>
<td>120</td>
<td>37.3</td>
<td>20.9</td>
<td>3.2</td>
<td>0.20</td>
</tr>
<tr>
<td>S-1000</td>
<td>7 days</td>
<td>120</td>
<td>36.7</td>
<td>25.9</td>
<td>3.2</td>
<td>0.15</td>
</tr>
</tbody>
</table>

### Table 4 Concrete Mechanical properties Mitani et al. (2011).

<table>
<thead>
<tr>
<th>Designation of Specimen</th>
<th>$b/h/l$</th>
<th>$c$ (mm)</th>
<th>$a$ (mm)</th>
<th>$d$ (mm)</th>
<th>$\rho_s$ (%)</th>
<th>$a/d$</th>
<th>$F_y/f_{ult}$ (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-250</td>
<td>300/350</td>
<td>200</td>
<td>750</td>
<td>250</td>
<td>1.03 (2D22)</td>
<td>3.0</td>
<td>1080/1230</td>
</tr>
<tr>
<td>S-500</td>
<td>300/580</td>
<td>500</td>
<td>1500</td>
<td>500</td>
<td>1.06 (2D32)</td>
<td>3.0</td>
<td>1080/1230</td>
</tr>
<tr>
<td>S-1000</td>
<td>300/1130</td>
<td>500</td>
<td>3000</td>
<td>1000</td>
<td>1.06 (4D32)</td>
<td>3.0</td>
<td>1080/1230</td>
</tr>
</tbody>
</table>

$b/h/l$: width/height/length of beam; $c$: distance between two loading points; $a$: shear span; $d$: effective depth; $\rho_s$: tension reinforcement ratio; $f_y$: yield strength of reinforcement; $f_{ult}$: ultimate strength of reinforcement.

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### 3.2 FE discretization and analysis

As indicated in Fig. 9, a fine mesh is set in the order of mm at the surface of the beams with the size of the elements increasing gradually as we go away from the surface. This is performed to capture the sharp hygral gradients caused by drying. To minimize the risk of shear locking, solid elements with eight nodes based on the enhanced strain formulation (Kasper and Taylor 1997) are used. To avoid the risk of divergence and ensure stability of the analysis, short time steps in the range of 0.01 days with a gradual change in ambient relative humidity, 1 to 2%, are set just after the specimens are exposed to drying. At the initial stage of drying, events are sharply changing and the consideration small time steps for the explicit time integration is necessary in the finite element analysis (Zienkiewicz and Taylor 2000) until the local moisture gradient becomes smaller. With this in mind, the authors carefully defined the sizing of the elements and the intervals of the time steps for the simu-
lation. This numerical treatment is indispensable for accurate analysis of RC members subjected to external drying unlike those of members subjected to self-desiccation.

Elements having size smaller than 1.5cm are modeled as discussed in section 2.2. Loading plates and support bearings are modeled as elastic elements. All steel bars were modeled as embedded smeared reinforcement inside the elements. The softening and stiffening parameters for the plain and reinforced concrete regions are obtained using the empirical formula proposed in the CEB-FIP Model Code.

Simulation of the experiments started from the onset of casting, just by the input of mix proportion and hygro-mechanical loading. At each time step, the thermo-hygral models responsible for computing cement hydration and the pore structure formation as well as moisture migration, are mutually linked with the mechanistic models. After the respective environmental exposures, the beams are subjected to a displacement controlled loading at a rate of 0.2mm/min until failure. Before discussing the shear response of the beams, it is imperative to check the reliability of the thermo-hygral component of the coupled multi-scale analysis using the meso-events measured in the experiment, which is the focus of the subsequent section.

3.3 Comparison of measured and computed meso-scale events
Prior to assessing the shear performance, the meso-scale events such as nominal compressive strength, elastic modulus, tensile strength and shrinkage of tested specimens obtained computationally will be compared with the measured results. These physical events mainly depend on the thermo-hygral modeling component and can directly serve as a verification of the subsystem.

With the intention of verifying the subsystem, Fig. 10 shows the computed development of compressive strength with time along with the experimentally measured results (Mitani et al. 2011) for the sealed samples. A fair agreement can be observed, indicating the reliability of the hydration model used in the multi-scale analytical framework (Maekawa et al. 2008). The compressive strength for the dry specimen is nearly 10% lower than that of the sealed specimen. This difference is due to micro-cracks at the surface of the specimen induced by drying. The evolution of shrinkage of the specimen is also indicated on Fig. 11. In the multi-scale analytical approach, the driving forces for shrinkage are quantified by considering the moisture state in the micro-nano size pores of concrete (Luan and Ishida 2013). Thus, the computed values include both drying and autogenous shrinkage without the need for separate treatments. This makes the multi-scale analytical approach ideal and highly suitable for understanding the structural performance of RC members subjected to coupled environmental and mechanical loads. It can be seen that, the computed shrinkage values both for the sealed and dry cases underestimate the measured values. This large value of drying shrinkage in the experiments of the sealed and dry cases can possibly be explained by the aggregate shrinkage. The drying shrinkage of aggregate is likely to be dependent on moisture states in the aggregate pores based on mechanisms such as capillary pressure and also on the increase in solid surface energy as well as in the concrete (Goto and Fujiwara 1979; Imamoto et al. 2006). Though no measurements are
provided for the sole aggregate shrinkage, use of an ultimate aggregate shrinkage strain of 500 micron gives consistent result by this hygral-mechanics simulation (Asamoto et al. 2008) with the experiments. The results with and without the consideration of aggregate shrinkage are indicated in Fig. 11a.

The evolutions of shrinkage indicated in Fig. 11a, are measured at the central core part of the specimen. To examine the spatial variation of shrinkage with in the specimen, Fig. 12 shows the comparison of shrinkage evolution measurement at the core and surface parts of the specimen by the analysis. The result indicates that, the difference is very small. This is attributed to the effect of relaxation brought by the dispersed micro-cracking at the surface of the specimen. The cracks after a drying period of 100 days for a quarter model are shown in Fig. 12a for illustration purpose. The cracks concentrate at the surface part of the specimen and their depth extends up to 15mm. The variation of the local tensile strength in the specimen is also indicated in Fig. 12b, with the outer most layer having a 35% increment in strength as a result of 100 days drying at a relative humidity of 46%. Note that, the evolution of the shrinkage in the specimen is the resultant of these combined factors.

3.4 Shear response

The high-cycle time-dependent model (Maekawa et al. 2006), is adopted for tension. For compression, the time-dependent elasto-plastic and fracture model (El-kashif and Maekawa 2004), and for shear, the contact density based model (Li and Maekawa 1989) is used. Three sets of RC beams are modeled and analyzed with effective depths of 250mm, 500mm and 1000mm (Mitani et al. 2011). Each set consists of one beam kept under sealed condition and its companion under drying condition. The dry case beams with effective depths of 250mm, 500mm and 1000mm are subjected to 125, 288, and 288 days of drying, respectively. The full FE discretization for each of the beams is indicated in Fig. 9. The dry case beams were subjected to a drying environment of relative humidity 46.1% and ambient temperature 15.2 °C, after seven days of wet curing. In the analysis of the beams a maximum of up to 35% increase in local tensile strength is observed at the surface of the beams due to drying.

First, the results of the beams with effective depth of 250mm are discussed. To illustrate the condition of the beam prior to shear loading, Fig. 13 shows the distribution cracks induced by drying. The depth of these cracks range from 7 to 12 cm, where the higher values are observed on the bottom side of the beam owing to the higher restraint provided by the reinforcing bars. Similar results are also observed for the beams with effective depths of 500mm and 1000mm. This is because the same amount of reinforcement ratio is provided for the beams.

Figure 14a shows the comparison of the load versus deflection curves for the sealed beam with effective depth of 250mm based on the coupled hygro-mechanical analysis and experiment. For reference, the shear capacity based on the JSCE design formula is in-
A fair agreement can be seen between the experiment and analysis in terms of stiffness, initial cracking load, maximum capacity, mode of failure, and post-peak response. Both results are also consistent with the shear capacity prediction based on the JSCE code. Figure 14b shows the corresponding results for the companion beam subjected to drying. Both the analytical and experimental results have a fair agreement in terms of capacity and post-peak softening response. However, the initial cracking load and stiffness are lower in the analysis. In the experiment, the stiffness for the sealed and dry cases are overlapping despite the significant amount of rebar strain (265 μ experimentally measured). Due to the self-equilibrated stresses caused by drying, a lower initial cracking load is expected. As a matter of fact, experimentally it is reported that the beam is cracked (through) prior to shear loading which is the case observed computationally as well. Thus, it appears that the response for the dry case is dubious to make a one to one comparative discussion with the analytically obtained result.

Figure 15 shows the principal strain contour plots for the sealed and dry beams before and after the formation of diagonal cracking. From the simulation result, it can be observed that the flexural cracks in the dry case are closely spaced and distributed over a wider range of the shear span as compared to those of the sealed case. The location of the diagonal crack for the dry beam is also closer to the loading points. Apart from the above mentioned differences, the effect of drying seems less evident for this case, effective depth of 250mm.

The less sensitivity of the shear capacity to drying for the beams with effective depth of 250mm is consistent with the current understanding of drying effect on ultimate states. Though the local behaviors are altered due to drying which are manifested in the conditions of cracking, the ultimate shear capacity still remains the same.

Figure 16 shows the load versus deflection curve for the beam with the effective depth of 500mm. The computed shear capacity is consistent with both the experiment and the JSCE code prediction. The initial stiffness of the dry specimen, which was analytically and experimentally cracked just before loading, is reasonably low and the load-deflection curve of both analysis and experiment is nearly linear, because the cracking has been already induced. The reasonably acceptable agreement in terms of stiffness, maximum capacity and mode of failure can be seen between the analysis and experiment as indicated in Fig. 16b. On the contrary, the initial stiffness of the sealed case is smaller than the prediction by the linear elastic analysis indicated as a linear line in Fig. 16a. Furthermore, a clear change of the tangential stiffness, which has been normally observed in virgin specimens before and after flexural cracking, cannot be seen in the experiment. In the analysis, clear change of flexural tangential stiffness is predicted as usual. This point should be discussed in view of both experiment and analysis in future although the crack pattern and the ultimate capacity of the sealed specimen are fairly predicted.

As the softening branch of the load-deflection curve is known to be dependent on the rigidity of loading devices, it is rational to dynamically analyze the loading system together with the target specimen in consideration of rate-effect and inertia. But, in this study, the machine rigidity is not known. Then, the authors conducted the nodal displacement control in the analysis, which corresponds to an infinitely rigid frame of loading machines. In this case, the computed softening of the load-displacement relation tends to be mild compared to the case of strict coupled system.
It is interesting to note that the diagonal cracking for the dry beam is closer to the loading points as shown in Figs. 16c and 16d. Furthermore, cracking just above the supports, which propagates from the top of the beam, can be observed. As a matter of fact, this type of cracking has been seen in experiment conducted in laboratories. This is due to the change of the diagonal compression flow. As the shrinkage introduces tensile stresses around the main reinforcing bars, cracking may occur comparatively easily accompanying less stiffness. On the other hand, the macroscopic stiffness reduction due to extended flexural cracks and occurrence of several cracks on the concrete surrounding the reinforcement may also lead to arch action, altering the stress path trajectory. This is observed after the formation of the diagonal cracks near to the loading points. The slight increment in load resistance after the diagonal crack is a possible manifestation for this phenomenon.

Figure 17 shows the load versus deflection curve for the large beam with the effective depth of 500mm. Except for the slight deviation in initial stiffness, the computed results based on the multi-scale analysis can reasonably reproduce the experimentally observed results. For reference, the initial stiffness based on a simple elastic analysis is indicated in Fig. 17a. For the dry case, reasonably acceptable agreement in terms of stiffness, first cracking load, maximum capacity, mode of failure, and post peak response can be seen between the analysis and experiment as indicated in Fig. 17b. Interestingly, the effect of drying is remarkable in terms of shear capacity as well. A reduction of nearly 18% in shear capacity is observed. The above result signifies size dependency of drying effect. In other words, the effect of drying gets more pronounced in beams with larger size.
Figures 16c and 17d show the principal strain plot for the beam with the effective depth of 1000mm, sealed and dry cases, respectively. Similar to the case of the beams with effective depth with 250mm and 500mm, the flexural cracks are more widely distributed over the shear span as compared to that of the sealed case. Besides, the formation of diagonal cracking is closer to the loading points.

The combined experimental and analytical observations indicate that, drying can have a remarkable impact on the stiffness, shear capacity and cracking pattern of RC beams. The impact of drying on the shear performance of arises from several factors having negative or positive influences. First, drying causes non-uniform shrinkage in side RC members. This will induce a self-equilibrated tensile stress on the surface concrete including the web regions thereby leading to a decrease in shear capacity with obvious flexural stiffness degradation. Second, the reduced flexural stiffness of RC beams caused by early cracking of concrete due to drying will impart higher strain field on the web region of the beam and tends to reduce the shear capacity. Third, the loss of moisture from the micro-pores of cement matrix entails increment in the local tensile strength. Unlike the abovementioned two factors, this mechanism tends to increase the shear capacity. Fourth, the micro-cracks and self-equilibrated tensile stresses in the concrete due to drying enable deeper penetration of the flexural cracks into the compression zone and deterioration of bond between concrete and reinforcing bar. These will alter the stress transfer mechanism in the beams from beam action to arch action, thereby tending to increase the shear capacity.

In general, the shear capacity of RC beams subjected to coupled hygral and mechanical loads involve several mutually interlinked factors having positive or negative roles coexisting together. Therefore, with the existing behavioral understanding, it is hardly possible to exclusively conclude that drying reduces the shear capacity of RC beams. The plus and minus factors for shear capacity of beams exist and both of them depend on detailed shape and dimension of members. For the case of beams discussed in this paper, the minus factor is found to be a little bit dominant. But, the authors consider that the plus factor could be dominant for different ambient boundary conditions and detailing of members. This is the reason why the authors do not use the term “capacity degradation” for structural concrete at this moment. There is a room for further investigation in future.

As a matter of fact, if drying in RC beams can be applied in a very gradual and extremely slow process without incurring an abrupt hygral gradient, the generality of the current observations will encounter severe challenge. This is because, in such ideal case the local tensile increment factor becomes predominant. Similar kind of investigations are valuable for improved understanding of drying. In the current paper the effect of drying is investigated by varying the depth of beams at constant width, however the effects of varying the widths is important as well. Drying effect is expected to diminish with the increase of width of beams. In general, several factors associated with the impact of drying under different ambient boundary conditions and detailing of members still remain unexplored and the multi-scale thermo-hygro mechanics based analytical platform is found to be a promising tool to tackle with this future challenge.

4. Conclusions

The effect of drying on reinforced concrete at material and structural levels is discussed based on the multi-scale analysis that provides an integration of the thermo-hygro and chemo-physics models for cementitious composites and the mechanical models of reinforced concrete. The simulation results are compared with experiments and the following conclusions are drawn from the study:

1. Apart from causing non-uniform shrinkage in concrete drying alters the mechanical properties of concrete as well. Thus, to replicate the structural response of RC beams, consideration of local mechanical properties is indispensable depending on the hygral states. This hygral-state dependent constitutive models are confirmed to play a key role for addressing the drying problems of shear critical beams.

2. Drying of concrete is always associated with surface cracking, and the effects of these cracks on the rate of moisture loss, progress of shrinkage and local tensile strength are found to be substantial. A simple linear crack-deformation dependent model is found to give consistency with experiments for practice.

3. At structural levels, drying is found to influence the stiffness and the shear capacity of RC beams with the effect getting pronounced for large size beams.

4. It is analytically observed that drying affects the distribution of flexural cracks and the pattern of shear cracks in RC beams as well as the capacity.

5. The multi-scale thermo-hygro and mechanics-based analytical platform is proved to be suited for assessing the performance of RC subjected to coupled mechanical loading and drying ambient actions.

6. Fundamental and well controlled experimental studies on drying at structural levels are necessary in the future for various detailing of members at different ambient boundary conditions.

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